Mathematical Modeling Development Of Medium Flow in Rotary-Pulse Device Channels

Yuliya Somova, Elena Moskvina, Mariya Gladysheva, Elena Gasanenko, Andrey Goncharov, Mikhail Sartakov, Stepan Krasnikov, Lubov Prokhasko

Abstract: This paper considers the issues of mathematical modeling development of medium flow in rotary-pulse device channels (RPD) which are used in the conditions of hydrodynamic cavitation for cleaning scale from technological lubricants.

Index Terms: Sludge deoiling, bottom sediments, laboratory installation, scientific research, automation, industrial enterprises

I. INTRODUCTION

A scientific search for progressive methods of treating fluids has shown that cavitation technologies, as well as devices implementing these technologies, can be one of the most effective methods. [1–6].

The basis of the oily sludge of bottom sediments is fine oily scale. Fine oily scale is laminal particles of different thickness, representing the ternary system “solid particles of iron oxides–water–technological lubricants”, which simultaneously have both hydrophilic (scale–water) and hydrophobic properties (oil–water), which makes it difficult to clean from oils.

In the conditions of hydrodynamic cavitation, the following model of descaling of process lubricants is proposed (Figure 1).

Oiled scale particles, falling into the cavitation cluster formed in the stator channel of the RPD, are under the influence of collapsing cavitation bubbles. Oil films that contaminate the surface of the scale are destroyed due to spherical shock waves arising from the collapse of many cavitation bubbles removed from the surface being cleaned or from each other, as well as from the cumulative jets arising from the asymmetric collapse of bubbles that are near the surface being cleaned.

II. MATERIALS AND METHODS

Mathematical modeling development of medium flow in rotary-pulse devices channels (RPD)

In order to determine the energy consumption in the process of destruction of pollution on the surface of the slurry particle, we can use the following dependence:

\[ \varepsilon = \frac{\eta \cdot E}{m_{DS}}, \]  

where \( \varepsilon \) – specific energy of destruction, J/kg;
**Mathematical Modeling Development of Medium Flow in Rotary-Pulse Device Channels**

\[ E = n \cdot E_{CM}, \]  
\[ n = \text{number of bubbles, pcs}; \]
\[ E_{CM} = \text{energy of a cumulative microjet, J}. \]

If the energy of the cumulative microjet is forced to destructible pollution due to the impact, we obtain:

\[ E_{CM} = \frac{m_{CM} \cdot v_{CM}^2}{2}, \]  
\[ m_{CM} = \text{mass of cumulative microjet, kg}; \]
\[ v_{CM} = \text{cumulative microjet speed, m/s}. \]

The mass of the cumulative microjet is determined from the expression:

\[ m_{CM} = \rho_{L} \cdot V_{CM}, \]  
\[ \rho_{L} = \text{density liquid, kg/m}^3; \]
\[ V_{CM} = \text{volume of cumulative microjet liquid, m}^3. \]

Considering the shape of a cumulative microjet cylindrical, its volume is equal to:

\[ V_{CM} = \frac{\pi \cdot d_{CM}^2 \cdot l_{CM}}{4}, \]  
\[ d_{CM} = \text{diameter of the cumulative microjet, m}; \]
\[ l_{CM} = \text{length of the cumulative microjet, m}. \]

The diameter and length of the cumulative microjet [6] are determined by the following relations:

\[ d_{CM} = 0.092 \cdot R_{MAX}; l_{CM} = 0.116 \cdot R_{MAX}, \]  
\[ R_{MAX} = \text{maximum radius of the cavitation bubble (at the moment of collapse), m}. \]

The speed of the cumulative jet [6] is determined by the expression:

\[ v_{CM} = k \cdot v_{c}, \]  
\[ v_{c} = \text{cavitation bubble collapse rate, m/s}; \]
\[ k = \text{energy ratio (k \approx 4)}. \]

The collapse of the cavitation bubble rate [6] is determined by the dependence:

\[ v_{c} = \frac{2}{3} \left( \frac{R_{MAX}^3}{R_{0}^3} - 1 \right) \frac{P}{\rho_{L}}, \]  
\[ R_{0} = \text{initial radius of the cavitation bubble (radius of the embryo), m}; \]
\[ P = \text{pressure acting on the cavitation bubble, Pa}. \]

The difficulty in the calculations is the determination of the initial and maximum radius of the cavitation bubble. From the works of I.M. Fedotkin [7, 8] it is known that the average radius of a cavitation bubble during hydrodynamic cavitation production is \( R_{AB} = 20 \cdot 10^{-6} \). According to experimental data from the works of M.A. Promtov [6, 9] ratio is \( R_{MAX}/R_{0} = 3 \div 6 \). Accept the ratio \( R_{MAX}/R_{0} = 4.5 \). Using dependencies (2)-(8), we can get an expression to determine the minimum value of the final radius of the cavitation bubble, at which the contamination of the slurry particle can be destroyed:

\[ R_{MIN} = 4.5 \cdot \frac{E}{33.8 \cdot n \cdot P}. \]

Practical use of expression (9) allows to preliminarily estimate the energy required to destroy the contamination of the slurry particle (1), as well as to determine the number of cavitation bubbles and the magnitude of their pressure at which they collapse.

In this case, the energy released during the collapse of cavitation bubbles participates in the destruction of hydrocarbon compounds, i.e. breaks chemical bonds between atoms of molecules. This leads to the fact that the desorbed oil droplets pass into light fractions and are easily removed by subsequent classification.

**III. RESULTS AND DISCUSSION**

When calculating the RPD distinguish two tasks:

- calculation and design of a universal device designed for carrying out hydromechanical and heat exchange processes in liquid media;
- calculation and design of the device intended for a specific technological process.

Considering the first task, when a multifunctional apparatus is being designed, it is necessary to consider natural and relative criteria firstly and then economic criteria. Moreover, it is primarily necessary to operate with criteria that show technical and technological efficiency, and then minimize the cost of manufacturing the device.

Solving the second task, when the technological chain and the technological cycle are fully defined, it is necessary to make calculations using economic criteria, for example, the payback period of capital investments and net present value. Universal RPD are usually used in low-tonnage production with a wide range of products and for solving research problems. In large-scale industrial production, and when the use of a device for carrying out only one technological process is justified, the use of a specially designed RPD for this technological process is most effective. Universal RPD are designed in such a way that the main factors affecting the liquid heterogeneous processable medium are involved and give the greatest return.

The design and calculation methodology for RPD are discussed in detail in [10, 11]. According to their recommendations, a single-stage RPD was designed and manufactured.

The mathematical model that fully reflects the basic laws of non-stationary hydro-mechanical processes in an RPD chopper formed by a rotor channel and a stator and a radial gap between them (Figure 2) is the Bernoulli equation, recorded considering non-stationarity of the flow [12-15]:

\[ \frac{\partial v}{\partial t} + \lambda \frac{v^2}{2} + \xi \frac{v^2}{2} + B \frac{v v}{2} = \frac{\Delta P}{P}. \]
where $\beta$ – coefficient of the amount of flow of the processed medium (for engineering calculations, it can be taken as equal to 1);

$t(t)$ – instantaneous value of the average over the channel cross-section velocity, m/s;

$\rho$ – density of the processed medium, kg/m$^3$;

$l$ – modulator (breaker) length, m;

$\lambda$ – hydraulic resistance coefficient;

$d$ – diameter of rotor and stator channels, m;

$\Delta P$ – differential pressure on the modulator, Pa;

$\zeta(t)$ – total coefficient of local hydraulic resistance;

$B(t)$ – coefficient of hydraulic resistance, considering the pressure loss, depending on the flow rate;

$\upsilon$ – medium viscosity index, m$^2$/s.

A similar effect is exerted by the pressure generated by the inlet of the rotor channel, since works in antiphase with an outlet. Then equation (11) takes the form:

$$
\frac{dv}{dt} + \frac{1}{2} \left( \lambda t^2 + \frac{1}{d} \left( \zeta(t) \upsilon^2 + B \frac{v^2}{d} \right) \right) = \frac{\Delta P}{\rho},
$$

where $\Gamma$ – the ratio of the area of the stator channel outlet to the area of the rotor channel entrance, in our case $\Gamma = 1$.

It can be seen from equation (13) that considering the effect of the generated variable pressure on the unsteady hydromechanical processes in the apparatus chopper reduces to increasing the effective length of the modulator to:

$$
l_e = l + (1 + \Gamma) \sqrt{S_0 / \upsilon},
$$

where $S_0$ – cross hole area of the stator channel, m$^2$.

It is also necessary to consider the pressure arising from the rotation of the medium in the rotor cavity and the centrifugal inertia forces acting:

$$
\Delta P_{ci} = \rho \omega^2 R_e^2 / 2.
$$

Then (13) is reduced to the form:

$$
l_e \frac{dv}{dt} + \frac{1}{2} \left( \zeta(t) \upsilon^2 + B \frac{v^2}{d} \right) = \frac{\Delta P}{\rho} + \frac{\omega^2 R_e^2}{2}.
$$

Analyzing this equation (16) without solving, it can be concluded that in the initial time interval, when the hydraulic resistance is large, and the speed of the processed medium is small, the dependence of speed on time is linear:

$$
\upsilon(t) = \left( \frac{\Delta P}{\rho l_e} + \frac{\omega^2 R_e^2}{2 l_e} \right) t + \upsilon(0).
$$

To bring equation (16) to the criterion dimensionless mind, the dimensional variables $\upsilon$ and $t$ must be replaced by the dimensionless variables $\omega$ and $\tau$ using the relations

$$
\upsilon = \upsilon_0 W_t; \quad \tau = t_0 \tau.
$$

Then equation (16) takes the form:

$$
l_e \frac{dv}{d\tau} + \frac{v_0}{t_0} + w^2 v_0^2 \left( \zeta(t) + \upsilon_0 W_t \frac{Bv}{2d} \right) = \frac{\Delta P}{\rho} + \frac{\omega^2 R_e^2}{2}.
$$

Multiplying the terms of both sides of equation (19) by the value $t_0 l_e / \upsilon v_0$, we get

$$
\frac{dw}{d\tau} + w^2 H_0 \omega \zeta(\tau) + \frac{Bw}{H_0} = H_0 E_B + \frac{1}{H_0}.
$$

The criteria included in equation (20) are as follows:

- criterion of “longitudinal” homochronicity

$$
H_{01} = \upsilon_0 t_0 / 2 l_e;
$$

- criterion of “transverse” homochronicity

$$
H_{02} = \omega^2 R_e^2 l_0 / 2 l_e \upsilon_0;
$$

- Euler criterion

$$
E_B = 2 \Delta P / \rho \upsilon_0^2;
$$

- Reynolds criterion

$$
R_e = 2 d l_e / \upsilon v_0.
$$
Mathematical Modeling Development of Medium Flow in Rotary-Pulse Device Channels

The homochronic criterion characterizes the non-stationary nature of fluid flow in spaces bounded by solid walls, and Euler's criterion considers the effect of pressure drops on the process, representing the ratio of pressure forces and inertia forces.

The scale of speed and time is chosen as follows (the rationale for the scale of speed and time is given below):

\[ t_0 = a_c/a_c R_c; \quad t_0 = \Delta P t_0 / \rho l_0, \]  

(25)

where \( a_c \) – stator channel width, m;
\( \omega \) – rotor angular speed, s-1;
\( R_c \) – rotor radius, m.

Considering (25) and the conversion of criteria (21)-(24), equation (20) takes the form:

\[ \frac{dw}{dt} + w^2 Ho(t) + wB(t) \frac{Ho}{Re} = \frac{Ho}{Re} (\frac{v}{r})^2. \]  

(26)

For the practical application of equation (26), it is necessary to know the coefficient of hydraulic resistance – the function \( \zeta(t) \) associated with the influence of resistance forces on the flow of the processed medium, the mechanism of action of which is complicated. As a result, in the calculations semi-empirical formulas are used that are valid in particular cases.

According to [14], shut-off valves and regulating devices, which constrict fluid flow, are hydraulically equivalent to the diaphragm. Therefore, to calculate the hydraulic resistance of the modulator RPD, the formulas used to calculate the diaphragms can be used.

Depending on the flow rate, two areas of change in hydraulic resistance can be distinguished (A.D. Altshul formula):

\[ \zeta' (t) = \zeta(t) + B(t)/Re, \]  

(27)

where \( \zeta'(t) \) – coefficient of hydraulic resistance in the quadratic domain of dependence of hydraulic losses on the flow velocity;
\( R_c \) – Reynolds number calculated by the speed in the stator channel;
\( B(t) \) – hydraulic loss coefficient determined using reference tables [12] and determining the linear dependence of losses on speed.

The coefficient \( \zeta(t) \) is calculated by the formula

\[ \zeta(t) = \left( \frac{S_0}{(S(t))^2} \right). \]  

(28)

where \( S(t) \) – the area of the channel system of the rotor and stator, free for the flow of the processed medium, m2.

In the process of opening the stator channel, the processed medium in the RPD chopper accelerates, and its speed increases in the initial time interval, which follows from equation (17), according to the law:

\[ v(t) = v(0) + \frac{\Delta P}{\rho l_e} t_0. \]  

(29)

The linear nature of the function \( v(t) \) is preserved for more than half of the expiration period, equal when \( a=e=a \) to \( 2t_0=2a/\omega R_c \), \( (a, e) \) is the width of the rotor channel. Consequently, considering the nature of the function \( v(t) \), the quantity \( \Delta P l_0 / \rho l_e \) can be called the average velocity.

The value \( \Delta P l_0 / \rho l_e \) is the characteristic speed of a nonstationary flow in RPD and therefore can be chosen as a scale of speed when building similarity criteria for the considered non-stationary hydromechanical process. This value contains information about the parameters of the modulator \( (t_0, l_0) \) and medium \( (\rho) \). As a time scale, it is advisable to choose a value equal to the half-period of modulation of the velocity and the flow area of the modulator. With this variant of time and speed, the product of the criteria \( E_u \) and \( H_0 \) is equal to 1 and, therefore, equation (26) is simplified:

\[ \frac{dw}{dt} + w^2 Ho(t) + wB(t) \frac{Ho}{Re} = 1 + \frac{1}{Ho} (\frac{a_c}{2l})^2. \]  

(30)

Since the \( E_u \) and \( H_0 \) criteria are independent, any one of them can be applied with equal success. In our case, we use the homochronicity criterion – it is equal to the ratio of the path traveled by the flow at an average speed during the time equal to the half-period of the rotor channel passing through the stator to the length of the liquid column in the modulator, or is the fraction of the liquid column in the RPD modulator leaving the modulator during the half-modulation period.

Denoting the ratio \( (a_c/2l) = R_0 \) as a rotation coefficient considering the rotation of the processed medium in the rotor cavity and the associated centrifugal effect, equation (30) of the movement of the treated fluid in the RPD channels will take the form:

\[ \frac{dw}{dt} + w^2 Ho(t) + wB(t) \frac{Ho}{Re} = 1 + R_0 Ho. \]  

(31)

Introducing the rotational coefficient \( R_0 \) into equation (31), it is necessary to keep in mind that the centrifugal effect depends not only on the angular velocity of the rotor and its geometrical parameters, but also on the degree of “dragging” the flow into rotational motion, therefore the rotational coefficient should take into account:

\[ R_0 = K_{rot} \frac{a_c}{2l} \]  

(32)

where \( K_{rot} \) – coefficient considering the measure of involvement of the flow of the processed medium in the rotational motion. \( K_{rot} \) determined experimentally, while \( K_{rot} < 1 \).

In order to solve the nonlinear differential equation (22), it is necessary to know the functions \( \zeta(t) \) – the coefficient of local hydraulic resistance and \( B(t) \) – the coefficient of hydraulic resistance, which considers the pressure loss.

In the work [18], using the table of N.E. Zhukovsky (for determining \( \zeta(t) \) and the table from the reference list of hydraulic resistances [15] (for determining \( B(t) \)), the Runge-Kutta method was used to solve equation (31) for wide intervals changes in the values of the homochronicity criteria, Reynolds criterion, the values of the dimensionless gap, the parameters \( A \) and the rotational coefficient \( R_0 \):

\[ \omega(0) = -\frac{\omega_0 Ho B(0)/Re_0}{Ho B(0)/Re_0 + 4Ho \zeta(0)(1 + Re_0/2Ho)}. \]  

(33)

The negative root of the quadratic equation is rejected, since the negative velocity (i.e. the flow of fluid towards the pressure drop) is impossible in this situation.

IV. CONCLUSION
The solutions obtained make it possible to determine the influence of various factors on the flow parameters of the treated medium in the rotor and stator channels of the RPD mainly on the acceleration of the fluid, since it is the acceleration that is responsible for the excitation of cavitation in the treated medium.

REFERENCES


