Network Reliability Evaluation and Analysis of Multistage Interconnection Networks

Deepak Kumar Panda, Ranjan Kumar Dash

Abstract— This paper proposes a new graph theoretic method for calculating network reliability of multistage interconnection networks (MINs). The proposed method converts MIN into its equivalent reliability logic graph (RLG). A new algorithm is proposed to find all the minimal cut-sets of the RLG and then uses inclusion-exclusion principle to evaluate the network reliability of the MIN. The proposed method is well illustrated by taking Extra stage shuffle exchange network (ESEN) as an example network. A comparative analysis is made among the values of network reliability computed by the proposed method as well as by some existing analytical methods. This comparison ensures the proposed algorithms to be quite competent in estimating the reliability values of MINs. Network reliability of nine different types of MINs of greater importance have been evaluated under different conditions (time dependent and time independent)and compared on the same functional environment.

Keywords: Reliability, Multistage interconnection network(MIN), Reliability logic graph(RLG), Minimal cut-set (MCS).

1. INTRODUCTION

Most of the parallel computer interconnection schemes can be classified into two groups: static networks and dynamic networks [1]. The dynamic networks are built on crossbar switches and are used in shared memory-multiprocessors system. Some of the important examples of fault tolerant candidates of dynamic networks (also called as tightly coupled systems) are Extra stage cube network, Multipath omega network, Shuffle exchange network [1-4]. The prime advantages associated with these networks are high bandwidth, minimum diameter, low latency, constant degree switches for which they have been used for various commercial machines including super computers [4].On the other hand a static network, has no crossbar type of switching elements and represents a fixed pattern of interconnection connecting a collection of standalone processors [5]. This type of architecture is used as distributed memory multi computers and is also called as loosely coupled system. Improved performance and increased reliability are the two main advantages attributed to interconnection topology [6]. With the increase in size and complexity of the parallel interconnection systems, their reliability becomes extremely important. There are many reliability measures of interest, out of which the network reliability is an important performance measure in parallel computer interconnection systems.

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Network reliability ensures at least one fault free path among a set of input-output nodes.

Reliability prediction and evaluation of parallel computer interconnection networks has been widely investigated by many researchers in the past. Existing methods for reliability calculation of multistage interconnection networks can be broadly classified into two different categories viz. analytical method [8-15] and simulation methods [16-21]. Analytical methods are primarily based on either continuous time Markov chains (CTMC) [7] or reliability block diagram (RBD) [14], [12]. The Continuous Time Markov Chains have been used effectively to compute the reliability of small sized networks. However, the exponential growth of the state space as the network size increases restricts its use. Reliability block diagram (RBD) is the graphical representation of the components of a system and the relationships among them, which can be used to determine the different reliability measures. Gunawan [12] used analytical method based to evaluate the reliability of shuffle exchange network and its two variants viz. SEN+ and SEN+2. He discussed the effect of reliability on adding extra stages to shuffle exchange network. In his subsequent work [13], he proposed a bound for reliability of gamma networks. Bistouni etal. [14], [15] addressed the same problem as reported in [12] and used reliability block diagram (RBD) to evaluate different reliability measures. Although the analytical methods compute the values of reliability of multistage interconnection networks with greater accuracy, still they are network specific i.e. for each network there is a separate formula for evaluation of reliability. It is quite difficult even sometimes impossible to make generalization of these reliability evaluation formula which would work for a similar group of MIN. So, in order to go for generalization, one may opt for simulation methods.

Some of important simulation methods are: Binary Decision Diagram (BDD) [17-18], Path-set/cut-set based methods [16], [18] [19] and, and Decomposition [20] etc. Out of these methods, the network properties of MINs allow methods based on path-sets/cut-sets for easier implementation on them to evaluate their reliability. However, the path-set/cut-sets based methods require all minimal paths/cut-sets to be generated in advance. Then the minimal path-sets/cut-sets are manipulated to get the counterparts in sum-of-disjoint product form. The main drawback associated with these methods is the requirement of enormous computational efforts for disjointing process. Furthermore,

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the reliability calculated by using these methods is generally a function of link reliability. However, the switching elements (nodes) are more prone to failure than links in MINs. This imposes restriction on the use of these methods for evaluating reliability of MINs as the switching elements are generally denoted by nodes in their equivalent reliability logic graph(RLG).

From the discussions carried out so far as well as the network properties of MINs, it is quite apparent that path-set/cut-set based methods are much easier to implement on MINs to evaluate their reliability. Moreover, cut-set based method should be preferred when the switching elements(node) may fail. However, the computational tasks required for generating valid cut-sets as well as the complex architecture of MINs do not attract more researcher to work on it. Hence, in this paper, an algorithm is proposed which finds all the valid minimal cut-sets using the basic architecture (stage) of MINs. Rest of the paper is organized as follows: Section II proposes a new method to evaluate the network reliability of interconnection network. A new algorithms have also been proposed in this Section. The proposed method is illustrated by taking ESEN as an example network in Section –III. Simulated results along with comparison of the proposed method against some existing methods are presented in Section IV. Section V concludes the paper with its future scope.

2. PROPOSED METHOD TO COMPUTE THE RELIABILITY OF INTERCONNECTION NETWORK

The following definitions are used for the rest of the paper:

Definition 1. Network Reliability- The Network reliability is defined as the probability that every node in the network is able to communicate with one another .

Definition 2. Minimal Cut-set - A Minimal Cut-set is a cut-set whose proper

subsets are no longer cut-sets.

Notation:

N Number of input

n Number of stages

u Number of Nodes

MCS Minimal cut-sets

RLG Reliability Logic graph

MIN Multistage Interconnection network

NR Network Reliability

A brief description of the proposed method is presented below:

2.1 Brief Description on the Proposed Method

The proposed method first converts the multistage interconnection network into its equivalent reliability logic (RLG) graph G(V,E). Each node the RLG graph represents the switching element (SE) of MIN while the links among them are represented by edge represents the links. The nodes are labeled by two digits where the first digit represents the stage and second digit is used to represent the SEs belonging to that stage. The adjacency matrix of G(V,E) is generated. The minimal cut-sets (MCS) are generated in stage wise manner. Each nodes belonging to the first and last stage is a valid MCS as failure of this node disconnects the input and output. The minimal cut sets for the intermediate stages are searched so that exclusion of that node(s) will disconnect the input and output. After successful generation of all MCS, reliability can be evaluated by applying inclusion-exclusion principle on these cut-sets.

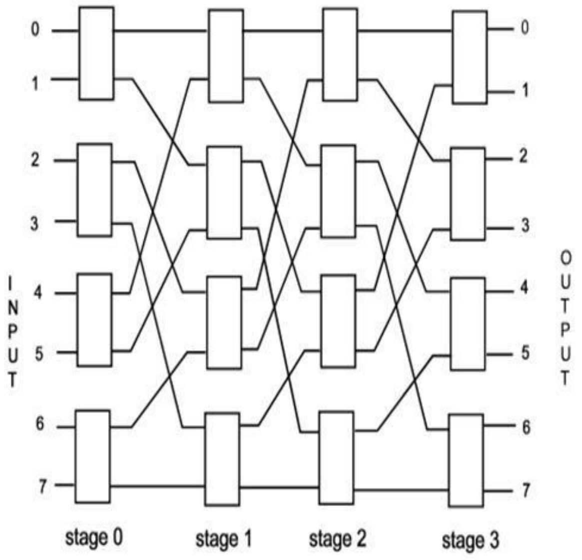

Fig. 1 : Extra stage shuffle exchange network

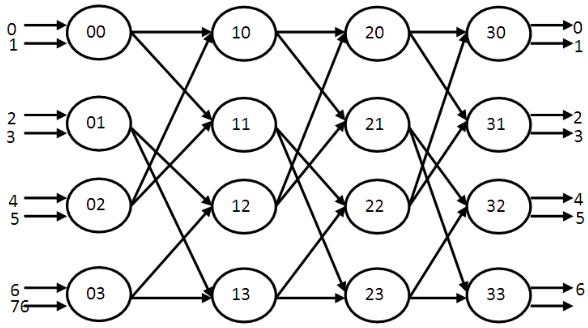

Fig.2 : Reliability logic graph of Extra stage shuffle exchange network

2.2 Algorithm for evaluation of Network reliability of MIN

The network reliability of MIN can be evaluated by using the following steps:

Input:

1. *Adjacency matrix (A) of RLG, G(V,E)*
2. *n – number of stgaes of MIN*

Network_Reliability(A,n)

$MCS = \emptyset$

For stages i=1 or n

For each node $u_{i,j} \in stagei$ *and j=1,2,...N/2*

$MCS = MCS \cup u_{i,j}$

End for

End for

J=1

For stage i=2 to n-1

For each node $u_{i,j} \in stagei$ *and j<=N/2*

If (Disconnect(G,$u_{i,j}$*)= True)*

$MCS = MCS \cup u_{i,j}$

Else If (Disconnect(G,$u_{i,j}$*,* $u_{i,j+1}$*)= True)*

$MCS = MCS \cup u_{i,j} \cup u_{i,j+1}$

$j = j + 1$

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End if

$$NR = P\left(\bigcup_i C_i\right), where C_i \in MCS$$

Return(NR)

Time Complexity of Network_Reliability(A, n)

The time complexity of the propsed algorithm depends largely on finding MCS in the intermediate stages as it superceeds all other operation viz. generation of cut-sets for first and last stages as well as the application of inclusion-exclusion principles on cut-sets.

The search operation to generate MCS for intermediate stages requires the following two functions:

1. Search among n-2 stages – O(n-2)
2. Search each node in each intermediate stage – O(N/2)

Thus, time complexity of the proposed algorithm is $O\left\{(n-2) \times \frac{N}{2}\right\} \approx O(n \times N)$

3. ILLUSTRATION

The proposed algoirhtm is applied on ESEN and the generated MCS are listed below:

Minimal Cut-sets Level Node sets

1. *C1* *1* *{00}*
2. *C2* *1* *{01}*
3. *C3* *1* *{02}*
4. *C4* *1* *{03}*
5. *C5* *2* *{10,11}*
6. *C6* *2* *{12, 13}*
7. *C7* *3* *{20,21}*
8. *C8* *3* *{22,23)*
9. *C9* *4 {30}*
10. *C10* *4 {31}*
11. *C11* *4 {32}*
12. *C12* *4 {33}*

By applying Inclusion-exclusion principle on above minimal cut-sets, the unreliability expression is

$$T = P(C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 \cup C_7 \cup C_8 \cup C_9 \cup C_{10} \cup C_{11} \cup C_{12})$$

Hence, the network reliability of ESEN is

$NR = 1 - QT$ =0.39 (when reliability of each SE=0.9)

4. RESULTS AND DISCUSSION

4.1 Comparison

The network reliability of SEN and ESEN is evaluated and compared against the same obtained from the existing methods [12] and [14]. The reliability of switching elements is varied from 0.9 to 0.99 for these networks. The comparison shows that two existing methods [12] [14] have same values of network reliability for these networks which is obvious as both use analytical methods to compute the reliability of said networks. The proposed method though being a simulation approach estimates reliability of these networks more accurately (refer Table 1).

Table 1 : Comparison of network reliability of SEN and ESEN

Switching Element	**Network Reliability of SEN Computed by**			**Network Reliability of ESEN Computed by**		
	Method [12]	**Method [14]**	**Proposed method**	**Method [12]**	**Method [14]**	**Proposed method**
0.9000	**0.2824**	**0.2824**	**0.2824**	**0.3887**	**0.4066**	**0.3940**
0.9100	**-**	**0.3224**	**0.3225**	**-**	**0.4489**	**0.4339**
0.9200	**0.3676**	**0.3676**	**0.3677**	**0.4799**	**0.4947**	**0.4775**
0.9300	**-**	**0.4185**	**0.4186**	**-**	**0.5493**	**0.5250**
0.9400	**0.4759**	**0.4759**	**0.4759**	**0.5863**	**0.5969**	**0.5767**
0.9500	**0.5403**	**0.5403**	**0.5404**	**0.6454**	**0.6538**	**0.6332**
0.9600	**0.6127**	**0.6127**	**0.6127**	**0.7086**	**0.7146**	**0.6946**
0.9700	**-**	**0.6938**	**0.6938**		**0.7796**	**0.7615**
0.9800	**0.7847**	**0.7849**	**0.7847**	**0.8468**	**0.8487**	**0.8344**
0.9900	**0.8863**	**0.8863**	**0.8864**	**0.9216**	**0.9221**	**0.9137**

4.2 Reliability analysis of Multistage Interconnection Network:

Here, nine numbers of different multistage interconnection networks have been considered for calculating as well as for analysing their network reliability. The network characteristics of these networks are presented in Table 5.3. The reliability analysis of MINs are classified into two types viz. time independent analysis and time dependent analysis.

Sl. No.	**Multistage Interconnection Network**	**No. of input (N)**	**No. of Switching element (SE)**	**No. of stages**
1	General Cube (GC)	8	12	3
2	SEN with an Extra Stage (ESEN)	8	16	4
3	Extra Stage Cube (ESC)	8	16	4
4	Phi Network (PHN)	8	10	3
5	Double tree network (DOT)	8	13	5
6	Four Tree Network (FT)	16	26	5
7	Fault-tolerant Double Tree (FDOT) Network	8	15	3
8	Extra Group Network (EGN)	8	12	2
9	Quad tree (QT)	16	26	5

Table-2 : Network characteristics of Multistage interconnection networks

4.3 Reliability analysis of MINs(Time independent)

For time independent reliability analysis, the reliability of each SEs are set to values ranging from 0.9 to 0.99. the network reliability of the said MINs are evaluated using the proposed method and plotted against switching element reliability (Fig. 3). From this figure, it is quite obvious to put the MINs on ascending order of their network reliability values as follows:

Thus, Quad tree though containing 16 input and 5 stages enjoys the highest values of network reliability because of the

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high degree of connectivity among the switching elements as well as between input and output.

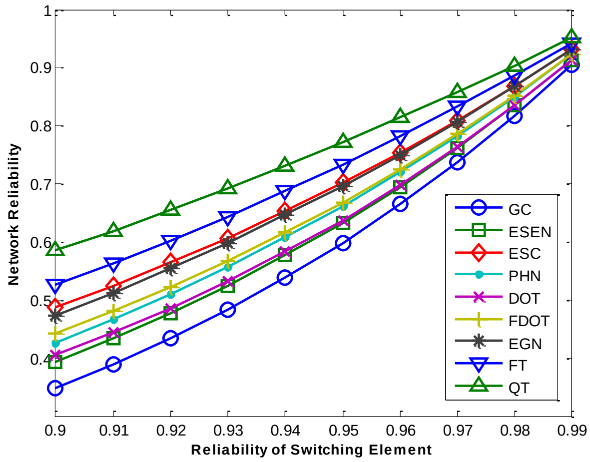

Fig-3 : Network reliability of Multistage Interconnection Networks (time independent)

4.4 Reliability analysis of MINs (Time dependent)

Under time dependent reliability analysis, the reliability of each SEs is obtained from the following equation

$$p(SE) = e^{-\lambda t}$$

Where,

p(SE)- probability of success of each SE

λ= switching failure rate

t= mission time in hours

The following parameters are set as per [14]:

The mission time is set values ranging from 0 to 1000 hours.

The switching failure rate of each switch is set to 0.001.

Setting all these parameters the reliability of said MINs are evaluated and plotted for the purpose of comparison (Figure 4).

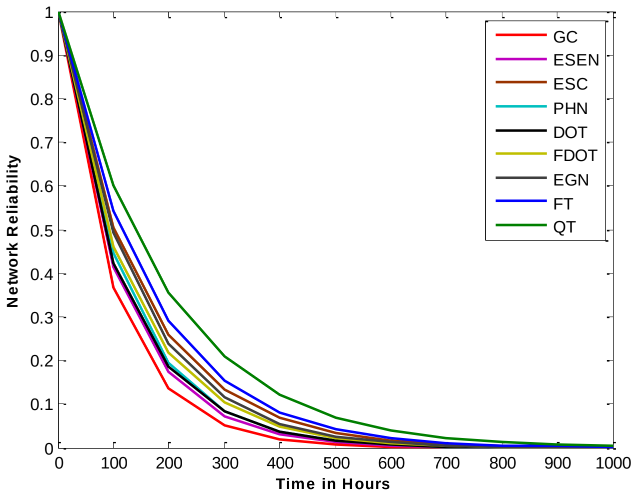

Fig- 4 : Network reliability of Multistage Interconnection Networks (time dependent)

The reliability of the said MINs are evaluated under different switching element failure rates(Table-2).

MIN	$\lambda = 10^{-2}$	$\lambda = 10^{-3}$	$\lambda = 10^{-4}$	$\lambda = 10^{-5}$	$\lambda = 10^{-6}$	$\lambda = 10^{-7}$	$\lambda = 10^{-8}$
GC	3.72007597602083e-44	4.53999297624849e-05	0.367879441171442	0.904837418035960	0.9900498337491 68	0.999000499833375	0.99990000499983 3
ESEN	1.85991976647910e-43	0.000128520380340689	0.412882255707261	0.914109453255573	0.9910423559040 51	0.999100424855541	0.99991000424985 5
ESC	9.02368336950362e-35	0.000949644300193257	0.504295525916241	0.932575689142461	0.9930264240226 22	0.999300264923852	0.99993000264992 4
PHN	1.63876532403834e-39	0.000201419678410874	0.445259878309756	0.923024952506497	0.9920309237966 00	0.999200309923680	0.99992000309992 4
DOT	2.97584122687283e-43	0.000182942601946655	0.422876712769717	0.914378205735755	0.9910453230851 35	0.999100454822559	0.99991000454982 2
FDOT	4.91607277431211e-39	4.91607277431211e-39	0.000398666259818696	0.459987322692932	0.9233868904898 46	0.992034883996265	0.99920034988370 0
FT	7.95071898667850e-31	0.00148830130320652	0.543841643465597	0.941671293425261	0.9940169710294 10	0.999400169971003	0.99994000169997 1
EGN	3.60962083556459e-35	0.000547515451718344	0.492088268493844	0.932301507502044	0.9930234509012 38	0.999300234950840	0.99993000234995 1
QT	1.75126239804194e-26	0.00404562238777820	0.601037968396643	0.951135247262550	0.9950114851746 36	0.999500114985167	0.99995000114998 5

Table -3 : Reliability analysis of MINs under different SE failure rates

From all these comparisons, it can be observed that Quad tree has the highest values of network while these Generilized Cube has the lowest values of network reliability.

CONCLUSION

In this paper, a new graph theoretic based method has been proposed. The proposed method is well supported by an algorithms for evaluating network reliability of MINs. The algorithm generates all the valid minimal cut-sets in stage wise manner in the reliability logic graph(RLG) of the MIN. The network reliability is then calculated by applying inclusion-exclusion principle on these cut-sets. The comparisons of the proposed algorithm against existing analytical methods ensure that the proposed algorithm is more efficient and competent with respect to its counterpart methods. Network reliability of nine number of fault tolerant MINs have been evaluated under the same environment. The work carried out in this paper may be extended to propose bounds on network reliability of MINs.

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