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Regular Filter, Associated Filter and Their Properties

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Abstract: ***The action $\sigma : L \times J \to J$ of a locale L on a join semi-lattice J, with bottom element 0_J, establishes the concept of L-slices (σ, J)which was introduced in [3]. Through the action , the join semilattice displays significant changes in their structure. Our study is based on the set of all elements of the locale which leaves $x \in (\sigma, J)$ fixed, under the action σ .This set is denoted as F_x . Also,$F_x = \{a \in L \,|\sigma(a, x) = x\}$is a filter on the locale L .In this study ,we illustrate the properties of the filter$F_x$of the locale generated by the L-slices . We put forth a study of two different types of filters, called the regular filter and the associated filter. The properties of these filters are studied. The collection of regular filters is separated into equivalence classes. Analogous to sequential continuity in topological spaces , we define the continuity of slice morphisms in terms of these filters. Also, the regular and associated filters on the locale L helps us characterise a new type of L-slices called the R-A slice.***

Index Terms: Locale, Filters, L-slices , R-A slice

I. INTRODUCTION

In algebra ,the action of a group Gon a set $X$is defined as a map $\varphi: G \times X \to X$ such that it is compatible with the group operation and it preserves the identity element of the group. This is the concept behind the definition of vector spaces and modules. The definition of vector spaces allows us to view vector spaces as the action of a field over an abelian group . Modules are the generalisation of vector spaces and the same theory is applied for the construction of modules also.Here the action is specified from a ring rather than from a field. This, is why modules become the generalisation of vector spaces. Locales are complete lattices which satisfy the infinite distributivity law .Thus the change of domain of action from algebra to lattices, more generally locales, motivated the idea of L-slices.The idea behind vector spaces and modules was to import some properties of field and rings to the abelian group, through the scalar multiplication. Analogously,in L-slices the properties of locale L is imparted to the join-semilattice J through the action $\sigma: L \times J \to J$ .The properties of action on the join semilattice J , the properties of L-slices (σ, J) and the morphisms between L-slices are studied in [3].

In this paper, we propose the study of filters that can be associated with an element $x \in (\sigma, J)$. In [3] ,to each $x \in (\sigma, J)$ a set $F_x = \{a \in L\,|\sigma(a, x) = x\}$ is defined and is shown to be a filter on L. This filter F_x forms the key point of our study.The paper is divided into seven sections. The II section is for the preliminaries on locales and L-slices. The main results are given from III to VI sections. In the second section ,a filter F in the locale L is characterised as regular filter and associated filter on the basis of its relation with F_x.Here ,we define regular filter and their properties are studied. Also, to each regular filter F we associate an element $x \in (\sigma, J)$ and call it as the knot point of the filter F. We study the properties of knot points.The third section deals with a detailed study on associated filters. The continuity of slice morphisms in terms of filters F_x is investigated. And the last section deals with the construction of a new L-slice based on regular and associated filters.

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II. PRELIMINARIES

Definition 2.1 [1] A join semilattice is a poset $(J, \leq)$ in which every finite subset has a join.

Example 2.2[1] The set of all natural numbers with the partial order less than or equal to .

Definition2.3[1] A lattice is a poset in which every finite subset has both a join and meet.

Definition 2.4[1] A subset I of a lattice J is said to be an ideal if

i) I is a sub –join semilattice of J ; i.e. 0 ∈ I , and a ∈ I ,b∈ I imply a ∨ b ∈ I ; and

ii)I is a lower set ;i.e. a ∈ I and b≤ a imply b∈ I.

Definition 2.5[1] A subset F of a lattice A is a set which satisfies the axioms dual to those defining an ideal ,is called filter of A.

Proposition2.6[1] Let I be an ideal of a lattice A .The following conditions are equivalent :

i) The complement of I in A is a filter
ii) 1 ∉ I , and (a ∧ b) ∈ I implies either a ∈ I or b ∈ I
iii) I is the kernel of a lattice homomorphism f: A ⟶ 2, where 2 denotes the two element lattice {0,1}.

Definition 2.7[1] An ideal satisfying the equivalent conditions of the above proposition is called a prime ideal ;its complement is called a prime filter.

Definition 2.8[2] A poset is a complete lattice if every subset has a join and meet.

Definition 2.9[2] A frame is a complete lattice L satisfying the distributivity law (⋁ A) ∧ b = ⋁ { a ∧ b | a ∈ A } , for any subset A ⊆ L and any b ∈ L.

Definition 2.10[2] Frame homomorphism h:L⟶ M between frames L and M are maps L⟶ M preserving all joins and all finite meets .

The resulting category will be denoted as **Frm.**

The dual category **Frm** ^{op} is called the category of locales ,and this category can be viewed as an extension of the category of sober spaces and hence the locales can be viewed as generalized spaces.

Examples 2.11[2]

i)The lattice of open sets of topological space.

ii)The Boolean algebra B of all open sets U of real line such that U= *int*(*cl*(U)).

Definition 2.12[1] In a locale L an element p is said to be meet irreducible if whenever $a \wedge b \leq p$ implies either $a \leq p$ or $b \leq p$.

Definition 2.13 L-slices [3]Let L be a locale with bottom element 0_L, top element 1_L and (σ, J)be a L - slice with bottom element 0_J. By the"action of L on J"we mean a function $\sigma: L \times J \rightarrow J$ such that the following conditions are satisfied 1.$\sigma(a, x_1 \vee x_2) = \sigma(a, x_1) \vee \sigma(a, x_2)$ for all $a \epsilon L$ and for all $x_1, x_2 \epsilon J$.

2. .$\sigma\left(a, 0_J\right) = 0_J$ for all $a \epsilon L$.

3. $\sigma(a \sqcap b, x) = \sigma(a, \sigma(b, x)) = \sigma(b, \sigma(a, x))$ for all ,$b \epsilon L$, $x \epsilon J$.

4. $\sigma(1_L, x) = x$ and $\sigma(0_L, x) = 0_J$ for all $x \epsilon J$.

5. $\sigma(a \sqcup b, x) = \sigma(a, x) \vee \sigma(b, x)$, for ,$b \epsilon L$, $x \epsilon J$.

If σ is an action of the locale L on a join semilattice J, then we call (σ, J) as L-slice.

Examples 2.14[3]

1.Let L be a locale and I be any ideal of L .Consider each $x \epsilon I$ and define $\sigma: L \times I \rightarrow I$ as $\sigma(a, x) = a \wedge x$, $a \in L$. It can be easily seen that (σ, I) is a L –slice.

2.Let L be a chain with top and bottom elements and J be any join semilattice with bottom element 0_J.Define $\sigma: L \times J \rightarrow J$ by $\sigma(a, j) = j$, for every $a \neq 0_L$ and $\sigma(0_L, j) = 0_J$. This is called a trivial L-slice.

3.Any locale L can be viewed as the meet L –slice$(\sqcap, L)$ where the action σ is defined as $\sigma(a, x) = a \sqcap x$

Proposition 2.15[3]Here we give some properties of the action $\sigma: L \times J \rightarrow J$.

1. For every $a \in L$ and $\in (\sigma, J)$, $\sigma(a, x) \leq x$

2. For $n, l \in (\sigma, J)$,if $l \leq n$,then $\sigma(a, l) \leq \sigma(a, n)$, for every $a \in L$.

3.If $a \leq b$, for $a, b \epsilon L$, then $\sigma(a, x) \leq \sigma(b, x)$ for every $x \in (\sigma, J)$.

Definition 2.16[3]Let (σ, J) be an L-slice.A subjoin semilattice $J'$of J is said to be L-subslice of J if $J'$is closed under action by eements of L.

Example2.17[3]Let (σ, J)be an L-slice and let $x \in (\sigma, J)$.Define $\langle x \rangle = \{\sigma(a, x): a \in L\}$.Then,$(\sigma, \langle x \rangle)$ is an L-subslice of $(\sigma, J)$and it is the smallest L-subslice of (σ, J) containing x.

Proposition2.18[3] The product $(\sigma, J_1 \times J_2)$ of two L-slices (σ_1, J_1) and (σ_2, J_2) of a locale L with action defined as $\sigma\left(a, (x, y)\right) = (\sigma_1(a, x), \sigma_2(a, y))$ is a L-slice.

Definition 2.19[3] Let (σ, J) and (μ, K) be L-slices. A map $f: (\sigma, J) \rightarrow (\mu, K)$ is said to be L-slice homomorphism if

i)$f(x_1 \vee x_2) = f(x_1) \vee f(x_2)$, for all $x_1, x_2 \in (\sigma, J)$.

ii)$f\left(\sigma(a, x)\right) = \mu(a, f(x))$ for all $a \in L$ and all $x \in (\sigma, J)$.

Proposition2.20[3] If $f: (\sigma, J) \rightarrow (\mu, K)$ is a L-slice homomorphism , then $f\left(0_J\right) = 0_K$

Proposition2.21[3]The composition of two L-slice homomorphism is a L-slice homomorphism.

Proposition2.22[3]Let(σ, J),(μ, K) be L-slices and let $f: (\sigma, J) \rightarrow (\mu, K)$ be a bijective L-slice morphism. Then the map $f^{-1}: (\mu, K) \rightarrow (\sigma, J)$ is an L-slice homomorphism.

Definition2.23[3]Let(σ, J), (μ, K)be two L-slices.A map $f: (\sigma, J) \rightarrow (\mu, K)$is said to be an L-slice isomorphism if

i)f is one-one

ii)f is onto

iii)f is a L-slice homomorphism.

Lemma2.24[3] If $f, g : (\sigma, J) \rightarrow (\mu, K)$are L-slice homomorphism,then the map $f \vee g: (\sigma, J) \rightarrow (\mu, K)$ defined by $f \vee g(x) = f(x) \vee g(x)$ for $x \in (\sigma, J)$is an L-slice homomorphism.

Proposition2.25[3] Let(σ, J)be an L-slice of a locale L.For each $x \in (\sigma, J)$,let $F_x = \{a \in L: \sigma(a, x) = x\}$is a filter in L.

Proposition 2.26[3] The filter$F_x$is proper for $x \neq 0_J$

Proposition 2.27[3]Let $x \in (\sigma, J)$be join irreducible element of (σ, J),then $F_x$is a prime filter in L.

Definition 2.28[3]An element $x \in (\sigma, J)$is said to be compact element of the L-slice(σ, J), if for any collection $\{a_\alpha\}$of L whenever $\sigma(\sqcup a_\alpha, x) = x$,then there exists a finite sub collection $\{a_1, a_2, \ldots . a_n\}$of a_α such that $\sigma(a_1, x) \vee \sigma(a_2, x) \ldots . . \sigma(a_n, x) = x$.

Example 2.29[3]Let (σ, J)be any L-slice.Then0_J is a compact element **.**

Proposition 2.30[3]Let $x \in (\sigma, J)$be join irreducible compact element of (σ, J),then $F_x$is a completely prime filter.

With these preliminaries, we now define regular and associated filters of a locale L.

III. REGULAR FILTER , IDLE POINTS AND $Knot_F$

Let L be a locale with top element 1_L and J be a join – semilattice with bottom element 0_J . On the locale L , we defined the L-slice (σ, J) .

For each $x \in (\sigma, J)$, $F_x = \{a \in L \,|\sigma(a, x) = x\}$ is a filter on L.

Definition 3.1A filter F on a locale L is said to regularise$x \in (\sigma, J)$, if $F \cap F_x$ is a non trivial filter .And F is called the regular filter at x .

For a regular filter F at$x$, the set of elements in $F \cap F_x$, is called the idle points of F with respect to the action on $x \in (\sigma, J)$. The intersection of any two filters is again a filter . Thus,the set of all idle points forms a filter of L.

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Proposition 3.2 Let $f:(\sigma,J) \longrightarrow (\mu,K)$ be a slice homomorphism , and let F regularise $x \in (\sigma,J)$ then F regularise $f(x) \in (\mu,K)$.

Proof: Let F regularise $x \in (\sigma,J)$, then $F \cap F_x$ is non trivial. In other words , $\sigma(a,x) = x$, for some $a \in F$, $a \neq 1_L$. $f(\sigma(a,x)) = f(x)$ implies $\mu(a,f(x)) = f(x)$. That is, $a \in F_{f(x)}$ and $F \cap F_{f(x)}$is non trivial. Thus, F regularise $f(x)$.

Theorem 3.3 Let $f:(\sigma,J) \longrightarrow (\mu,K)$ be a one-one slice homomorphism then F regularise $x \in (\sigma,J)$ if and only if F regularise $f(x) \in (\mu,K)$.

Proof: From proposition 2.3 it follows that if F regularise $x \in (\sigma,J)$ then F regularise $f(x) \in (\mu,K)$. To prove the converse part , suppose that F regularise $f(x)$, then $F \cap F_{f(x)}$is non trivial .

Let $s \in F \cap F_{f(x)} \Rightarrow \mu(s,f(x)) = f(x)$

$$\Rightarrow f(\sigma(s,x)) = f(x)$$
$$\Rightarrow \sigma(s,x) = x$$
$$\Rightarrow s \in F_x \cap F$$

Thus F regularise $x \in (\sigma,J)$.

Theorem 3.4 An Equivalence relation on regular filters at $x \in (\sigma,J)$.

Let F and G be any two regular filters at $x \in (\sigma,J)$.We define $F \sim_x G$, if F and G has same idle points at $x \in (\sigma,J)$.

Let (σ,J)be an L-slice and $x \in (\sigma,J)$. Then the relation $\sim_x$ on all regular filters at x is an equivalence relation.

Proof: Since , $F \sim_x F$, the relation is reflexive. $F \sim_x G$ if and only if $G \sim_x F$, hence symmetric. Also, if $F \sim_x G$, then $F \cap F_x = G \cap F_x$ and $G \sim_x H$, implies$G \cap F_x = H \cap F_x$ Thus the relation is transitive.

For each $\in (\sigma,J)$, the regular filters at x is partitioned into equivalence classes with respect to their idle points .

$Knot_F$ for a filter F

Definition 3.5 Let F be a filter of the locale L,then $x \in (\sigma,J)$is called a knot point of F, if $F_x \cap F$ is non trivial. Or, in other words , x is called a knot point , if F regularises x .Also, we define the collection of knot points of filter F as the non empty set $Knot_F = \{ x \in (\sigma,J) | F$ regularises $x \}$.

Proposition 3.6 If $F \sqsubseteq G$ then $Knot_F \subseteq Knot_G$

Proof: Let $x \in Knot_F$, then $F_x \cap F$ is nontrivial implies $F_x \cap G$ is also nontrivial. Therefore , $x \in Knot_G$.

Proposition 3.7 Let F and G be filters of the locale L , then $Knot_{F\cap G} \subseteq Knot_F \cap Knot_G$

Proof: Let $x \in Knot_{F\cap G}$,then $F_x \cap (F \cap G)$ is nontrivial. Therefore , there exists $1 \neq b \in (F_x \cap F) \cap G$, i.e , $F_x \cap F$ is nontrivial.Hence, F regularises$x$.Similarly , $(F_x \cap F) \cap G = F \cap (F_x \cap G)$ is nontrivial and thus G regularises$x$.

Definition 3.8A subset$J'$of the L-slice (σ,J) is said to be a semi-slice if $(a,x) \in J'$, for $x \in J'$ and any $a \in L$

Theorem 3.9 For any filter F of the locale L , $Knot_F$is a semi slice of the L-slice (σ,J)

Proof: Let $x \in Knot_F$,then $F_x \cap F$ is nontrivial .Let $b \in F_x \cap F$ and $a \in L$,then $\sigma(b,\sigma(a,x)) = \sigma(a,\sigma(b,x)) = \sigma(a,x)$ .Thus, $b \in F_{\sigma(a,x)}$ and F is a regular at $\sigma(a,x)$. Therefore, $(a,x) \in Knot_F$, for every $a \in L$.

IV. ASSOCIATED FILTER

Definition4.1 Consider a filter F of L .Let $x \in (\sigma,J)$ be such that $F_x \subseteq F$. Then, F is called the associated filter of $x \in (\sigma,J)$ and the pair (F,x) is the associated filter -element with respect to x.

Also, if we impose an additional condition that $F_x \neq \{1_L\}$ and $F_x \subseteq F$, then such a filter F is said to be strongly associated to x.

Note : From now on , the tuple (F,x) would suggest that F is an associated filter of the element $x \in (\sigma,J)$ and will be addressed as the associated filter element with respect to x.

Remarks4.2. Consider the meet L-slice $(\sqcap,L)$,where $\sqcap(a,x) = a \wedge x$, for $a \in L, x \in J$, then a filter F is said to be associated with x , if $\uparrow x \subseteq F$.

Remarks 4.3 For $0_J \in (\sigma,J)$, $F_{0_J} = L$ .Therefore, the only associated filter of 0_J is the locale L

We have the following observations on $(\sqcap,L)$

Observation 4.3 Consider the L-slice $(\sqcap,L)$ and $x \in (\sigma,J)$ with $x \leq y$. Now , for any filter F on L , if (F,x) ,then (F,y).

Proof: Since, (F,x) , $\uparrow x \subseteq F$. Also, $x \leq y$ implies $\uparrow y \subseteq \uparrow x$. Hence $\uparrow y \subseteq F$.

Observation 4.4 If (F,x) and (G,y) in L-slice $(\sqcap,L)$ then $(F \cap G,\ x \wedge y)$.

We can know generalise the above results for any L-slice.

Lemma 4.5 For any two filters F , G on locale L and $x \in (\sigma,J)$, if $(F,x)$and (G,x)then $(F \cap G,x)$

Proof: Since, F and G are associated to x , we have$F_x \neq \{1_L\}$, $F_x \subseteq F$ and $F_x \subseteq G$. Thus , $F_x \subseteq F \cap G$ and this completes the proof.

Lemma 4.6 Let F and G be filters on locale L with $F \subseteq G$. If (F,x) then (G,x) , for some $x \in (\sigma,J)$.

Proof: (F,x)implies $F_x \subseteq F$. Also , $F \subseteq G$. Hence the proof.

Lemma 4.7 Every strongly associated filter of $x \in (\sigma,J)$ regularize x.

Proof: Consider a filter F on L such that $F \looparrowright x$, for some $x \in (\sigma,J)$. Then , F_x is non trivial and $F_x \subseteq F$. Hence ,$F_x = F \cap F_x$, is non trivial

Definition 4.8 For $x \in (\sigma,J)$, then $[F:x]_L$ is the set of all filters of L that are associated with x.

Theorem 4.9 Let $\mathfrak{F}(L)$ denote the collection of all filters on the locale L, then $[F:x]_L$ is a filter on $\mathfrak{F}(L)$.

Proof: Follows from the above two lemma.

Proposition 4.10 If $\sigma_b: (\sigma,J) \rightarrow (\sigma,J)$is one-one and (F,x) then $(F,\sigma(b,x))$

Proof: For $a \in F_{\sigma(b,x)}$, we have $\sigma(a,\sigma(b,x)) = \sigma(b,x)$

$$\Rightarrow \sigma(a \sqcap b,x) = \sigma(b,x)$$
$$\Rightarrow \sigma(b,\sigma(a,x)) = \sigma(b,x)$$
$$\Rightarrow \sigma(a,x) = x \Rightarrow a \in F_x$$

Thus, $F_{\sigma(b,x)} \subseteq F_x \subseteq F$.

Theorem 4.11 Let $f:(\sigma,J) \longrightarrow (\mu,K)$ be a one-one slice homomorphism , then if a filter F on L is associated with $x \in (\sigma,J)$ then F is associated with $f(x)$.

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Proof: A filter F on L associated to $x \in (\sigma, J)$implies F_x is non trivial and $F_x \subseteq F$. Since , f is a slice homomorphism , $F_{f(x)}$ is non trivial.

Now, $r \in F_{f(x)} \Rightarrow \mu(r, f(x)) = f(x)$

$\Rightarrow f(\sigma(r, x)) = f(x)$

$\Rightarrow \sigma(r, x) = x$

$\Rightarrow r \in F_x$.

i.e, $F_{f(x)} \subseteq F_x \subseteq F$. Thus (F ,$f(x)$).

We observe from Theorem 4.9 that for each $x \in (\sigma, J)$, the collection of all associated filters form a filter on the power locale $\mathfrak{F}(L)$.

V. CONTINUITY OFL-SLICE MORPHISM WITH RESPECT TO ASSOCIATED FILTERS

Definition 5.1 A slice homomorphism $f :(\sigma, J) \longrightarrow (\mu, K)$ is said to be semi-continuous at $x \in (\sigma, J)$,if for any filter F associated to x implies F is associated to $f(x)$

Remarks

1.A slice homomorphism is said to be semi-continuous on (σ, J) , if it is semi continuous at every $x \in (\sigma, J)$.

2.Every slice isomorphism is semi-continous on (σ, J).

Theorem 5.2 Composition of semi-continuous slice morphism is semi-continous.

Proof: Consider any two semi –continuous slice morphisms $f :(\sigma, J) \longrightarrow (\mu, K)$ and $g :(\mu, K) \longrightarrow (\delta, M)$. Let the filter F on L associated to x . Since f is semi-continous , F associated to $f(x)$. Also, the semi-continuity of g ensures that F is associated to $g(f(x))$. Thus F is associated to $g \circ f\ (x)$.

Definition 5.3A slice homomorphism $f :(\sigma, J) \longrightarrow (\mu, K)$ is said to be continuous at $x \in (\sigma, J)$ if $F_{f(x)} \subseteq F_x$.

A slice morphism is said to be continuous on (σ, J) if it is continuous at every $x \in (\sigma, J)$.

Remark : For the meet –slices $(\sqcap, L)$ and $(\sqcap, M)$,continous L-slice homomorphism from $f: (\sqcap, L) \rightarrow (\sqcap, M)$ is precisely identity morphisms.

Proposition 5.4 Every continuous slice morphisms are semi continuous.

Proof:Let $f :(\sigma, J) \longrightarrow (\mu, K)$ be continuous at some $x \in (\sigma, J)$. i.e, $F_{f(x)} \subseteq F_x$. Let F be a filter that is associated to$x \in (\sigma, J)$,then $F_x \subseteq F$. Thus, $F_{f(x)} \subseteq F_x \subseteq F$ implies that F is associated$f(x)$ .Hence , f is semi-continous at $x \in (\sigma, J)$.

Theorem 5.5 If$f :(\sigma, J) \longrightarrow (\mu, K)$ is continuous and F regularises $f(x) \in (\mu, K)$ then F regularises $x \in (\sigma, J)$.

Proof: F regularises $f(x)$ implies $F \cap F_{f(x)}$ is non trivial .The continuity of f shows that $F_{f(x)} \subseteq F_x$. Hence , F regularises x.

Theorem 5.6 Composition of two continous slice morphisms on $(\sigma, J)$is continous .

Proof: Consider any two continuous slice morphisms $f :(\sigma, J) \longrightarrow (\mu, K)$ and $g :(\mu, K) \longrightarrow (\delta, M)$, then $F_{f(x)} \subseteq F_x$ and $F_{g(f(x))} \subseteq F_{f(x)}$, for some $x \in (\sigma, J)$. Thus , $g \circ f$ is continous at $x \in (\sigma, J)$.

Simple algebraic calculations will give the proof, so we omit the proof of the following theorem.

Theorem 5.7 Let $f: (\sigma, J) \longrightarrow (\mu, K)$ be bijective L-slice morphism , then $f^{-1}: (\mu, K) \longrightarrow (\sigma, J)$ is continous.

Theorem 5.8 If $f: (\sigma, J) \longrightarrow (\mu, K)$ is a continous morphism of L-slices and $x \in (\sigma, J)$ a compact element of (σ, J) then $f(x)$ is a compact element of (μ, K).

Proof: Let there exist a collection $\{a_\alpha;\ \alpha \in I,$ some indexed set $\}$elements of the locale L , such that $\mu(\sqcup a_\alpha, f(x)) = f(x)$. Then, $\sqcup a_\alpha \in F_{f(x)} \subseteq F_x$ which implies $(\sqcup a_\alpha, x) = x$. Since x is a compact element of (σ, J), we can find a finite sub collection $\{a_1, a_2, \dots \dots . a_n\}$ such that $\sigma(\sqcup a_n, x) = x$. Thus , $f(\sigma(\sqcup a_n, x)) = f(x)$ implies $\mu(\sqcup a_n, f(x)) = f(x)$, showing that $f(x)$ is a compact element of (μ, K).

VI. R-ASLICE

Definition 6.1 A L-slice (σ, J) is said to be R-A slice if every regular filter at x is associated to x.

Remark 6.2 : Let (σ, J) be an L-slice having property that $\sigma_x: (\sqcap, L) \rightarrow (\sigma, J)$ defined as $\sigma_x(a) = \sigma(a, x)$ is one-one for every $x \in (\sigma, J)$ then (σ, J) is a R-A. slice.

Remark 6.3 Any subslice of R-A slice is a R-A slice.

Theorem 6.4 Let $f: (\sigma, J) \longrightarrow (\mu, K)$ be a slice morphism and (μ, K) be a R-A slice , then f is semi-continous at every $x \in (\sigma, J)$.

Proof:Let F be associated to x. Then, Fregularises x. Since, f is a slice morphism F regularises $f(x)$.Also, (μ, K) is a R-A slice would imply that F is associated to $f(x)$ .Thus, f is semi- continous at every x of (σ, J).

Theorem6.6 The product of two R-A slices is a R-A slice.

Proof:Let (σ_1, J) and (σ_2, K)be any two R-A slices. For an element (x, y) of the product slice $(\sigma, J \times K)$, $F_{(x,y)} = \{a \in L :\ \sigma(a, (x, y)) = (x, y)\} = \{a \in L :$ $(\sigma_1(a, x), \sigma_2(a, y)) = (x, y)\} = F_x \cap F_y$. Let F be a regular filter at (x, y). Then,$F \cap F_{(x,y)} = F \cap (F_x \cap F_y)$ is non-trivial. Hence, $F \cap F_x$and $F \cap F_y$ is non-trivial. Since,$(\sigma_1, J)$and $(\sigma_2, K)$are R-A slices, F is associated to x and y.That is,$F_x \subseteq F$and $F_y \subseteq F$.Thus,$F_x \cap F_y \subseteq F$ implies F is associated to $F_{(x,y)}$.

Theorem 6.5 Let $f: (\sigma, J) \longrightarrow (\mu, K)$be a continous L-slice morphism and let (σ, J) be a R- A slice, then (μ, K) is also a R-A slice.

Proof: Let F regularises $f(x)$, then $F_{f(x)} \cap F$ is non trivial .Since, f is continous at x , we have $F_{f(x)} \subseteq F_x$, which impiles $F_x \cap F$ is non trivial .Hence,F regularises x and since $(\sigma, J)$is a R- A slice , F is associated to x .

Thus, $F_{f(x)} \subseteq F_x \subseteq F$, implies that F is associated to$f(x)$,and thus (μ, K) is also a R-A slice.

Thus the inverse image of R-A slice is R-A and also the product of two R-A slices is again a R-A slice.

VII. RESULTS AND DISCUSSION

Sequences and their generalisation called nets ,play an important role in describing the concept of convergence in a topological space. On similar lines , the convergence of filter, cluster point of filter etc. were introduced in topological spaces.

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Topology on a space enables us to define two different types of sequence, namely convergent and Cauchy sequences. Similarly, in L-slices we have succeeded in defining two different types of filters, called associated and regular filters. In classical topology ,we have that all convergent sequences are Cauchy. Here, we obtain that all associated filters are regular. The knot points behave very much identical to the cluster point defined in classical topology.The next section gives a definition of continuity of slice morphisms, similar to the sequential continuity in topological spaces. Also , we observed that many results pertaining to continuous maps in topological spaces are obtained in this domain of L-slices. The theorems 5.6,5.7 and 5.8 illustrates this fact. We find the R-A slice can be an analogoue to complete spaces. The productive property of complete spaces is reflected in the theorem 6.5.Thus, new concept of regular filter,associated filter , knot points and R-A slice introduced in this paper will have a far-reaching application in many fields relating to topology.

VIII. CONCLUSION

For any L-slice$(\sigma, J)$the filter F_x acts as a fixed set in the locale $L$with respect to the action σ.In this study, we have characterised any filter of the locale L with respect to the L-slice (σ, J). Our idea was to define an analogue of nets and sequences in classical topology , to the language of L-slices.Here , we have defined regular filter, associated filter and knot points of a regular filter. We succeeded in defining a continuity aspect to L-slice morphism, using the filters F_x. Also, these investigations led us to the construction of the L-slice, R-A slice.

The theory of domains in computer science made it possible to give a mathematical foundation to the semantics of programming languages. The idea of expressing open sets in a realm outside that of topological spaces witnessed the development of Locale theory. In [4] Steven Vickers has defined a locale in the language of computer science.He developed the idea of topological systems.The L-slice is a look alike of topological systems .Since, our domain of study includes that of locales , we definitely will have an application side for our theory in computer science. .Hence , we propose our future studies in the area of computer science.

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