Design and Development of Process and Product Control Sampling Plans

V. Jemmy Joyce, K. Rebbeca Jebaseeli Edna

ABSTRACT--- In this paper process and product control sampling plans are developed as quality control tools in acceptance sampling. The process control part refers to a measurable process characteristics using process capability index which is based on normal distribution and chi square distribution. The lot control part is performed by an attribute sampling plan based on Poisson distribution. The O.C. function and associated measures are derived and tables are constructed and presented for easy selection of the plans. Instead of fixing the first stage sample size in mixed sampling plans, this plan helps to find the first stage sample size.

Keywords: Process capability measure , Process and Product Control, producer's risk, consumer's risk, Operating Characteristic function,

I. INTRODUCTION

Process capability measure has become an important tool in the applications of statistical process control which will monitor the continuous improvement of quality and productivity. In this section process and product sampling plan, based on $\hat{C}_{pmk}$ is introduced. The first sampling step controls the production process by means of variable sampling plan based on $\hat{C}_{pmk}$. If the process is judged to be sufficiently good, the lot is immediately accepted. Otherwise the lot is controlled by an attribute sampling plan and accepted, if the number of non-conforming items does not exceed an acceptance number.

II. LITERATURE REVIEW:

Vannman (2007) had developed distribution and moments in simplified form for a general class of capability indices. CheinWei and Pearn (2008) have developed a Variable Sampling Plan based on $\hat{C}_{pmk}$ for product acceptance determination. Negrin, Parmet and Schechtman (2010) have developed a Sampling plan based on C_{pk} - Unknown Variance. ChuanWu and Lin (2003) have developed sample size determination for the estimate of process capability indices. Lin and Pearn (2003) have developed Distributions of the estimated process capability index c_{pk}. Pearn, Lin and Chang (2003) have developed Distributions of the estimated process capability Index c_{pmk}. Suresh, K.K. and Devaarul, S. (2002) have developed Sampling plans combining Process and Product Control for reducing sampling costs. Devaarul and Jemmy Joyce(2010) have developed mixed sampling plans for second quality lots. Jemmy Joyce, Rebecca Jebaseeli Edna (2018) have developed process and product control sampling plans based on minimum angle.

III. FORMULATION OF THE PROCESS AND PRODUCT CONTROL SAMPLING PLAN:

The design of mixed sampling plan using $\hat{C}_{pmk}$ assuming that standard deviation is known is specified by four parameters, n_1, k, n_2, c

n_1 is the first stage sample size

n_2 is the second stage sample size, if the process has not been accepted in first stage sampling

k is the critical acceptance value.

c is the acceptance constant for the attribute sampling plan for deciding on the lot, in case the process could not be accepted.

Moreover, we distinguish different mixed sampling plans. The first one is characterized by the fact that the process control and the lot control are independent, i.e. based on two different samples. The second one, dependent mixed sampling plan, is used for an early rejection of the lot by attribute the sample that was drawn from variable sampling. The sampling procedures for the two versions of a mixed sampling are given below.

IV. OPERATING PROCEDURE OF THE PLAN

INDEPENDENT MIXED SAMPLING PLAN (n_1, k, n_2, c):

Step 1: Take a random sample of size n_1 from the lot (assumed to be large).

Step 2: Determine

$$\hat{C}_{pmk} = \min\left\{ \frac{USL - \bar{X}}{3\sqrt{S^2_{n_1} + (\bar{X} - T)^2}}, \frac{\bar{X} - LSL}{3\sqrt{S^2_{n_1} + (\bar{X} - T)^2}} \right\}$$

Step 3: If the process capability measure $\hat{C}_{pmk} \geq k$ then the lot or entire process is accepted.

Step 4: If $\hat{C}_{pmk} < k$ then take a second sample of size n_2.

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Step 5: Count the number of non-conformities, let it be'd'

Step 6: If the number of non-conforming items'd' in the second sample is less than or equal to *c*, then accept the lot, otherwise reject the lot.

DEPENDENT MIXED SAMPLING PLAN (n_1, k, n_2, c):

Step 1: Take a random sample of size n_1 from the lot (assumed to be large).

Step 2: If the process capability measure $\hat{C}_{pmk} \geq k$ then the lot is accepted.

Step 3: If $\hat{C}_{pmk} < k$ then examine the first sample for the number d_1 of non- conforming items.

Step 4: If d_1 > c, then reject the lot.

Step 5: If $d_1 \leq c$, take a second sample of size n_2 from the lot and determine the number d_2 of non-conforming items therein.

Step 6: If in the combined sample of size $n_1 + n_2$, the total number of non conforming item $d_1 + d_2 \leq$ c, then accept the lot, otherwise reject the lot.

V. CONDITIONS FOR APPLICATION:

1. The production process should be steady and continuous.
2. The process characteristic must be measurable and the produced items must be classified as conforming and non-conforming units.

Theorem:1 (Independent plan) Operating characteristic function based on the Poisson distribution is given by

$$P_a(p) = P_{n_1}(\hat{C}_{pmk} \geq k) + P_{n_2}\left(\hat{C}_{pmk} < k\right)\sum_{j=0}^{c} \frac{e^{-n_2 p}(n_2 p)^j}{j!}$$

Proof: For mixed plans in which the two stages are kept independent the probability of acceptance is given by the complement of the product of the two probabilities of rejection for a given percent defective.

$$P_a(p) = 1 - P_{n_2}\left(\hat{C}_{pmk} < k\right)\sum_{j=c+1}^{n_2} P_{n_2}(j; n_2)$$

$$= P_{n_1}(\hat{C}_{pmk} \geq k) + P_{n_2}\left(\hat{C}_{pmk} < k\right) -$$

$$P_{n_2}\left(\hat{C}_{pmk} < k\right)\sum_{j=c+1}^{n_2} P_{n_2}(j; n_2)$$

$$= P_{n_1}(\hat{C}_{pmk} \geq k) + P_{n_2}\left(\hat{C}_{pmk} < k\right)\sum_{j=0}^{c} P_{n_2}(j; n_2)$$

$$= P_{n_1}(\hat{C}_{pmk} \geq k) + P_{n_2}\left(\hat{C}_{pmk} < k\right)\sum_{j=0}^{c} \frac{e^{-n_2 p}(n_2 p)^j}{j!}$$

Hence the proof.

VI. MEASURES OF INDEPENDENT MIXED SAMPLING PLAN

1. Operating characteristic function is given below

$$P_a(p) = P_{n_1}\left(\hat{C}_{pmk} \geq k\right) + P_{n_2}\left(\hat{C}_{pmk} < k\right)\sum_{j=1}^{c} P(j; n_2)$$

$$P_a(p) = P_{n_1}(\hat{C}_{pmk} \geq k) + P_{n_2}\left(\hat{C}_{pmk} < k\right)\sum_{j=0}^{c} \frac{e^{-n_2 p}(n_2 p)^j}{j!}$$

2. Average Sample Number:

ASN= $n_1 + n_2 P\left(\hat{C}_{pmk} < k\right)$

3. Average Total Inspection

ATI=ASN + (N - n_1 - n_2) (1 - P_a (p))

4. Average Outgoing Quality;

ASN=P. P_a (p)

Properties of the dependent mixed sampling plan

1. Operating characteristic function:

$$P_a(p) = P_{n_1}\left(\hat{C}_{pmk} \geq k\right) + P_{n_2}\left(\hat{C}_{pmk} < k\right)\sum_{i=1}^{c}\sum_{j=1}^{c-i} P_{n_1}\left(i, \hat{C}_{pmk} < k\right) P_{n_2}(j; n_2)$$

2. ASN= $n_1 + n_2 \sum_{i=1}^{c} P_{n_1}\left(i, \hat{C}_{pmk} < k\right)$

VII. DESIGNING AND SELECTION OF THE SAMPLING PLAN (n_1, k, n_2, c) GIVEN (p_1, β_1), (p_2, β_2)

The procedure for determining the acceptance sampling plan satisfying the above mentioned conditions assuming that the mixed sampling plan is independent is as follows:

1. Split the probability of acceptance that will be assigned to the first stage. Let it be β_1^{1} and β_2^{1} respectively, such that $\beta_1 \geq \beta_1^{1}$ and $\beta_2 \geq \beta_2^{1}$.
2. The required sample size n_1 and critical acceptance constant value k of are calculated.
3. Now determine $\beta_1^{``}$ and β_2^{11} the probability of acceptance assigned to the attributes plan associated with second stage sample as

$$\beta_1^{''} = \frac{\beta_1 - \beta_1^{1}}{1 - \beta_1^{1}} \quad \beta_2^{''} = \frac{\beta_2 - \beta_2^{1}}{1 - \beta_2^{1}}$$

4. Determine the appropriate second stage sample of size n_2 and acceptance number from

$$\sum_{j=0}^{c} \frac{e^{-n_2 p_1}(n_2 p_1)^j}{j!} = \beta_1^{''} \text{ for fraction defective } p_1$$

$$\sum_{j=0}^{c} \frac{e^{-n_2 p_2}(n_2 p_2)^j}{j!} = \beta_2^{''} \text{ for fraction defective } p_2$$

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VIII. RESULT:

TABLE :1

Values of n_1, k, n_2, c given, $(p_1, \beta_1), (p_2, \beta_2)$

Let $\beta_1^{'} = .9$, $\beta_2^{'} = .9$

p_1	$\beta_1^{'}$	$\beta_1^{''}$	p_2	$\beta_2^{'}$	$\beta_2^{''}$	n_1	k	n_2	c
.001	.9	.9	.05	.025	.076	102	1.2001	55	0
.002	.925	.86	.06	.01	.09	135	1.2034	43	0
.003	.9	.75	.03	.01	.09	126	1.2027	90	0
.004	.9	.5	.03	.01	.09	126	1.2027	70	0
.005	.9	.75	.05	.025	.076	101	1.2001	55	0
.006	.9	.75	.04	.01	.09	127	1.2028	50	0
.006	.925	.86	.05	.01	.09	127	1.2028	45	0

IX. EXAMPLE:

Consider the industrial manufacturing of fiber glass sheets. The Target value T is set to .6mm with respect to the thickness of the fiber glass. The USL of fiber glass thickness is .7mm and the LSL is .4mm. Determine the process and product control sampling plan given (p_1, β_1) & (p_2, β_2) as (.002,.99) & (.06,.1)

Solution:

The parameters are n_1, k, n_2, c, from the table n_1 =135, k =1.2034, n_2=43, c =0

Where $\beta_1^{'}$ = .925, $\beta_1^{''}$=.86, $\beta_2^{'}$ = .01, $\beta_2^{''}$=.09

Take a sample of 135 sheets.

Compute $\hat{C}_{pmk}$

If $\hat{C}_{pmk}$ < 1.2034 then reject the lot, otherwise take second stage sample of size 43, find the number of non-conformities d, if d = 0 then accept the lot, otherwise reject the lot.

X. CONCLUSION:

The process and product control sampling plan helps to find the first stage sample size using a process capability index. Table is constructed and Operating characteristic function is derived , which can used in practical applications.

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