Abstract: This article proposes a mathematical model and a solution method for a heterogeneous fleet vehicle routing problem (HFVRP), which is applied in a draft beer manufacturing company. The objective is to determine the optimal set of customers and delivery route for a fleet of vehicles so that the minimum delivery cost is achieved. There are 12 vehicles separated into 3 types provided to deliver draft beer to 14 customers. The delivery cost depends on the fuel consumption rate of the vehicle and the travel distance. The delivery planning program according to the model is developed using Microsoft Excel. Branch and bound algorithm in OpenSolver is used to solve the problem. The results show that the delivery cost can be reduced by 8.15% after implementation.

Keywords: Heterogeneous fleet vehicle routing, branch and bound algorithm, transportation

Transportation and distribution management is a crucial element in logistics management to enhance customer satisfaction and the company’s core competency. Transportation occupies one-third of the logistics costs and significantly influences the performance of the logistics system (Tseng et al., 2005). The efficient distribution of products to customers at the lowest cost is considered to be one of the most challenging problems in logistics management. This problem involves finding the optimal assignment of customers to be visited by a fleet of vehicles, selecting the vehicle route, and calculating the number of vehicles needed to serve the customers. This type of problem is called the Vehicle Routing Problem (VRP).

VRP has been extensively studied since it was first introduced by Dantzig and Ramser (1959) in 1959. However, current VRP models are much different from the first model as they increasingly aim to incorporate real-life complexities, such as time-dependent travel time windows for pickup and delivery, depot visiting after serving the last customer (Ren, 2011), and input information that changes dynamically over time. HFVRP is a well-known variant of VRP, and is also known as the mixed fleet vehicle routing problem (Braeckers et al., 2016). HFVRP designs optimal delivery routes to be used by a fleet of vehicles to serve a set of customers with minimum delivery cost subject to the vehicle capacity constraint. The fleet of vehicles is based at a central depot and is characterized by different capacities and costs.

HFVRP is an NP-hard problem and most HFVRP models are integer programming problems. Classical exact algorithms such as the branch and bound algorithm, the branch and cut algorithm, branch and the price algorithm, etc., are only efficient in instances of small problems. Therefore, most research focuses on heuristics and metaheuristics (Baldacci et al., 2008) such as the genetic algorithm (Mohammed et al., 2017), sequential insertion (Arvianto et al., 2015), the sweep and swarm intelligence algorithm (Akhand et al., 2017) and so on. Even though an exact algorithm takes more computational time than heuristic and metaheuristic algorithms, it guarantees the optimal solution. Moreover, a significant increase in the performance of computers due to new technology or the parallel computing technique dramatically extends the limitation of problem size to be solved by an exact algorithm. There is much software supporting the exact algorithms. Most are commercial software. Some are freeware such as OpenSolver, software developed by Erdoğan (2017), etc.

The purpose of this research is to develop a delivery planning program to solve HFVRP which represents a real problem in the case study company. To solve the problem, OpenSolver is employed because of its free software license and user familiarity. The answer, which is a delivery plan, identifies the optimal assignment of customers to be served by the available vehicles and delivery route to minimize the delivery cost under the delivery policy of the case study company. This program is intended to facilitate better delivery planning and to reduce the delivery cost.

RESEARCH METHODOLOGY

The research methodology has four steps, which are studying the current delivery planning system, creating a mathematical model, developing a delivery planning program, and program implementation.

The first step is executed to understand the delivery characteristics restrictions and data collection, which can be summarized as follows. The delivery plan is a weekly plan. There is no existing travel distance data. Therefore, the distance between each pair of customers and the distance between each customer and the factory are estimated from a geometric information system (GIS). The delivery cost considers only the fuel cost, which depends on the type of vehicle and the distance. Delivery time is not restricted to a specific time interval.
Each customer is served by only one vehicle. The maximum order for any customer does not exceed the capacity of the available vehicles. Draft beer is in 30-liter stainless steel barrels each weighing forty kilograms. The maximum number of customers is 14. These customers are located in 14 provinces as shown in Table 1. Because of confidentiality, the customers are referred to by the name of the province. The maximum number of available vehicles is 12. They are divided into three types: 6-wheel trucks, 10-wheel trucks, and trailers. All vehicles start and finish at a factory.

In the second step, the mathematical model representing the case study company delivery problem is developed. The notation used in the formula is as follows.

\[ V = \{0, 1, \ldots, N\} \] is the set of \( N+1 \) nodes. Node 0 represents the factory, while the remaining node set \( V' = V \setminus \{0\} \) corresponds to the \( N \) customers.

\[ M = \{1, \ldots, K\} \] is the vehicle set. The vehicles are indexed by parameter \( k \) running from 1 to 12 and each type of vehicle has a different capacity \( C_k \) as shown in Table 2.

### Table 1. Customer information

<table>
<thead>
<tr>
<th>Customer Number</th>
<th>Province</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nakhon Pathom</td>
</tr>
<tr>
<td>2</td>
<td>Prachuap khiri khan</td>
</tr>
<tr>
<td>3</td>
<td>Suphanburi</td>
</tr>
<tr>
<td>4</td>
<td>Ratchaburi</td>
</tr>
<tr>
<td>5</td>
<td>Kanchanaburi</td>
</tr>
<tr>
<td>6</td>
<td>Singburi</td>
</tr>
<tr>
<td>7</td>
<td>Saraburi</td>
</tr>
<tr>
<td>8</td>
<td>Lopburi</td>
</tr>
<tr>
<td>9</td>
<td>Ayutthaya</td>
</tr>
<tr>
<td>10</td>
<td>Rayong</td>
</tr>
<tr>
<td>11</td>
<td>Chonburi</td>
</tr>
<tr>
<td>12</td>
<td>Chachoengsao</td>
</tr>
<tr>
<td>13</td>
<td>Prachinburi</td>
</tr>
<tr>
<td>14</td>
<td>Trat</td>
</tr>
</tbody>
</table>

### Table 2. Vehicle information

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Number of vehicles</th>
<th>Index</th>
<th>Fuel consumption rate (kilometers/liter)</th>
<th>Capacity (barrels/trip)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-wheel Truck</td>
<td>5 trucks</td>
<td>( k = 1 ) to ( k = 5 )</td>
<td>7.0</td>
<td>150</td>
</tr>
<tr>
<td>10-wheel Truck</td>
<td>5 trucks</td>
<td>( k = 6 ) to ( k = 10 )</td>
<td>5.5</td>
<td>350</td>
</tr>
<tr>
<td>Trailer</td>
<td>2 trailers</td>
<td>( k = 11 ) to ( k = 12 )</td>
<td>4.0</td>
<td>500</td>
</tr>
</tbody>
</table>

\( d_{ij}^k \) is the traveling cost from node \( i \) to node \( j \). It is calculated by dividing the distance from node \( i \) to node \( j \) at fuel consumption rate of the \( k^{th} \) vehicle. \( d_{ii}^k = \infty \) if demand (the number of barrels of draft beer) at node \( i \). The decision variables and the mathematical model for this problems are as follows.

\[
\begin{align*}
    x_{ij}^k &= \begin{cases} 1 & \text{if vehicle } k \text{ travels from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases} \\
y_{ij}^k &= \begin{cases} 1 & \text{if node } i \text{ is visited by vehicle } k \\ 0 & \text{otherwise} \end{cases} \\
u_{ik}^k &= \text{arbitrary real numbers which satisfy constraints (7)}
\end{align*}
\]

Minimize \[ Z = \sum_{k \in M} \sum_{i,j \in V} d_{ij}^k x_{ij}^k \quad (1) \]
subject to
\[ \sum_{k \in M} y_{ij}^k = 1 \quad \forall i \in V' \quad (2) \]
\[ \sum_{i \in V} q_i y_{ij}^k \leq C^k \quad \forall k \in M \quad (3) \]
\[ \sum_{j \in V} x_{ij}^k \geq y_{ij}^k \quad \forall i \in V', \forall k \in M \quad (4) \]
\[ \sum_{i \in V} x_{ij}^k \leq 1 \quad \forall k \in M \quad (5) \]
\[ \sum_{i \in V} x_{ij}^k - \sum_{j \in V} x_{ij}^k = 0 \quad \forall p \in V', \forall k \in M \quad (6) \]
\[ u_{ij}^k - u_{ij}^k + n x_{ij}^k \leq n - 1 \quad \forall (i, j) \in V', i \neq j, \forall k \in M \quad (7) \]
\[ x_{ij} \in \{0, 1\} \quad \forall (i, j) \in V, \forall k \in M \quad (8) \]
\[ y_{ij} \in \{0, 1\} \quad \forall i \in V', \forall k \in M \quad (9) \]

The objective function (1) states the total travel cost is to be minimized. Constraint (2) ensures that every customer is
visited at least once. Constraint (3) ensures that the total quantity of product loaded on a vehicle is not over the truckload. Constraint (4) states that if a customer is visited, at least one vehicle must travel to that customer. Constraint (5) states that each vehicle can be used at most once. Route continuity is represented by constraint (6), i.e., if a vehicle enters a demand node, it must exit from that node. Constraint (7) is the subtour-breaking constraint. Constraints (8)-(9) are logical constraints.

In the third step, a program corresponding to the mathematical model is developed using Microsoft Excel because of user friendliness and familiarity. Since the problem size is too large for Excel Solver, OpenSolver is used to solve the problem. Since the mathematical model for the problem is a mixed integer programming problem (MIP), the branch and bound algorithm, which is an exact algorithm for that kind of problem, is used to solve the problem in this program. The program is verified by testing with thirty randomly created data sets to ensure the accuracy of the program before implementing with the real data sets in the last step.

In the final step, the delivery plan obtained from the program will be compared with the original plan of the case study company.

RESULTS AND DISCUSSION

The developed delivery planning program is divided into three parts. They are the data input part, the solver part and the report part. Examples are shown in Figure 1-3 respectively.

The data input spreadsheet shown in Figure 1 is created to input the required data, which are customer number, or customer code, and their demand. Users input the data into two columns, which are customer number and demand. The customer’s name will appear automatically. The solver spreadsheet is a spreadsheet in which the related cells contains formula representing the mathematical model. The data used in the formula are imported from the data input spreadsheet. These cells are then referred in the OpenSolver shown in Figure 2 and the problem is solved. The solutions are translated into the descriptive format shown in the report spreadsheet (Figure 3). The report spreadsheet informs the vehicle number to be used and their delivery routes.

For example, a problem with 7 customers is demonstrated. The planner inputs seven customer numbers and their demand as shown in the first and third columns in Figure 1. The solver spreadsheet is then created according to the input data and the problem is solved by OpenSolver. The solution from the program is as follows.

\[
\begin{align*}
x_{99} = x_{90} = x_{91} = x_{20} = x_{03} = x_{26} = x_{12} = x_{70} = 1, \text{ and } \ y_{6} = y_{7} = y_{1} = y_{12} = y_{7} = y_{12} = y_{7} = 1 .
\end{align*}
\]

According to the value of \(y^k\), vehicle \(k = 6\) is assigned to serve customer number 9, vehicle \(k = 7\) is assigned to serve customers number 1 and 2, and vehicle \(k = 12\) is assigned to serve customers number 3, 6, 7, and 8. The delivery sequence of each vehicle can be read from \(x^k_{ij}\). The sequence of nodes to be visited for vehicle numbers 6, 7, and 12 are 0-9-0, 0-1-2-0, and 0-3-6-8-7, respectively. They are translated into a delivery route for each vehicle as shown in Figure 3. After implementing the program with real data sets, the program gives delivery plans within 5 minutes and the delivery cost is reduced by 8.15% on average.

CONCLUSION

In this research, a delivery planning program is developed to determine vehicle usage and to provide an optimal delivery route for a fleet of vehicles in the case study company. After program trial, the program facilitates the planners to make decisions on delivery planning and the delivery cost is decreased by 8.15%. The new mathematical model applied in the program can represent the real problems. The branch and bound algorithm in OpenSolver can solve the model, which is a MIP, in an acceptable computing time.
For future research, there are two main extensions of this research. The first area is to find an efficient heuristic algorithm to reduce the computational time. The second extended version is to expand the scope or objective of the problem such as considering the fixed costs of using the vehicle, the working time limit of drivers, the planning period, the most economical number of vehicles, and so on.

REFERENCES


