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A New Higher Order Theory for Analysis of Orthotropic Cylindrical Shell Under Electromechanical Load

Muhammad Rahman Adedi, J.S. Mohamed Ali, Meftah Hrairi

Abstract: In this work, the effect of electrical loads on the bending behavior of a simply supported cylindrical shell made of composite and piezoelectric layups have been considered. A new 8 terms higher order shear deformation theory (HSDT8) is proposed and used to analyze the problems. The HSDT8 is the extensional of FSDT by incorporating Murakami zig-zag function and higher order terms in the displacement model. Results are presented for mechanical and electromechanical loading for various layups and validated against available elasticity solutions. HSDT8 proves to be an accurate model for all cases, thin and thick laminate problems.

Index Terms: Composite, Cylindrical shell, Higher order shear deformation theory, piezoelectric.

I. INTRODUCTION

Smart composite materials with piezoelectric layers are widely used in most of engineering applications such as civil and aerospace engineering. Due to high specific strength and stiffness compared to other material, they have become more popular and become the first choice in material selection. Apart from that, its ability to perform health monitoring of the structures in real-time is paid special attention to their high potential applications in structures which requires intelligent functions such as aircraft and marine applications. However composite materials have significant effect due to moisture, temperature variations and electrical effect for piezo material. This effect should be considered in structural analysis for any structures made of composite materials. Hence, a detail analysis should be conducted to study the combined effect of mechanical and electrical loads on composite structure.

The analysis of such structures can be done through experimental or mathematical modelling. The experimental result has been recognized as the prime benchmark for such analysis but due to several limitations and required a lot of efforts, researchers have proposed analytical solution which can be used to accurately analyze such problems. The elasticity solutions are obtained by directly solving the 3D equilibrium equations. Pagano [1] presented exact solution for a composite laminated plate under mechanical load. Bhaskar and Varadan [2], derived the exact solution for laminated anisotropic shell subjected to mechanical transverse load. Exact solution for laminated piezoelectric plates has been presented by Hayliger [3]. Exact solution for laminated orthotropic cylindrical shell under mechanical loading for infinite shell and shell panel cases have been presented by Ren, [4], [5] respectively. Chen et al [6] and Dumir et al [7] both presented exact solution for piezoelectric cylindrical shell under cylindrical bending. The elasticity solutions have been recognized as benchmark results, but its only limited to simply supported structures and some loadings conditions.

For thin laminates, 2D theories such as CST, FSDT and higher order shear deformation theories can be used to overcome these limitations of elasticity solutions. These 2D theories should be validated against 3D elasticity solutions, in order to determine the range of its applicability. Such study can be found in [8] – [11]. Carrera and Brischetto [8], [9] used their refined 2D model, Carrera unified formulation model, to analyze smart plate and smart shell subjected to mechanical, thermal and electrical loads. Jam et al [10] applied Generalized Differential Quadrature (GDQ) on FSDT in order to solve for different boundary conditions. Nath and Kapuria [11] used Improved third order theory and Improved zigzag theory to analyze piezo-composite and piezo-sandwich laminates subjected to thermoelectric loading. It appears that, from all these references, developing 2D model to analyze smart structures subjected to various loading conditions still an active field of research and proves that there is still room for some improvement in these 2D shear deformation theories to predict accurate results.

Based on accurate model for a cylindrical shell under cylindrical bending for hygrothermal loading by Mohamed et al [12] and Saleh et al [13], in the present work, the HSDT8 model will be extended to electromechanical loading and the accuracy will be assessed against the available elasticity solutions.

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* Correspondence Author (s)
Muhammad RahmanAdedi, Department of Mechanical Engineering, International Islamic University Malaysia, PO Box 10, 50728 Kuala Lumpur, Malaysia.
J.S. Mohamed Ali, Department of Mechanical Engineering, International Islamic University Malaysia, PO Box 10, 50728 Kuala Lumpur, Malaysia (Email: jaffar@iium.edu.my).
MeftahHrairi, Department of Mechanical Engineering, International Islamic University Malaysia, PO Box 10, 50728 Kuala Lumpur, Malaysia (Email: meftah@iium.edu.my).

II. FORMULATION

A. Geometrics definition

Consider an infinitely long laminated cylindrical shell as in Figure 1. The panel coordinate system is such that $0 \leq x \leq \infty, 0 \leq \theta \leq \theta_m$ and $-h/2 \leq z \leq h/2$. The mean radius of the shell strip is considered as R and the load is uniform along the x axis so that the shell undergoes cylindrical bending($\varepsilon_x = 0$).

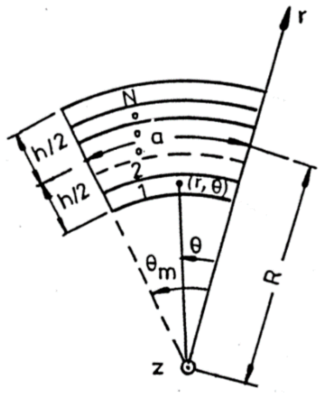

Figure 1

B. Displacement model

Higher order shear deformation theory – HSDT8

$$v(x,\theta,z) = v_o(x,\theta) + z\emptyset_\theta(x,\theta) + z^2\lambda_\theta(x,\theta) + z^3\zeta_\theta(x,\theta) + \psi_k S_\theta \qquad (1)$$
$$w(x,\theta,z) = w_o(x,\theta) + zw_1(x,\theta) + z^2\,\Gamma(x,\theta)$$

Where $v(x,\theta)$ and $w(x,\theta)$ are the displacements at any point in the laminate. The parameter v_o are the in-plane displacements, and w_o is the transverse displacement of a point (x,θ) on the middle plane respectively. The functions ϕ_θ is the rotations of the normal to the middle plane about θ. The other parameters $\lambda_\theta, \zeta_\theta, w_1$ and Γ are unknown higher-order terms which are function of x and θ only. ψ_k is the zigzag function term as defined by Murakami[14].

$$\psi_k = 2(-1)^k z_k/h_k \qquad (2)$$

Where z_k a local transverse is coordinate with its origin at the center of the kth layer and h_k is the corresponding layer thickness.

C. Stress-strain relations

$$\begin{Bmatrix}\sigma_x\\ \sigma_\theta\\ \sigma_z\\ \tau_{\theta z}\\ \tau_{xz}\\ \tau_{x\theta}\end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}^k \begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \varepsilon_{zz}\\ \gamma_{\theta z}\\ \gamma_{xz}\\ \gamma_{x\theta}\end{Bmatrix} - \begin{Bmatrix}\Delta E_z e_1\\ \Delta E_z e_2\\ \Delta E_z e_3\\ \Delta E_\theta e_4\\ \Delta E_x e_5\\ 0\end{Bmatrix} \qquad (3)$$

where C_{ij}'s are the stiffness coefficients ,ΔE is the electric field and its components, e_1,e_2, e_3, e_4 and e_5 are the linear piezoelectric stress constants.

D. Equilibrium equations and boundary conditions

Equilibrium equations can be derived from the principle of virtual work with the understanding that the potential energy is zero. This can be written as

$$\delta\Pi = \delta(U - W) = 0 \qquad (5)$$

where U is the strain energy of the laminate and W represents the work done by external forces. These are evaluated as follows:

$$\delta\Pi = \iiint(\sigma_x\delta\varepsilon_x + \sigma_\theta\delta\varepsilon_\theta + \sigma_z\delta\varepsilon_z + \tau_{xz}\delta\gamma_{xz} + \tau_{x\theta}\delta\gamma_{x\theta} + \tau_{\theta z}\delta\gamma_{\theta z})dzdA - \iint q\,\delta w dA = 0 \qquad (6)$$

in the above equations q is the distributed transverse load.

Thus, one can obtain governing equilibrium equations by substituting (1) - (5) in (6) then perform integration by parts. Total number of equilibrium equations are dependent on the unknown of model. The following equilibrium equations are obtained:

$$N_{\theta,\theta} + N_{\theta z} = 0;\; N_{\theta z,\theta} - N_\theta = -qR_0;$$
$$\frac{M_{\theta,\theta}}{R_0} - N_{\theta z} = 0;\; K_{\theta,\theta} - T_{\theta z} = 0;$$
$$J_{\theta,\theta} - J_{\theta z} = 0;\; H_{\theta,\theta} - H_{\theta z} = 0; \qquad (7)$$
$$\frac{M_{\theta z,\theta}}{R_0} - \frac{M_\theta}{R_0} - N_z = 0;\; \frac{K_{\theta z,\theta}}{R_0} - \frac{K_\theta}{R_0} - 2M_z = 0;$$

Boundary conditions are given as, at $\theta =$ constant: one from each of the following bracketed quantities should be specified.

$$(N_\theta, v_0), (N_{\theta z}, w_0), (H_\theta, S_\theta), (M_{\theta z}, w_1), (M_\theta, \phi_\theta), (K_\theta, \lambda_\theta), (J_\theta, \zeta_\theta), (K_{\theta z}, \Gamma) \qquad (8)$$

The stress resultants in (7) and (8) are defined as follow:

$$(N_\theta, M_\theta, K_\theta, J_\theta, H_\theta) = \sum_{k=1}^{N}\int \sigma_\theta^{(k)}\,(1, z, z^2, z^3, \psi_k)dz^{(k)}$$

$$(N_z, M_z) = \sum_{k=1}^{N}\int \sigma_z^{(k)}\,(1 + \frac{z}{R})(1, z)dz^{(k)} \qquad (9)$$

$$(N_{\theta z}, M_{\theta z}, K_{\theta z}, H_{\theta z}, J_{\theta z}, T_{\theta z}) = \sum_{K=1}^{N}\int \tau_{\theta z}^{(k)}\,(1, z, z^2, \frac{2(-1)^k(R+z)}{h_k}, (3Rz^2 - 2z^3), (3Rz - 2z^2))dz^{(k)}$$

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where the integration between $-h_k/2$ to $h_k/2$ is being carried out over the N layers of shells.

E. Navier Solutions

The solutions that satisfy simply supported edge boundary conditions in (8) and subjected to sinusoidal loading are given in Fourier series as:

$$(v_0, \emptyset_\theta, \lambda_\theta, \zeta_\theta, S_\theta) = (P_1, P_2, P_3, P_4, P_5)\cos(\pi\theta/\theta_m)$$
$$(w_0, w_1, \Gamma) = (P_6, P_7, P_8)\sin(\pi\theta/\theta_m) \quad (10)$$

whereP$_1$ to P$_8$ are the displacement amplitudes. Substituting (10) in the (7) by using the expression (3) and (9), lead to linear algebraic equations which can be solve simultaneously. Hence unknown P$_1$ to P$_8$ and generalized displacement can be obtained. Consequently, strains and stresses can be calculated from the constitutive relations (3).

III. RESULTS AND DISCUSSION

To assess the accuracy of HSDT8, it is validated against available 3D elasticity solutions for different case studies with various layup and loads condition. The following load conditions are considered:

Load Case 1: Cylindrical bending due to mechanical load given by

$$q = \bar{q}_0\left(1 + \frac{h}{2R_0}\right)\sin(m\pi\theta/\theta_m) \quad (11)$$

Load Case 2: Cylindrical bending due to electromechanical load given by

$$q = \bar{q}_0\left(1 + \frac{h}{2R_0}\right)\sin(m\pi\theta/\theta_m)$$
$$E_z = V^B\left(\frac{z_{k+1} - z}{h}\right)\sin(m\pi\theta/\theta_m) + V^T\left(\frac{z - z_k}{h}\right)\sin(m\pi\theta/\theta_m) \quad (12)$$

The piezoelectric material considered in this assessment is an isotropic with uniaxially polarized. The properties are as follow [6]:

$$E = 2\ Gpa\ ;\ v = 0.29\ ;\ e_1 = 0.046(\mathrm{C/m^2})\ ; \quad (13)$$

The material properties of the graphite-epoxy are those used by Bhaskar and Varadan [2]

$$\frac{E_L}{E_T} = 25; \frac{G_{LT}}{E_T} = 0.5\ ; \frac{G_{TT}}{E_T} = 0.2;$$
$$v_{LT} = v_{TT} = 0.25\ and\ \frac{\alpha_T}{\alpha_L} = 1125 \quad (14)$$

Where subscript L denotes the fiber direction and T refer to perpendicular direction to the fiber orientations.

Three different cases are considered for the numerical examples:

1 – 3 layered graphite-epoxy with (90/0/90) orientation under applied load case 1. θ_m =1 and m=1.

2 – Single layered graphite-epoxy (90) orientation and piezoelectric materials at the top and bottom under applied load case 2. $\theta_m = \pi/3$ and m =1. $H_c = H_p$*100. Where H_c denotes the thickness of the graphite-epoxy and H_p stand for piezoelectric layer thickness.

2 – Three layered graphite-epoxy (90/0/90) orientation and piezoelectric materials at the top and bottom under applied load case 2. $\theta_m = \pi/3$ and m =1. $H_c = H_p$*100. Where H_c denotes the thickness of the graphite-epoxy and H_p stand for piezoelectric layer thickness.

All variables are non-dimensionalized using the following expressions. For the first case, the following dimensionless expression are considered.

$$\bar{w} = \frac{100E_T w}{q_0 h S^4},\quad (\bar{v}) = \frac{100E_T(v)}{(q_0 h S^3)} \quad (15)$$

While for the second and third cases, h in (15) is replaced with H_c and S = R/ H_c.

Numerical results are presented in Table 1 to Table 3. The results for case 1 are presented in Table 1. As for composite substrate with piezoelectric layers at top and bottom, case 2 and case 3, results are presented in Table 2 and Table 3 respectively.

Table.1: Nondimensional maximum displacements under load case 1 for a cross-ply (90/0/90) cylindrical shell.

R/h	Theory	$\bar{w}$ (-h/2)	$\bar{v}$ (-h/2)
4	Exact [13]	4.019	6.944
	HSDT8	4.122	7.081
10	Exact	1.221	4.981
	HSDT8	1.226	4.999
50	Exact	0.659	11.401
	HSDT8	0.659	11.401
100	Exact	0.639	21.26
	HSDT8	0.639	21.26

From this table one can note that HSDT8 provide very accurate results even for thick laminate when R/h = 4.

Table 2 presented electromechanical analysis on a laminated composite shell with PVDF layers. V1 = 0 implies that no electric field is applied to the laminates while when V1 is not zero, the laminate is subjected to couple of electric field and mechanical loadings. The same trend can be noted as previous where HSDT8 produced accurate results for both coupled and uncoupled loadings cases.

Table 3 is similar to table 2 except that the composite substrate consists of 3 layers of cross-ply laminates (PVDF/90/0/90/PVDF) rather than one ply only. Results calculated by HSDT8 are very close to the elasticity results for all cases except for in-plane displacement $\bar{v}$ at R/h = 4 and voltage applied is 100 volts.

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Table.2: Nondimensional maximum displacements under load case 2 for laminates (PVDF/90/PVDF) cylindrical shell.

S	Theory	V1 = 0		V1=100	
		$\overline{w}$ (0)	$\overline{v}$ (0)	$\overline{w}$ (0)	$\overline{v}$ (0)
4	Exact [8]	0.306	1.030	-0.164	-0.387
	HSDT8	0.322	1.064	-0.163	-0.511
10	Exact	0.112	0.374	-0.034	-0.061
	HSDT8	0.115	0.383	-0.034	-0.112
50	Exact	0.075	0.251	0.048	0.163
	HSDT8	0.077	0.256	0.049	0.164
100	Exact	0.074	0.247	0.060	0.202
	HSDT8	0.076	0.252	0.062	0.205

Table.3: Nondimensional maximum displacements under load case 2 for laminates (PVDF/90/0/90/PVDF) cylindrical shell.

S	Theory	V1=0		V1=100	
		$\overline{w}$ (0)	$\overline{v}$ (0)	$\overline{w}$ (0)	$\overline{v}$ (0)
4	Exact [8]	0.459	1.549	-0.163	-0.159
	HSDT8	0.438	1.440	-0.167	-0.494
10	Exact	0.144	0.480	-0.023	-0.011
	HSDT8	0.137	0.456	-0.025	-0.080
50	Exact	0.081	0.269	0.052	0.176
	HSDT8	0.081	0.268	0.052	0.172
100	Exact	0.079	0.262	0.064	0.215
	HSDT8	0.079	0.262	0.064	0.214

For the sake of comparison, results for case 2 (S=4) are made in graphical form illustrating in Fig 2, through the thickness variation of in-plane displacement $\overline{v}$ for both electromechanical and mechanical load. When there is no voltage applied (V=0), the in-plane displacement is almost linear through the thickness. But, when the voltage is increase to 100 volts, the variations is observed to be the opposite and noticeable warping can be seen. This figure is similar to those presented by Chen et al [6]. The deformation of the PVDF at the top and bottom of the laminate increase when the voltage is applied.

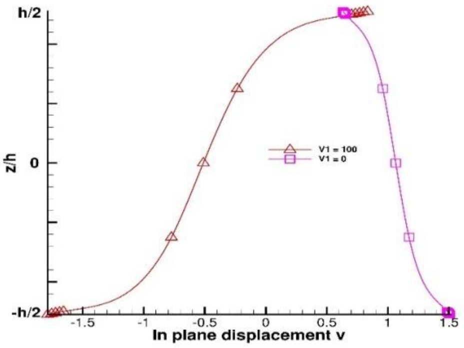

Figure 2: Displacement $\overline{v}$ for (PVDF/90/PVDF) cylindrical shell under electromechanical loading – case 2

IV. CONCLUSION

In the present work, the accuracy of HSDT8 has been assessed with respect to the exact 3D elasticity solutions. Numerical results have been presented for various layups subjected to mechanical loading and electromechanical loading in graphical and tabular forms. It can be observed that HSDT8 model predicts accurate results for both mechanical and electromechanical load cases even for thick laminates.

V. ACKNOWLEDGMENT

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