International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-8 Issue-4, April 2019

Time Comparison for the Evaluation of Stiffness Matrix Of The Family Of Brick Elements

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***Abstract*: The past two decades researchers much attention about the bricks element using Finite element analysis(FEA). In three dimensional finite models, the coefficients of stiffness matrix have to be integrated either analytically or numerically, thus optimization of stiffness matrices and computational time for the calculation of stiffness matrices is important in finite element analysis. This paper presents a reduced integration method and an overview of formulation of stiffness for the family of brick elements such as 8-node, 20-node, 27-node and 32- node brick elements. The comparison of CPU computational time for evaluating element stiffness matrix using Gauss 3x3x3 integration method and reduced integration method is also presented.**

***Keywords*: quadrature, numerical integration, Brick elements.**

I. Introduction

For the cases of realistic problems, we use three dimensional elements for solving problems in FEM. This paper presents an overview of various numerical integration techniques in finite element method for the family of brick elements on application of functionally graded materials. Computational effort for calculating the element stiffness matrix for element stiffness matrix is more. G. Zlokoviet.al [1] for the derivation of element stiffness matrices of brick elements and quadrilateral and also in G-invariant subspaces introduced monomial shape functions. K. S. Surana et.al [2] presents a 27- node brick element in application of heat conduction and using Lagrange interpolation functions. They derived the approximation functions for the elements and nodal variables. A simple 5-point integration scheme for quadrilateral element was developed by Shengrong Huet.al [3]. A stable midpoint quadrature for the quadrilateral element was proposed by Jeyakarthikeyan P.V.et.al [4].

Jeong et.al [5] had modified the conventional gauss sampling points of 16 node and 6 node elements and tested successfully on beams at various conditions. C.K.Yew et.al [6] had developed a computer algebra for finite element environment. Y. J Chiang et.al [7] had evaluated the accuracy in modeling the 20-noded brick element using statistical factorial design.

Manuscript published on 30 April 2019.
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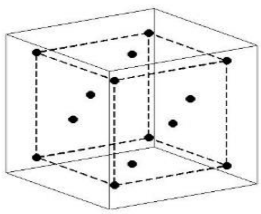
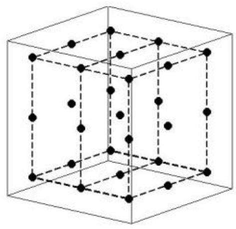
II. Numerical Integration

Numerical integration is the important method for evaluating integrals over the brick elements. For general case of finite elements, the expressions for the integration of element stiffness matrices and load vectors cannot be done analytically. Instead of that load vectors and element stiffness matrices are evaluated by using some integration rule numerically. For estimation of element matrices and vectors usually we use numerical integration methods for different types of finite elements. Gauss numerical integration is an approximation of the definite integral of a function and can be stated within the domain of integration as the weighted sum of functional values at points specified. While calculating element stiffness matrix for the brick elements a lot of effort has to be taken. While calculating for large number of elements computational time for gauss integration is more thus a reduced integration method with 14 sampling points and weights are introduced and the points are shown in table I. In reduced integration method numbers of points to be integrated are reduced to almost half of the full integration. Thus it will reduce the computational time for evaluating the stiffness matrix. The Gauss 3x3x3 numerical integration is shown in fig.1(b) which shows that there are 27 points for evaluating and fig.1(a) shows the reduced 14-point integration.



(a)



(b)

Fig.1. (a) reduced 14-point integration (b) Gauss 3x3x3 integration

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A. *Derivation of new set of points*

For the purpose of deriving the new set of points, undetermined coefficients method is used. The main aim is to determine a new set of sampling points and weights. The typical brick is considered with an interior cube where 14 sampling locations were chosen. Eight points are assumed to be located at the corners of the cube and six sampling points are assumed to be located at the axis of the cube. Fig.1 (a) shows the assumed new set of sampling points and Fig.1 (b) shows the gauss 3-point quadrature sampling point locations. Each sampling point has four unknowns and they are the weights and coordinates so there will be 56 unknowns for the new set of sampling points. In order to integrate at the same time reducing the error in calculation, a polynomial should be chosen and it should contain the terms in r, s and t. Based on the above assumptions we have a total of four unknowns they are the weights and the points, namely a, W_a, b and W_b. The following polynomial is assumed to integrate

$$\Phi(r,s,t)=a_1+a_2r+a_3s+a_4t+a_5r^2+a_6s^2+a_7t^2+\ldots\ldots \quad (1)$$

On integrating (1) we get the following equation

$$\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\Phi(r,s,t)drdsdt=a_1+a_2r+a_3s+a_4t+a_5r^2+\ldots. \quad (2)$$

on performing the numerical integration on (2) we get

$$=a_1(8)+a_5\left(\frac{8}{3}\right)+a_6\left(\frac{8}{3}\right)+a_7\left(\frac{8}{3}\right)+\ldots..$$

(3)

Evaluation of the function Φ can be done by using the following numerical form

$$\Phi=\sum_{i=1}^{14}W_1\Phi(r,s,t) \quad (4)$$

Again substituting (1) in (4) we get

$$=W_1(a_1+a_2r+a_3s+a_4t+\ldots\ldots..)+W_2(a_1+a_2r+a_3s+a_4t+\ldots\ldots..)+\ldots\ldots\ldots..+W_{14}(a_1+a_2r+a_3s+a_4t+\ldots\ldots..)$$

$$=(W_1+W_2+.W_3+\ldots\ldots\ldots+W_{14})a_1+(W_1+W_2+.W_3+\ldots\ldots\ldots+W_{14})a_2+(W_1+W_2+.W_3+\ldots\ldots\ldots.+W_{14})a_3+\ldots\ldots\ldots. \quad (5)$$

Based on the locations of sampling points shown in Fig. 1 (a) there can be only two weights Wa and Wb. For each integration points, the coordinates and the weights are substituted simultaneously and simplifying we get a set of simultaneous equations as the follows.

$$8W_a+6W_b=8$$

$$8a^2W_a+2b^2W_b=\frac{8}{3}$$

$$8a^4W_a+2b^4W_b=\frac{8}{5}$$

$$8a^4W_a=\frac{8}{9} \quad (6)$$

On substituting the points and solving the (6) the set of points are found and shown in table I. For three dimensional brick element the reduced integration will be

$$I=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}f(r,s,t)drdsdt=\sum_{i=1}^{14}f(r_i,s_i,t_i)W_i \quad (7)$$

Table I: Brick element with new set of integration points

Integration point location	Integration points	Weighting function
Corner points	s =±0.75878 t =±0.75868 r =±0.75868	0.33516
Center points	s =±0.79582 t =±0.79582 r =±0.79582	0.88645

III. ELEMENT STIFFNESS MATRIX FOR THE BRICK ELEMENTS.

Strain energy (U) can be defined as

$$U=\frac{1}{2}\int_v\sigma^T\varepsilon dv=\frac{1}{2}[u\quad v\quad w]^T[K][u\quad v\quad w]$$

(8)

$$\begin{Bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{zz}\\\tau_{xy}\\\tau_{yz}\\\tau_{zx}\end{Bmatrix}=\frac{E}{(1+\mu)(1-2\mu)}\begin{bmatrix}1-\mu&\mu&\mu&0&0&0\\\mu&1-\mu&\mu&0&0&0\\\mu&\mu&1-\mu&0&0&0\\0&0&0&\frac{1-2\mu}{2}&0&0\\0&0&0&0&\frac{1-2\mu}{2}&0\\0&0&0&0&0&\frac{1-2\mu}{2}\end{bmatrix} \quad (9)$$

$$\{\sigma\}=[D]\{\varepsilon\} \quad (10)$$

The material matrix (D) can be defined as a function of Poisson's ratio (μ) and Young's modulus (E). In the formulation of stiffness matrix for brick elements the function for defining the material functions (E and μ), field geometry defining functions (x, y and z) and variable defining function (u, v and w) are described in (11).

$$u=\sum_{i=1}^{n}N(r\quad s\quad t)_iu_i \qquad v=\sum_{i=1}^{n}N(r\quad s\quad t)_iv_i$$

$$w=\sum_{i=1}^{n}N(r\quad s\quad t)_iw_i$$

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$$x=\sum_{i=1}^{n}N(r\quad s\quad t)_i x_i \qquad y=\sum_{i=1}^{n}N(r\quad s\quad t)_i y_i$$

$$z=\sum_{i=1}^{n}N(r\quad s\quad t)_i z_i$$

$$E=\sum_{i=1}^{n}N(r\quad s\quad t)_i E_i \quad \mu=\sum_{i=1}^{n}N(r\quad s\quad t)_i \mu_i \quad (11)$$

For the unique mapping of finite elements, for each set of corresponding non-dimensional coordinates there should be only one set of Cartesian coordinates. Jacobian matrix is used for this mapping of elements and it will be

$$J=\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial z}{\partial t} \end{pmatrix} \quad (12)$$

The general strain equation is

$$\{\varepsilon\}=\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}=\begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z}+\frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x}+\frac{\partial u}{\partial z} \end{Bmatrix} \quad (13)$$

$$\{\varepsilon\}=[B]*\lfloor u \quad v \quad w\rfloor^T \quad (14)$$

Where [B] is the strain displacement matrix which can be defined in terms of plane strain and plane stress as shown in (15)

$$[B]_{6\times 3n}=[B_1]_{6\times 9}\times[B_2]_{9\times 9}\times[B_3]_{9\times 3n} \quad (15)$$

$$[u \quad v \quad w]^T=[u_1 \quad v_1 \quad w_1 \quad u_2 \quad v_2 \quad w_2 \quad . \quad . \quad . \quad u_n \quad v_n \quad w_n]^T \quad (16)$$

Where 'n' is defined as nodes per element

$$[B_1]=\begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&1&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&1&0 \\ 0&0&1&0&0&0&1&0&0 \end{bmatrix}$$

$$[B_2]=\begin{bmatrix} JI_{11}&JI_{12}&JI_{13}&0&0&0&0&0&0 \\ JI_{21}&JI_{22}&JI_{23}&0&0&0&0&0&0 \\ JI_{31}&JI_{32}&JI_{33}&0&0&0&0&0&0 \\ 0&0&0&JI_{11}&JI_{12}&JI_{13}&0&0&0 \\ 0&0&0&JI_{21}&JI_{22}&JI_{23}&0&0&0 \\ 0&0&0&JI_{31}&JI_{32}&JI_{33}&0&0&0 \\ 0&0&0&0&0&0&JI_{11}&JI_{12}&JI_{13} \\ 0&0&0&0&0&0&JI_{21}&JI_{22}&JI_{23} \\ 0&0&0&0&0&0&JI_{31}&JI_{32}&JI_{33} \end{bmatrix}$$

$$[B_3]=\begin{bmatrix} \frac{\partial N_1}{\partial r}&0&0&\frac{\partial N_2}{\partial r}&0&0&.\,.\,.&\frac{\partial N_n}{\partial r}&0&0 \\ \frac{\partial N_1}{\partial s}&0&0&\frac{\partial N_2}{\partial s}&0&0&.\,.\,.&\frac{\partial N_n}{\partial s}&0&0 \\ \frac{\partial N_1}{\partial t}&0&0&\frac{\partial N_2}{\partial t}&0&0&.\,.\,.&\frac{\partial N_n}{\partial t}&0&0 \\ 0&\frac{\partial N_1}{\partial r}&0&0&\frac{\partial N_2}{\partial r}&0&.\,.\,.&0&\frac{\partial N_n}{\partial r}&0 \\ 0&\frac{\partial N_1}{\partial s}&0&0&\frac{\partial N_2}{\partial s}&0&.\,.\,.&0&\frac{\partial N_n}{\partial s}&0 \\ 0&\frac{\partial N_1}{\partial t}&0&0&\frac{\partial N_2}{\partial t}&0&.\,.\,.&0&\frac{\partial N_n}{\partial t}&0 \\ 0&0&\frac{\partial N_1}{\partial r}&0&0&\frac{\partial N_2}{\partial r}&.\,.\,.&0&0&\frac{\partial N_n}{\partial r} \\ 0&0&\frac{\partial N_1}{\partial s}&0&0&\frac{\partial N_2}{\partial s}&.\,.\,.&0&0&\frac{\partial N_n}{\partial s} \\ 0&0&\frac{\partial N_1}{\partial t}&0&0&\frac{\partial N_2}{\partial t}&.\,.\,.&0&0&\frac{\partial N_n}{\partial t} \end{bmatrix}. \quad (17)$$

The element stiffness matrix for brick element can be written as

$$[K^e]=\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B^e(r,s,t)]^T[D^e(r,s,t)][B^e(r,s,t)]\det[J^e]drdsdt \quad (18)$$

Suitable numerical quadrature can be defined following equation.

$$[K^e]=\sum_{i=1}^{P}\sum_{j=1}^{Q}\sum_{k=1}^{R}W_iW_jW_kK^e(r_i,s_j,t_k)drdsdt \quad (19)$$

Here P, Q and R represents the sampling points number along the r, s and t directions and Wi, Wj, Wk represents the sampling weights for the respective Quadrature.

IV. CPU TIME ESTIMATION

The computational time required for the family of brick elements such as 8-noded, 20-noded, 27-noded and 32-noded and the proposed method is verified using typical elements. The CPU time comparison for family of brick elements is shown in fig. (2-5) and the table (II-V) shows the CPU time. It is found that computational time is lowered for the proposed method. The computational time are lowered due to the fact that number of points to be integrated is reduced to almost half of the full integration.

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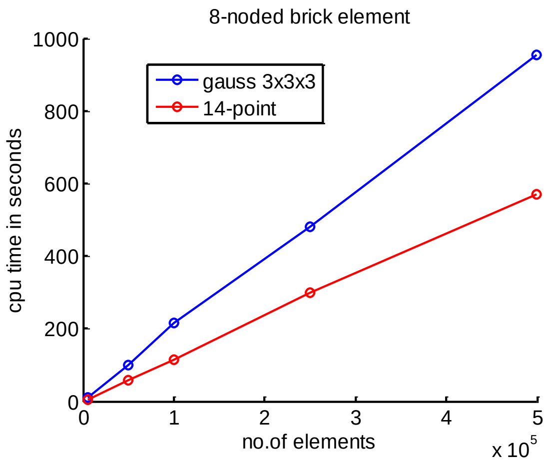

Fig. 2. CPU time comparison for 8-node brick element

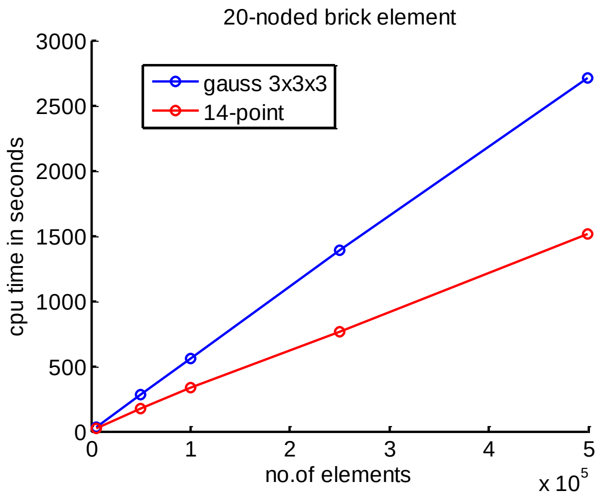

Fig. 3. CPU time comparison for 20-node brick element

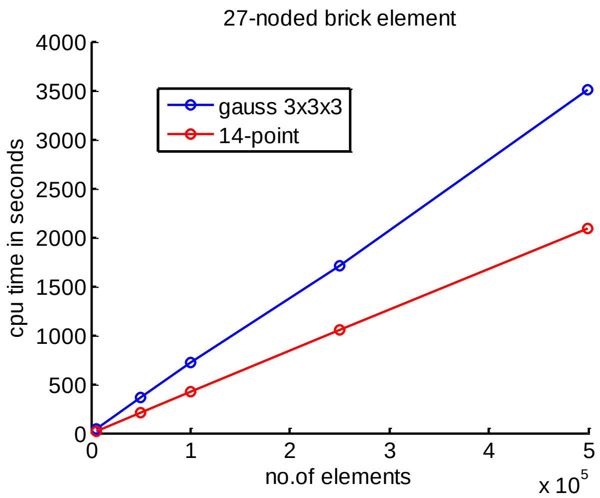

Fig. 4. CPU time comparison for 27-node brick element

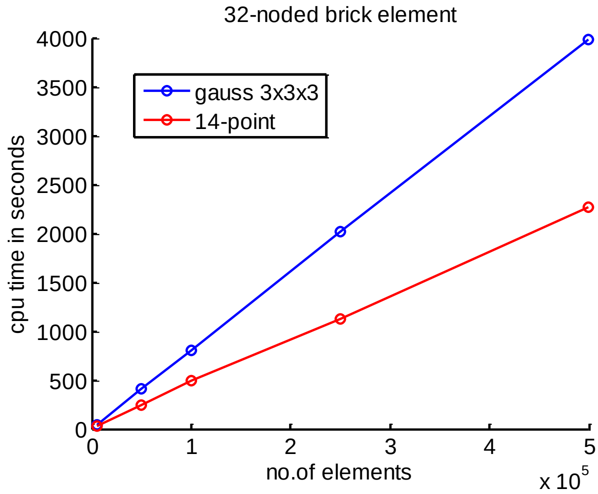

Fig. 5. CPU time comparison for 32-node brick element

TABLE II: 8-NODE BRICK ELEMENT TIME ESTIMATION

No. of elements	3x3x3 gauss integration	14-point integration
5000	10.616189	5.813401
50000	99.22307	57.8168
100000	215.049	113.748
250000	479.2075	298.3489
500000	955.261222	571.093299

TABLE III: 20-NODE BRICK ELEMENT TIME ESTIMATION

No. of elements	3x3x3 gauss integration	14-point integration
5000	27.3932	17.8912
50000	283.105	170.247
10000	561.992	333.278
25000	1386.66	759.972
50000	2707.28	1517.10

TABLE IV: 27-NODE BRICK ELEMENT TIME ESTIMATION

No. of elements	3x3x3 gauss integration	14-point integration
5000	36.879966	21.3692
50000	367.533812	212.05047
100000	724.729551	418.173235
50000	1704.2367	1054.3843
500000	3508.053156	2094.36382

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ISSN: 2249-8958 (Online), Volume-8 Issue-4, April 2019

TABLE V:32-NODE BRICK ELEMENT TIME ESTIMATION

No. of elements	3x3x3 gauss integration	14-point integration
5000	41.903084	24.893294
50000	411.292567	239.2307926
100000	807.855097	492.0118724
250000	2012.2562	1120.0234
500000	3986.904376	2273.249092

V. CONCLUSION

A reduced integration method has applied on the family of brick elements such as 8-node, 20-node, 27 node and 32 node brick elements and the CPU execution time of proposed set of points for the evaluation of global stiffness matrix is found to be reduced to almost half of the execution time taken by the gauss quadrature method. Reasons for reduction in computational time is due to the fact that number of points to be integrated is reduced to almost half of the full integration. On computing element stiffness matrix shows that time for higher order elements is found to be more this is because number of nodes per element is more.

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