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Systematic Model Generation for Shear Stress using Elementary Mathematical Equation

Yatharth Joshi, Yuvraj Joshi, Gagan Bansal, Jasmeet Kalra

***Abstract:** In engineering mechanics we deal with the forces, tensions, deflections, stresses, etc. acting on a system in various directions. There are basically of two types of stresses i.e. normal stresses and shear stresses. Normal stresses further can be compressive or tensile. Also from these elementary stresses we derive other stresses such as bending, torsion, their combinations etc. In the current research model of shear stress we will develop that shear stresses are actually nothing but the combination of normal stresses acting on different directions and generations. We will look on various 'generations' of stresses acting in different directions due to shear forces. As through algebraic mathematics, any function can be represented by any polynomial equation which may be in the form of any finite or infinite series, similarly in this shear stress model we will further derive an equation in which shear stress can be represented as an infinite series of compressive and tensile stresses.*

***Keywords:** Normal Stress, Direct Stress, Shear Stress, Strength of Material, Engineering Mechanics, Mechanical*

I. INTRODUCTION

Basically we say that if the load is parallel to the cross-section of any member is shear force and that causes shear stress into the member [1]. Till now we used to study shear stress as shown below:

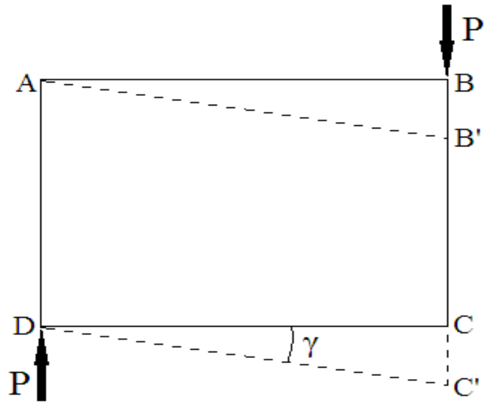

Figure 1: Representation of shear stress in classical model.

In this model shear stress,

$$\boxed{\tau = \gamma G} \quad \ldots (1)$$

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Where,

$$\tau \leftrightarrow shear\ stress$$
$$\gamma \leftrightarrow angle\ of\ deflection$$
$$G \leftrightarrow Bulk\ Modulus$$

Here ABCD is a member upon which shear force P is acting. ABCD is its initial position without any force acting and after P is acting, some deflection takes place and the position changes to AB'C'D.

On further investigating this classical model we found that there are some other stresses involved. Let us see following diagrams:

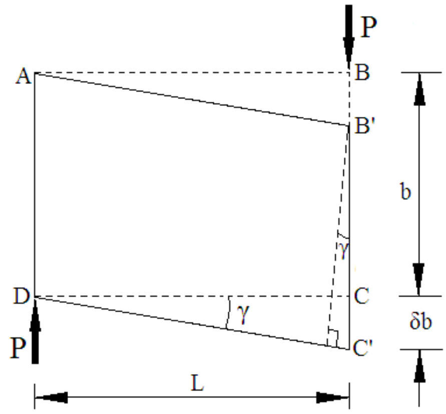

Figure 2a Dimensions of the elements under shear

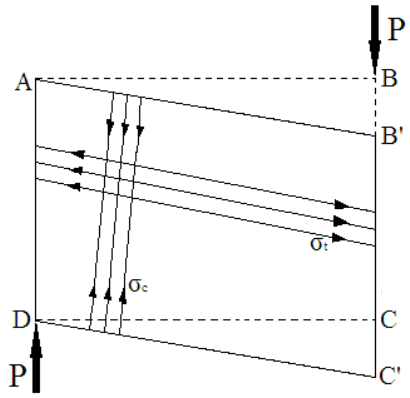

Figure 2b Stresses acting on the on the element under shear

From fig 2a,

$$AB = DC = l$$
$$AD = BC = B'C' = b$$

From geometry,

$$AB' = DC' = \frac{l}{\cos\gamma}$$

And,

$$BB' = CC' = \delta b$$

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By fig 2b,

$\sigma_t = Tensile\ stress$

$\sigma_c = Compressive\ stress$

And σ_t and σ_c are perpendicular in direction.

The stresses σ_t and σ_c arises due to the tension and compression. If we look at AB' and DC' then, AB'>AB and DC'>DC and the fibers of material of this member parallel to AB' will experience pure tensile stress.

Similarly B'E<B'C' or B'E<BC, i.e. the fibers of material of this member parallel to B'E will experience pure compressive stress.

Now as we know that,

$\sigma \propto \epsilon$

i.e. stress is directly proportional to strain or,

$$\boxed{\sigma = E\epsilon} \quad \dots (2)$$

$$\boxed{\sigma_t = E\epsilon_t} \quad \text{or} \quad \boxed{\sigma_c = E\epsilon_c} \quad \dots (3)$$

Where,

$\epsilon_t = Tensile\ strain$

$\epsilon_c = Compressive\ strain$

Now there is a big contradiction in my head that is Young's modulus is same or not for compression an tension. We know that there is term *strength differential* [2], which denotes the difference in the compressive about Young's modulus can be determined only by experiments.

Now,

$$\epsilon_t = \frac{AB' - AB}{AB}$$

$$\epsilon_t = \frac{\frac{l}{cos\gamma} - l}{l}$$

$$\epsilon_t = \frac{1}{cos\gamma} - 1$$

$$\boxed{\epsilon_t = \frac{1 - cos\gamma}{cos\gamma}} \quad \dots (4a)$$

And,

$$\epsilon_c = \frac{B'C' - B'E}{B'C'} = \frac{BC - B'E}{BC}$$

$$\epsilon_c = \frac{b - bcos\gamma}{b}$$

$$\boxed{\epsilon_c = 1 - cos\gamma} \quad \dots (4b)$$

Let there be the two cross-section areas, one perpendicular to σ_t and one perpendicular to σ_c as shown below:

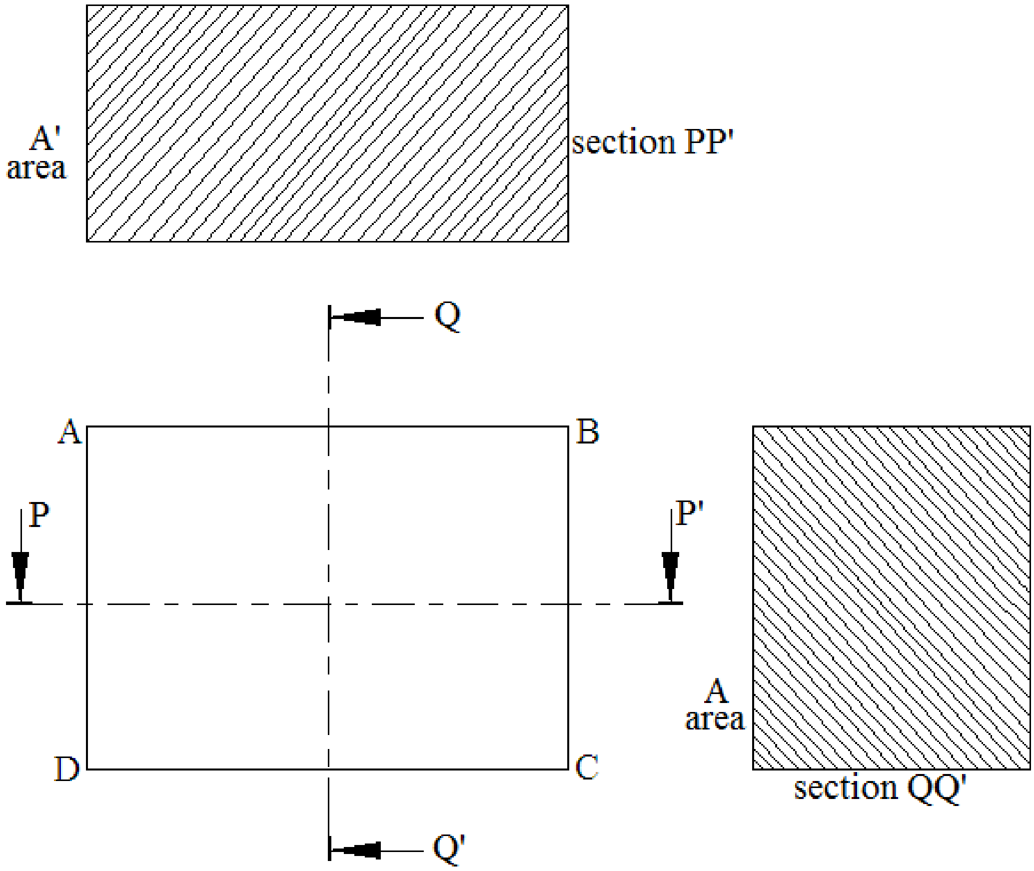

Figure 3: Representation of two different cross-sections of the member.

So, from equation 3& 4 we can write:

$$\boxed{\sigma_t = \left(\frac{1 - cos\gamma}{cos\gamma}\right)E} \quad \dots (5a)$$

$$\boxed{\sigma_c = (1 - cos\gamma)E} \quad \dots (5b)$$

So the internal forces that arise due to these stresses are F_t and F_c.

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Where, F_t = Internal force due to tension
F_c = Internal force due to compression

But as shown in figure 3 and the direction of σ_t & σ_c in figure 2b, the area perpendicular to σ_t is A and area perpendicular to σ_c is A'.

So F_t & F_c are:

$$F_t = \left(\frac{1-\cos\gamma}{\cos\gamma}\right) EA \quad \dots (6a)$$

$$F_c = (1-\cos\gamma)EA' \quad \dots (6b)$$

This means that the internal forces F_t and F_c are compressive and tensile caused by the shear force P. So it can be also said that the resultant of F_t and F_c is equal and opposite to the external shear force P. This is due to the material of member would resist the external load.

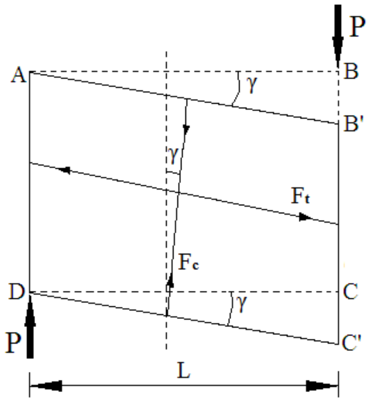

Figure 4a. Representation of F_t & F_c induced on the member.

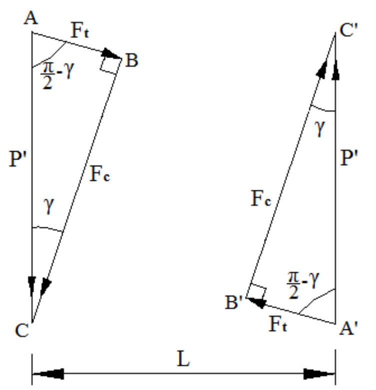

Figure 4b. Force triangle of F_t & F_c and their resultant P' equal and opposite to P.

From the force triangle shown in fig 4b, by the properties of triangles we can write,

$$P' = \sqrt{F_t^2 + F_c^2} \quad \dots (7a)$$

Since the horizontal component of the resultant will be zero as clearly shown in fig 4b

$$P' = F_t\sin\gamma + F_c\cos\gamma \quad \dots (7b)$$

These equations can be further resolved as:

$$P' = \sqrt{\left(\frac{1-\cos\gamma}{\cos\gamma}\right)^2 E^2A^2 + (1-\cos\gamma)^2E^2A'^2}$$

$$P' = \left(\frac{1-\cos\gamma}{\cos\gamma}\right) E\sqrt{A^2 + \cos^2\gamma A'^2} \quad \dots (8)$$

And from eq (7b), we get,

$$P' = F_t\sin\gamma + F_c\cos\gamma$$

$$P' = \left(\frac{1-\cos\gamma}{\cos\gamma}\right) EA\sin\gamma + (1-\cos\gamma)EA'\cos\gamma$$

$$P' = \left(\frac{1-\cos\gamma}{\cos\gamma}\right) E[A\sin\gamma + A'\cos^2\gamma] \quad \dots (9)$$

As we know that shear stress,

$$\tau = \frac{P}{A} \quad \dots (10)$$

Now dividing equation (8) by A, we will get the value of τ as in eq (10).

$$\tau = \frac{P}{A}$$

$$\tau = \frac{1}{A}\left(\frac{1-\cos\gamma}{\cos\gamma}\right) E\sqrt{A^2 + \cos^2\gamma A'^2}$$

$$\tau = \left(\frac{1-\cos\gamma}{\cos\gamma}\right).E.\sqrt{1 + \cos^2\gamma\left(\frac{A'}{A}\right)^2} \quad \dots (11)$$

By dividing equation (9) by A, we will get,

$$\tau = \frac{P}{A}$$

$$\tau = \frac{1}{A}\left(\frac{1-\cos\gamma}{\cos\gamma}\right) E[A\sin\gamma + A'\cos^2\gamma]$$

$$\tau = \left(\frac{1-\cos\gamma}{\cos\gamma}\right) E\left[\sin\gamma + \frac{A'}{A}\cos^2\gamma\right] \quad \dots (12)$$

So the equation (11) & (12) shows a tremendous relation between the shear stress and the normal stress according to the classical shear model.

II. SHEAR STRESS MODEL- YJ'S MODEL:

We saw the classical model and realized that the shear stress was actually a combined result of normal stresses. But in this we are trying to show how this combined result of normal stresses actually [3]. Let us see how this model looks like:

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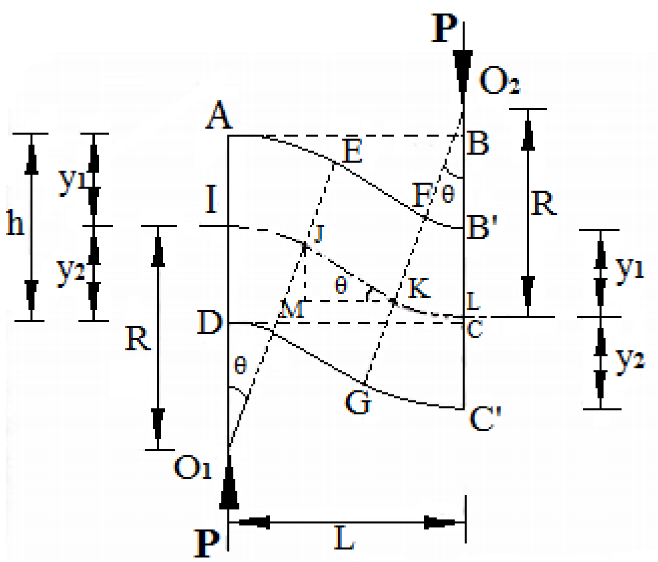

Figure 5- Representation o9f YJ's Shear Stress Model

In this model we think some bending also takes place at the ends. In figure 5, AEHD an FB'C'G represents that bending region. And in that bending IJ and KL represents the neutral axis and as we know that at neutral axis stress is zero in magnitude. In AEJI and KLC'G tensile stresses are distributed linearly. And in DHJI and FB'LK compressive stresses are distributed linearly, from zero at neutral axis to maximum at the end. Also,

$\theta \leftrightarrow Angle\ of\ bending\ and\ angle\ of\ deflection = \angle AO_1E = \angle C'O_2C$

$R \leftrightarrow Radius\ of\ bending = O_1I = O_1J = O_2L = O_2K$

$L \leftrightarrow Horizontal\ length = AB = CD$

$h \leftrightarrow Vertical\ length = AD = BC = B^{'}C$

$y_1 \leftrightarrow Vertical\ distance\ from\ NA\ to\ upper\ surface$

$y_2 \leftrightarrow Vertical\ distance\ from\ NA\ to\ lower\ surface$

y_1 and y_2 are not necessarily equal i.e. they may or may not be equal, it depends upon geometry such as triangular cross-section, T-shaped cross-section, etc. Now, every dimension which is shown in figure 6 are constant except R and θ somehow depends upon the load P and some other properties of material like Young's modulus, area moment of inertia, etc. Now let us investigate the relation between R & θ.

2.1 Radius and angle of Bending:

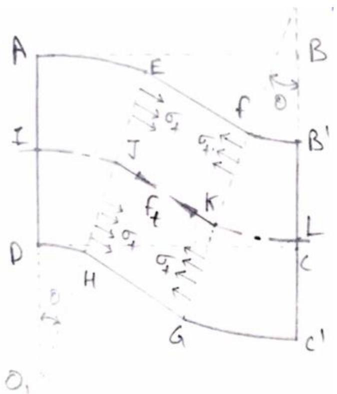

Figure 6a. Representation of tensile force & tensile stress acting in region EFGH.

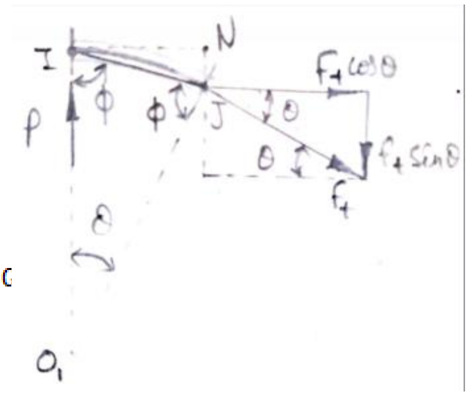

Figure 6b Forces acting for bending region.

As we can see figure 6b represents that is bending region two forces are acting viz. P which is external load and F_t which is internal tensile force. This is clear that this is like a cantilever beam [4]. The bending is also caused by the internal forces as well, so the moment acting at J would be internal moment,

$$\boxed{M_1 = (\overline{IN}).F_t.sin\theta} \quad \text{… (13)(a-)}$$

But, the total moment acting would be external moment,

$$\boxed{M_2 = P.l} \quad \text{… (13)(b-)}$$

From the geometry, length of arc IJ is greater than length of line IJ, so in ΔO_1JI, since $\overline{IO_1} = \overline{JO_1} = R$

$\theta + \phi + \phi = \pi$

$$\boxed{\phi = \frac{\pi}{2} - \frac{\theta}{2}} \quad \text{… (14a)}$$

$\phi = \angle O_1IJ = \angle O_1JI$, because ΔO_1IJ is isosceles.

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By law of triangle,

$$\frac{IJ}{sin\theta} = \frac{R}{sin\phi}$$

$$\overline{IJ} = \frac{Rsin\theta}{sin\phi} = \frac{Rsin\theta}{\sin\left(\frac{\pi}{2}-\frac{\theta}{2}\right)} = \frac{Rsin\theta}{\cos\left(\frac{\theta}{2}\right)}$$

$$\overline{IJ} = \frac{R\left[2.sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right]}{\cos\left(\frac{\theta}{2}\right)}$$

$$\boxed{\overline{IJ} = 2.R.sin\left(\frac{\theta}{2}\right)} \quad \text{… (14b)}$$

Since, arc

$\widehat{IJ} = R.\theta$

So from above equations,

$\widehat{IJ} \neq \overline{IJ}$

Now in ΔO_1IJ and ΔIJN

$$\angle O_1IN = \frac{\pi}{2}$$

$$\angle JIN = \frac{\pi}{2} - \phi = \frac{\pi}{2} - \frac{\pi}{2} + \frac{\theta}{2} = \frac{\theta}{2}$$

Now,

$$\overline{IN} = \overline{IJ}\cos(\angle JIN)$$

$$\overline{IN} = \left(2R.sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}\right)$$

$$\boxed{\overline{IN} = R.sin\theta} \quad \text{… (14c)}$$

Similarly,

$$\overline{JN} = \overline{IJ}\sin(\angle JIN)$$

$$\overline{JN} = \left(2R.sin\frac{\theta}{2}\right)\left(\sin\frac{\theta}{2}\right)$$

$$\boxed{\overline{JN} = R.(1-cos\theta)} \quad \text{… (14d)}$$

So from equation (13a) & (14c)

$$M_1 = (\overline{IN}).F_t.sin\theta$$

$$M_1 = (R.sin\theta)(F_t sin\theta)$$

$$\boxed{M_1 = F_t R.sin^2\theta} \quad \text{… (15a)}$$

Now from equation (14a), F_t is internal force as so we need to find σ_t first, and

$$\boxed{F_t = \sigma_t A} \quad \text{… (15b)}$$

Where, $A \leftrightarrow area \perp to\ \sigma_t$

$$\boxed{\sigma_t = E\epsilon_t} \quad \text{… (15c)}$$

Where, $E \leftrightarrow Young's\ modulus$

And, $\epsilon_t \leftrightarrow tensile\ strain\ at\ region\ EFGH$

$$\epsilon_t = \frac{\overline{JK} - \overline{MK}}{\overline{MK}} = \frac{\overline{JK} - \overline{JK}cos\theta}{\overline{JK}cos\theta}$$

$$\boxed{\epsilon_t = \frac{1-cos\theta}{cos\theta}} \quad \text{… (15d)}$$

By combining equations (15b), (c) & (d)

$$\boxed{F_t = EA\left(\frac{1-cos\theta}{cos\theta}\right)} \quad \text{… (15e)}$$

From equation (15a)

$$\boxed{M_1 = EAR\left(\frac{1-cos\theta}{cos\theta}\right)\sin^2\theta} \quad \text{… (15f)}$$

Now from the deflection equation,

$$EI\frac{d^2y}{dx^2} = M_1 \quad \text{… (16a)}$$

$$EI\frac{dy}{dx} = M_1x + C_1 \quad \text{… (16b)}$$

$$EIy = \frac{M_1x^2}{2} + C_1x + C_2 \quad \text{… (16c)}$$

Applying boundary conditions i.e.

At, $x = 0, y = 0$

From equation (16c), $C_2 = 0$

At, $x = 0, \frac{dy}{dx} = 0$

From equation (16b), $C_1 = 0$

$$\Rightarrow \quad \boxed{EIy = \frac{M_1x^2}{2}} \quad \text{… (16c)}$$

And here, $= \overline{IN} = Rsin\theta$, $y = \overline{JN} = R(1-cos\theta)$

$$EIR(1-cos\theta) = \frac{M_1R^2sin^2\theta}{2}$$

$$EI(1-cos\theta) = \frac{M_1Rsin^2\theta}{2}$$

$$EI(1-cos\theta) = \left[EAR\left(\frac{1-cos\theta}{cos\theta}\right)\sin^2\theta\right]\left(\frac{R^2sin^2\theta}{2}\right)$$

$$\frac{2Icos\theta}{A} = R^2sin^4\theta$$

$$R^2 = \frac{2I\ cos\theta}{A\ sin\theta}$$

Or,

$$\boxed{R = \frac{1}{sin^2\theta}\sqrt{2I\frac{cos\theta}{A}}} \quad \text{… (17)(a-)}$$

This equation shows that R and θ are inter dependent variables. Let us find upon what they both depends:

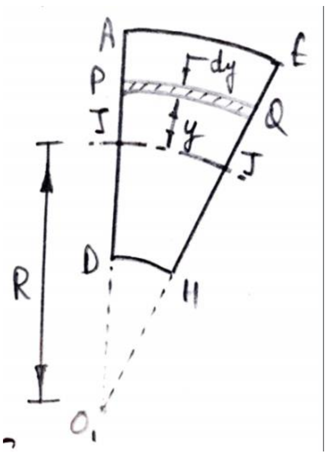

Figure 7- Bending region.

As shown in fig let us take an element PQ, which is y distance from neutral axis and has dy thickness. So strain at PQ

$$\epsilon = \frac{\overline{PQ} - \overline{IJ}}{\overline{IJ}} = \frac{[(R+y)\theta - R\theta]}{R\theta}$$

$$\boxed{\epsilon = \frac{y}{R}} \quad \text{… (17)(b-)}$$

So stress at PQ, $\sigma = El$

$$\boxed{\sigma = \frac{Ey}{R}} \quad \text{… (17)(c-)}$$

Also for bending stress can be determined by,

$\sigma = \frac{M_2 y}{I}$ $\quad$ [*Since, both moments* M_1 & M_2 *are acting and* M_2 *is internal*]

From equation (13)(b-),

$$\boxed{\sigma = \frac{Ply}{I}} \quad \text{… (17d)}$$

So from equation (17c & d),

$$\frac{Ey}{R} = \frac{Ply}{\cdot}$$

…(17b)

$$\boxed{R = \frac{EI}{Pl}} \quad \text{… (17e)}$$

For θ combining all equations,

…(17c)

$$\boxed{\frac{\sin^4\theta}{\cos\theta} = \frac{2}{AI}\left(\frac{Pl}{E}\right)} \quad \text{… (17f)}$$

2.2 Forces due to Bending:

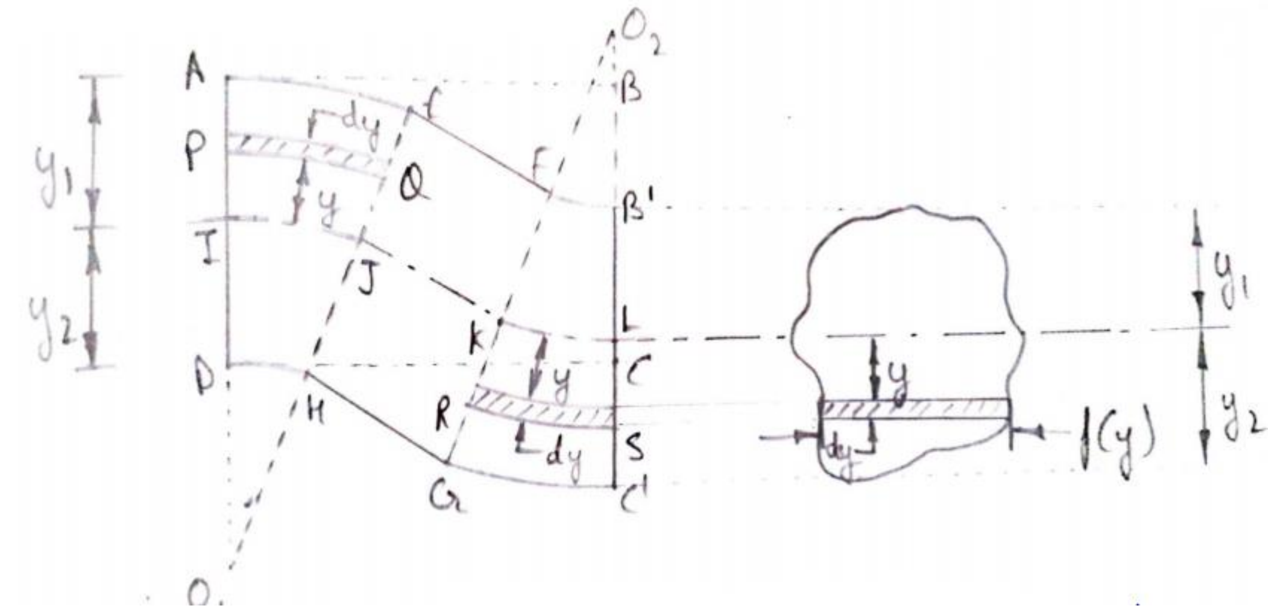

Figure 8a. Front View

Fig shows the front and side views, fig shows that the cross-section can be of any geometry and at y distance from neutral axis, the width of the cross-section would be $f(y)$ which is the *width function* of that geometry [5]. It is different for different geometries and depends only upon the geometry, for example for rectangle $f(y)$ will be constant, for circle $f(y) = \sqrt{R^2 - y^2}$, for triangle $f(y) = \left(\frac{2h}{3} - y\right)\frac{b}{h}$ or $(y) = \left(\frac{2h}{3} + y\right)\frac{b}{h}$, etc.

Now, let $\sigma_1 = \textit{stress at tension}$

$\sigma_2 = \textit{stress at compression}$

$$=> \boxed{\sigma_1 = \frac{Ply}{I}} \quad \text{… (18a)}$$

And $\boxed{\sigma_2 = \frac{Ply}{I}}$

i.e. $\boxed{df_1 = \sigma_1 dA}$ … (18b)

& $\boxed{df_2 = \sigma_2 dA}$

Where, df_1 = elementary force at area of dA (tensile)

df_2 = elementary force at area of dA (compressive)

also,

$$\boxed{dA = f(y)dy} \quad \text{… (18c)}$$

for upper part, from eq 22, a, b & c

$$F_{1u} = \frac{Pl}{I}\int_0^{y_1} f(y).y.dy \quad \& \quad F_{2u} = \frac{Pl}{I}\int_0^{y_1} f(y).y.dy$$

And for lower part,

$$F_{1L} = \frac{Pl}{I}\int_0^{y_2} f(y).y.dy \quad \& \quad F_{2L} = \frac{-Pl}{I}\int_0^{y_2} f(y).y.dy$$

Now due to function $f(y)$ & the limits y_1 & y_2 , it's a fact that $F_{1u} = F_{1L}$ & $F_{2u} = F_{2L}$, so tensile force,

$$\boxed{F_1 = \frac{Pl}{I}\int_0^{\bar{y}} f(y).y.dy} \quad \text{… (18d)}$$

And the compressive force,

$$\boxed{F_2 = \frac{Pl}{I}\int_0^{\bar{y}} f(y).y.dy} \quad \text{… (18e)}$$

Where, $\bar{y} \leftrightarrow \textit{centroidal distance}$

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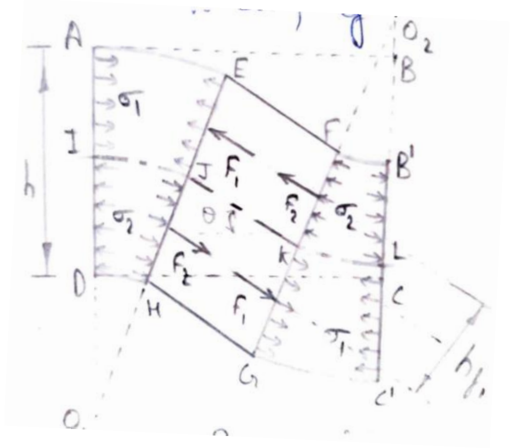

Figure 9a. Representation of forces & stress.

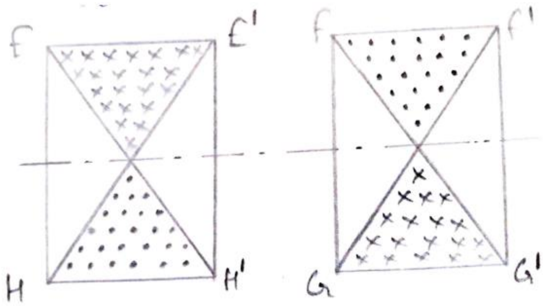

Figure 9b. Representation of stresses in side view w.r.t. neutral axis.

Now from the above figures we know the directions of each force which is θ from the horizontal. But we should also find out their positions as well. From figure 9b we can see that the stress varies with the distance from the neutral axis i.e. it is zero at neutral axis and maximum at the ends. As we know that the stress equation for bending.

$$\sigma = \frac{Ply}{I} \quad \text{or} \quad \sigma = ky$$

Where, $k = \frac{Pl}{I}$

& force $F = \int \sigma . dA$

$$F = k \int y . dA$$

$$F = k \int y . f(y) . dy$$

So function $(\int y . f(y) . dy)$ gives the force distribution function. As we find centroid of any geometry by:

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

Here,

$$A = \int y . F(y) . dy$$

So the distance between the resultant of F_1 & F_2 and these twin forces would be h_{f1} distance apart.

$$h_{f1} = \frac{\int_0^{\bar{y}} y^2 . f(y) . dy}{\int_0^{\bar{y}} y . f(y) . dy} \quad \text{... (19a)}$$

And the resultant of F_1 & F_2 will be F_{R1} and from figure

$$F_{R1} = F_1 + F_2$$

$$F_{R1} = \frac{2Pl}{I} \int_0^{\bar{y}} y . f(y) . dy \quad \text{... (19b)}$$

Now we got the magnitude, direction and the position of F_{R1}.

2.3 Forces at region without Bending:

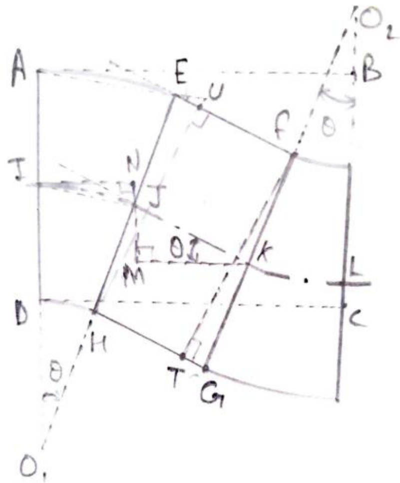

Figure 10a. Shear stress model with some perpendicular line segment FT, JM & IN.

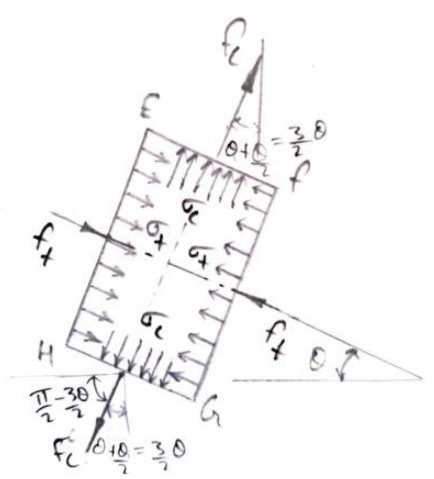

Figure 10b. Stress & Forces acting on region EFGH.

As we saw in the classical shear stress model that there were also normal stresses involved, similarly in region EFGH tensile stress σ_t and compressive stress σ_c are involved. So just like equations 4a & 4b the respective tensile and compressive strains (ϵ_t & ϵ_c) can be determined as [6]:

Compressive strain,

$$\epsilon_c = \frac{\overline{FG} - \overline{FT}}{\overline{FG}}$$

For this first we need to find $\angle GFT$, from equation (17a)

$$\angle O_1IJ = \phi = \frac{\pi}{2} - \frac{\theta}{2}$$

By further investigation we found that,

$$\Delta INJ = \Delta EHU = \Delta GFT$$

$$\angle NIJ = \angle GFT$$

$$\angle NIJ + \angle O_1IJ = \frac{\pi}{2}$$

$$\angle NIJ + \frac{\pi}{2} - \frac{\theta}{2} = \frac{\pi}{2}$$

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$$\angle NIJ = \frac{\theta}{2} \; \& \; \angle GFT = \frac{\theta}{2}$$

$\Rightarrow$

$$\epsilon_c = \frac{\overline{FG} - \overline{FG}\cos\angle GFT}{\overline{FG}} = \frac{\overline{FG} - \overline{FG}\cos\frac{\theta}{2}}{\overline{FG}} \quad \dots (20a)$$

$$\boxed{\epsilon_c = 1 - \cos\frac{\theta}{2}}$$

Now Tensile strain,

$$\epsilon_t = \frac{\overline{JK} - \overline{MK}}{\overline{MK}} = \frac{\overline{JK} - \overline{JK}\cos\theta}{\overline{JK}\cos\theta}$$

$$\boxed{\epsilon_t = \frac{1 - \cos\theta}{\cos\theta}} \quad \dots (20b)$$

Where θ can be determined by equation (20b)

$\Rightarrow$

$$F_t = E\left(\frac{1-\cos\theta}{\cos\theta}\right) A'$$

$$\boxed{F_t = E\left(\frac{1-\cos\theta}{\cos\theta}\right).\int_0^h f(y).dy} \quad \dots (21a)$$

&

$$F_c = E\left(1 - \cos\frac{\theta}{2}\right) A'$$

Where,

$$A' = (area \perp \sigma_c) = \overline{JK}.f(\bar{y})$$

$$\left(\begin{matrix} \because f(\bar{y}) = width\ at\ centroidal\ distance \\ \& \quad \overline{JK} = l - 2R\theta \end{matrix}\right)$$

$$A' = (l - 2R\theta).f(\bar{y})$$

$$\boxed{F_c = E\left(1 - \cos\frac{\theta}{2}\right).(l - 2R\theta).f(\bar{y})} \quad \dots (21b)$$

So equation (21a & b) gives the value of forces F_t & F_c , now we need to find the direction and its position. From above equations in most cases F_c would be rather smaller as compared to F_t . Also F_t makes θ angle with horizontal and F_c makes $\frac{\pi}{2} - \frac{3\theta}{2}$ with the horizontal.

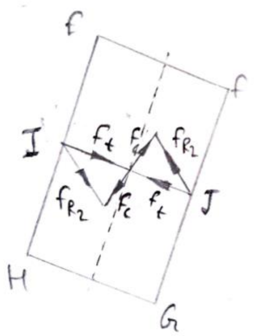

Figure 11a. Position of forces F_t & F_c .

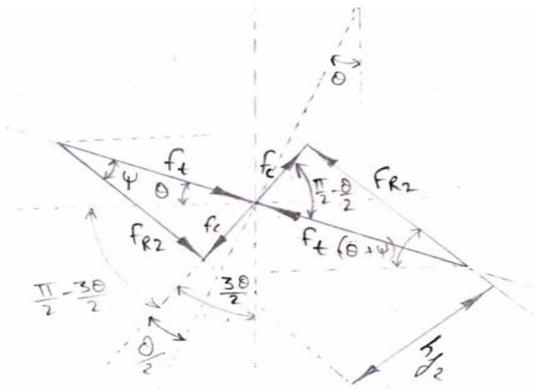

Figure 11b Detailed representation of F_t & F_c with their resultant F_{R2} .

Here,

$$\boxed{h_{f2} = (l - 2R\theta).\sin\psi}$$

Now the angle between F_t & F_c is $\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$. By applying parallelogram law,

$$F_{R2} = \sqrt{F_t^{\,2} + F_c^{\,2} - 2F_t.F_c.\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right)}$$

$$\boxed{F_{R2} = \sqrt{F_t^{\,2} + F_c^{\,2} - 2F_t.F_c.\sin\frac{\theta}{2}}} \quad \dots (22a)$$

For ψ which is the angle between F_{R1} & F_{R2} ,

$$\psi = \tan^{-1}\left[\frac{F_c\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)}{F_t - F_c\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right)}\right]$$

$$\boxed{\psi = \tan^{-1}\left[\frac{F_c.\cos\frac{\theta}{2}}{F_t - F_c.\sin\frac{\theta}{2}}\right]} \quad \dots (22a)$$

2.4 Generated Force:

Now from the previous equations we know the magnitude, direction and position of F_{R_1} & F_{R_2}. The resultant of their force would be a generated force F_g as shown below:

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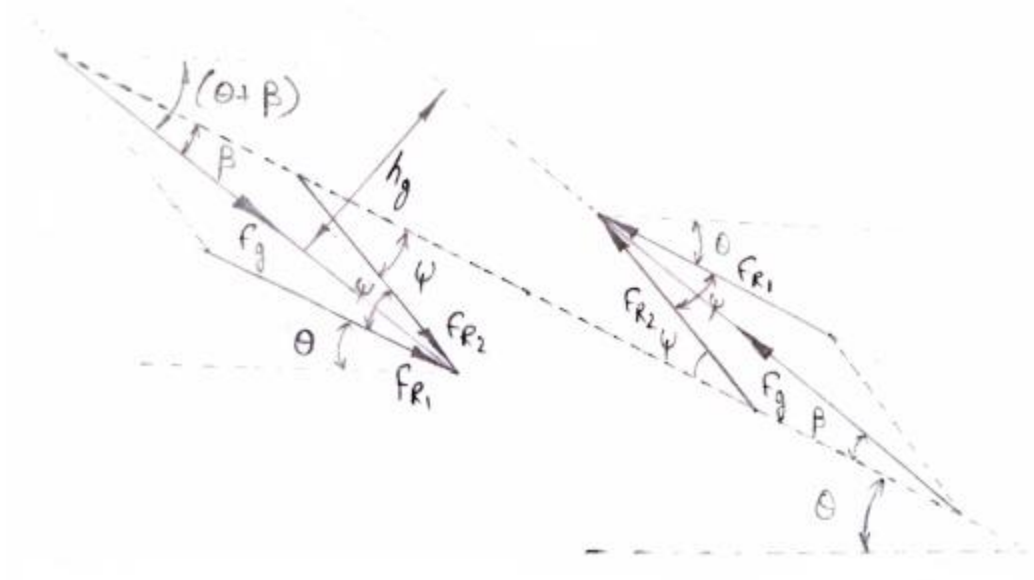

Figure 12-Representation of Generated force .

Since F_{R_1} was already inclined at θ angle from horizontal and F_{R_2} was inclined at $\theta + \psi$ angle, so the angle between F_{R_1} & F_{R_2} is ψ.

So the parallelogram law

$$F_g = \sqrt{F_{R_1}{}^2 + F_{R_2}{}^2 + 2.F_{R_1}.F_{R_2}.\cos\psi} \quad \ldots$$

(23a)

And the angle β between F_{R_1} and F_g would be:

$$\beta = \tan^{-1}\left[(F_{R_2}.\sin\psi)/F_{R_1} + F_{R_2}.\cos\psi\right] \quad \ldots$$

(23b)

If h_g is the perpendicular distance between F_g & its twin then,

$$F_{R_1}.h_{f_1} + F_{R_2}.h_{f_2} = F_g.h_g$$

$$h_g = (F_{R_1}.h_{f_1} + F_{R_2}.h_{f_2})/F_g$$

$$or \quad h_g = \frac{F_{R_1}.h_{f_1} + F_{R_2}.h_{f_2}}{\sqrt{F_{R_1}{}^2 + F_{R_2}{}^2 + 2.F_{R_1}.F_{R_2}.\cos\psi}} \quad \ldots \textbf{(24a)}$$

Now the force F_g is inclined at $(\theta + \beta)$ angle from horizontal. Let $\theta + \beta = \omega$ **… (24b)**

Basically, F_g is a totally internal force which is caused by external shear force 'P'. Similarly, F_g will also cause some other internal force $F_g{}'$ at distance h_g' and inclined at ω' angle from horizontal. And as we found F_g, h_g and ω with respect to some quantities like P, L, E, I, f(y), θ, etc. Similarly we can find F_g', h_g' & ω'. And $F_g{}'$ will then cause some other force $F_g{}''$ and then we will get:

$$P = F_g \sin\omega + F_g' \sin\omega' + F_g'' \sin\omega'' \ldots\ldots\ldots\ldots \quad \textbf{… (25a)}$$

$$or \quad \tau = \frac{1}{A}[F_g \sin\omega + F_g' \sin\omega' + F_g'' \sin\omega'' + F_g''' \sin\omega''' \ldots\ldots\ldots\ldots\ldots] \quad \textbf{… (25b)}$$

All these equations can give us some important information about the stress distribution due to shear. The force acting or 'internal force' acting per unit area is what we call stress and could be found using those equations. We can also find the stress concentration in shear stress by this theory. As described in the last section that there is an infinite series of stresses, this is what I propose in the introduction part, to understand this let me give you an example of **Taylor's expansion** of any function in the introduction [7]. I think equation (25b) wouldn't look exactly like what I mentioned in the introduction, but it could give impressive results as I think. The part where I mentioned in this page about 'stress concentration' [8] was about to calculate the distance between various generations of forces, magnitude of the forces and forces $F_g, F_g{}', F_g{}'', F_g{}'''$ etc. These are not single acting [9], these are the generations of forces and their positions i.e. $h_g, h_g{}', h_g{}'', h_g{}'''$ etc. represents that they are concentrated at that point i.e. the stress is distributed over that region above & below it. It should be visualized clearly then only this theory could be understood properly [10].

III. CONCLUSION

The elaborative algebraic and derived equation for the shear stress is developed. The assumptions and the conditions used for the analysis and modeling are the general assumptions used normally. The efforts finally conclude that the developed model gives the equation similar to Taylor's expansion. All these equations can give us some important information about the stress distribution due to shear. This one equation finally leads to solve the large range of mathematical problems that falls under shear stress model.

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Further this model can be developed for uneven distribution of loads, force generations and complex geometry. The complete derivation is formulated using the understanding of classical geometry of the authors.

IV. ACKNOWLEDGEMENT

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