

Convective Heat Transfer through Nano Fluid in A Vertical Wavy Channel with Travelling Thermal Wave

P.Venkataramana, G.V.P.N srikanth, G. Srinivas

Abstract— The effect of radiation on free convective flow of heat transfer through a porous medium in a vertical wavy channel has been studied. The resultant differential equations are solved by RK 6th order method. The numerical computations are presented graphically to show the salient features of the fluid flow and heat transfer characteristics. The Nusselt numbers are also analyzed for various of governing parameters.

Keywords: Nano-fluid, Free convection, Radiation, Travelling Thermal Waves, Porous Medium, RK 6th order method.

I. INTRODUCTION

Study of heat transfer through a porous medium in a wavy channel play an important role in various applications. As a result of which buoyancy affect the flow in the presence of radiation. Hence heat transfer with radiation play an important role in manufacturing industries for the design of fins, steel rolling , nuclear power plants, gas turbines and various propulsion devices for aircraft, missile, satellites, food processing, agricultural, heat and military applications. If the temperature of surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of radiation. The study of heat transfer in the presence of radiation has attracted the attention of a large number of scholars due to diverse applications. It is applied to study the stellar and solar structures, radio propagation through the ionosphere, etc. Radiative flows are encountered in many industrial and environmental processes i.e, heating and cooling chambers fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. The two dimensional free convection flow and heat transfer of a Cu and water nano-fluid in a vertical wavy channel with Traveling thermal waves was investigated by P.Venkataramana, S.V.Raganayakulu and G.Srinivas [1]. The Impact of Thermal Radiation and Heat source /sink on Eyring –powell fluid over an unsteady Oscillatory porous Stretching Surface was discussed by Abdullah Dawar, Zahir

Shah ,Muhammad Idress ,Waris khan ,Saeed Islam and Taza Gul[2]. The study Heat transfer with Radiation and temperature dependent Heat source in MHD free Convection flow in a porous medium between two vertical wavy walls was presented by M.S Dada and A.B Disu[3].Chudhary et.al.[4] presented the paper , the thermal radiation effects of fluid on an exponentially extending surface. G.V.P.N Srikanth et.al[5] attempted the effect of chemical reaction on heat transfer through the Nano- fluid. Molla, and Hossain M. A.[6] studied “Radiation Effect on Mixed Convection Laminar Flow Along a Vertical Wavy Surface,”. The effects of heat and mass transfer on MHD mixed convection flow of a vertical surface with radiation, heat source and chemical reaction has been is discussed by S.K Reddy etc [7]. In the view of above all literature survey we wants to investigate the effect of radiation on free convective flow of heat transfer through a porous medium in a vertical wavy channel with travelling thermal waves.

II. MATHEMATICAL MODELLING

We consider the flow of a Cu – water nano fluid between the two vertical wavy walls. We choose the x – axis along the direction of flow and y – axis perpendicular to it. The two wavy walls are at $y = d + a\text{Cos}\lambda x$ and $y = -d + a\text{Cos}(\lambda x + \zeta)$. The upward flow is assumed due to buoyancy. The buoyancy force is due to the density variation and temperature difference along the flow. The thermal wave is imposed on the two plates. We assume that the wave length of the wavy wall is proportional to $1/\lambda$. The flow is assumed to follow Bounessq approximation. In view of the above the governing equations in dimensional form are as follows:

Equation of Continuity,

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} = 0 \quad (1)$$

Equations of Momentum,

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{-1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] - \frac{\mu_{nf}}{\rho_{nf}} u + g \beta_r (T - T_0) u \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{-1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] - \frac{\mu_{nf}}{\rho_{nf} k} u \quad (3)$$

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Equation of Energy,

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] -$$

$$\frac{1}{(\rho c_p)_{nf}} \frac{\partial q_r}{\partial z} \text{ Where } \alpha_{nf} = \frac{k_{nf}}{\rho_{nf}}$$

The boundary conditions are:

$$u = 0, v = 0, T = T_0(1 + \varepsilon \cos(\lambda x + \omega t)) = T_0'$$

at $y = d + a \cos \lambda x$

$$u = 0, v = 0,$$

$$T = T_1[1 + \varepsilon \cos(\lambda x + \omega t)] = T_1' \text{ at } y = -d + a \cos(\lambda x + \zeta)$$

We introduce the following non-dimensional parameters:

$$X = \frac{x}{d}, Y = \frac{y}{d}, t' = \frac{t\gamma}{d^2}, \rho = \frac{\rho d^2}{\rho \gamma^2}, \theta = \frac{T - T_0'}{T_1' - T_0'}$$

$$U = \frac{ud}{y}, V = \frac{vd}{y}, \lambda' = \lambda d, \varepsilon = \frac{a}{d}$$

$$q_r = - \frac{4\sigma_1}{3\delta} \frac{\partial T^4}{\partial y}$$

We assume that the temperature differences within the flow are sufficiently small so that the T^4 can be expressed as a linear function after using Taylor series to expand T^4 about the free stream temperature T_∞ and neglecting higher-order terms. This result is the following approximation:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4$$

$$\text{By using above, we obtain } \frac{\partial q_r}{\partial z} = - \frac{16\sigma_1}{3\delta} \frac{\partial^2 T^4 T_\infty^3}{\partial z^2}$$

After introducing above non-dimensional parameters, the stream function the governing equations becomes,

$$\frac{\partial^3 \psi}{\partial y^2 \partial t} + \frac{\partial^3 \psi}{\partial x^2 \partial t} - \frac{\partial \psi}{\partial y} \left[\frac{\partial^3 \psi}{\partial y^2 \partial x} + \frac{\partial^3 \psi}{\partial x^3} \right] + \frac{\partial \psi}{\partial x} \left[\frac{\partial^3 \psi}{\partial x^2 \partial y} + \frac{\partial^3 \psi}{\partial y^3} \right] = \frac{\mu_{nf} \rho_f}{\mu_f \rho_{nf}} \left[2 \frac{\partial^4 \psi}{\partial y^2 \partial x^2} + \frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^4 \psi}{\partial y^4} \right] - Da \frac{\mu_{nf} \rho_f}{\mu_f \rho_{nf}} \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right] - G \frac{\partial \theta}{\partial Y} \quad (5)$$

$$\frac{\partial T}{\partial t} - \frac{\partial \psi}{\partial Y} \frac{\partial T}{\partial X} + \frac{\partial \psi}{\partial X} \frac{\partial T}{\partial Y} = \frac{k_{nf} \rho_f}{k_f \rho_{nf}} \frac{1}{Pr}$$

$$\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] - \frac{1}{Pr} \frac{4}{3} \frac{1}{Ra} \frac{\partial^2 \theta}{\partial z^2} \quad (6)$$

The non – dimensional boundary conditions are:

$$-\frac{\partial \psi}{\partial Y} = 0, \frac{\partial \psi}{\partial X} = 0, \theta = 0 \text{ at } Y = 1 + \varepsilon \cos \lambda' X$$

$$-\frac{\partial \psi}{\partial Y} = 0, \frac{\partial \psi}{\partial X} = 0, \theta = 1 \text{ at } Y = -1 + \varepsilon \cos (\lambda' X + \zeta)$$

$$\text{Where, } G = \frac{d^3 g \beta_T (T_1' - T_0')}{\nu^2}, \text{ Grashoff Number,}$$

$$Pr = \frac{\nu}{\alpha} \text{ Prandtl Number, Darcy number } Da = \frac{k}{d^2}$$

$$(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s \quad (7)$$

$$(\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_s \quad (8)$$

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1 - \phi)^{0.25}}, \quad \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}$$

III. DISCUSSION AND RESULTS

Figures 1- 15 exhibits the variations of u, v and θ for $Pr = 0.71, \lambda = 0.02, \lambda x = \pi/2, \omega t = \pi/4, \zeta = 0$ in the region $-1 \leq y \leq 1$. From Fig.1 the velocity u increases with increase in solid volume fraction, the reverse effect is observed for v from Fig. 6. Both figures stress the importance of solid part in the fluid. 5% or above solid volume fraction accelerates the flow. Magnitude of u increases gradually with Darcy parameter Da (Fig.2). Similarly velocity v shows gradual variations with Da from Fig.7. Interestingly the reduction of pore diameter is inversely proportional to the velocity. The buoyancy force shows a gradual effect on u and v from Figs. 3 & 8. The buoyancy is proportional to the velocity. The natural convection is significant even in wavy channel. The variation of u for ε is very clear from Fig.4, the pattern for v is reversed in Fig. 9. The frequency parameter is more significant on the flow field. Variations of u and v with Ra are presented in Fig.5 and Fig 10 respectively. Absolute values of u and v are directly proportional to the radiation parameter Ra. The variations of temperature (θ) with $\phi, Da, Gr \in$ and Ra are displayed from Figures 10 to 15. The significance of nano fluid is found well from Fig.11. 5% of metal presence showed great significance in temperature enhancement. Fig.12 represents the variation of temperature with porous parameter Da. Absolute temperature is inversely proportional to the pore diameter. Absolute temperature is directly proportional to the buoyancy, wave frequency parameter and radiation from Figs.13, 14 and 15 respectively.



The wavy walls of the channel reverse the effects in either parts of the channel. Wavy channel shows its cooling trend in the left half of the channel and heating trend in the right half of the channel.

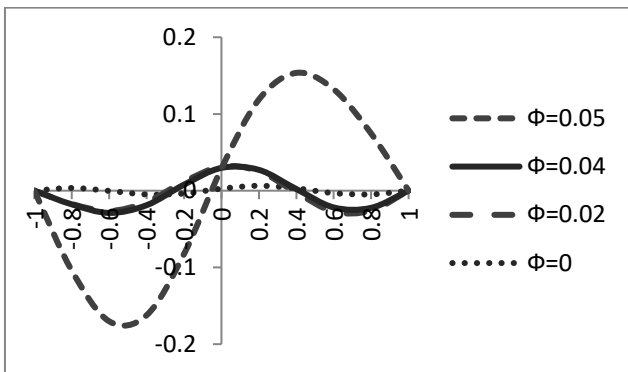


Fig.1 Variation of U with ϕ

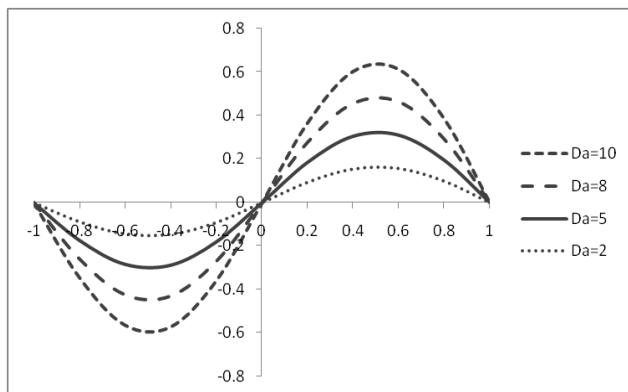


Fig.2 Variation of U with Da

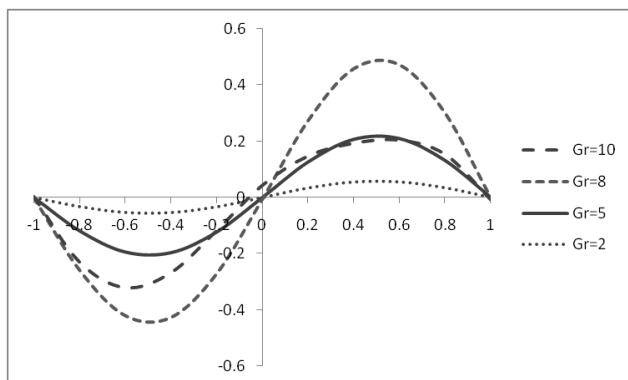


Fig.3 Variation of U with Gr

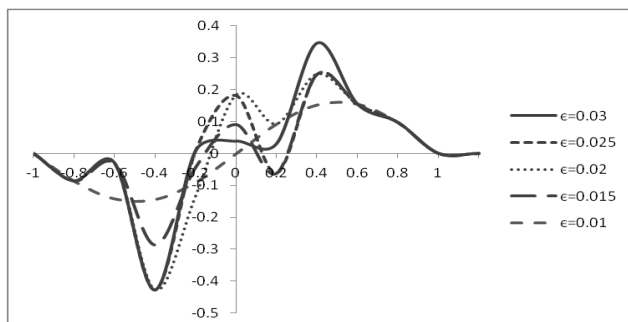


Fig.4 Variation of U with ϵ

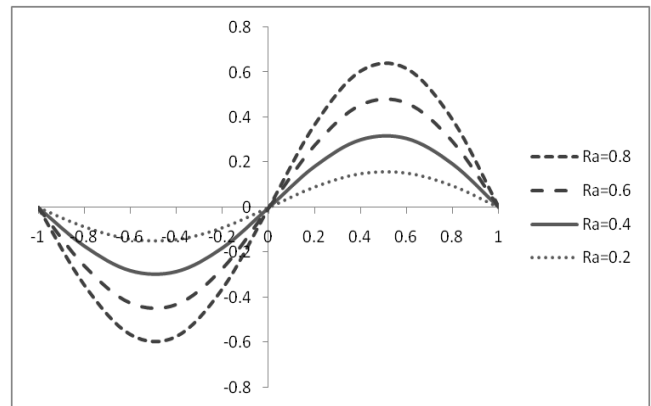


Fig.5 Variation of U with Ra

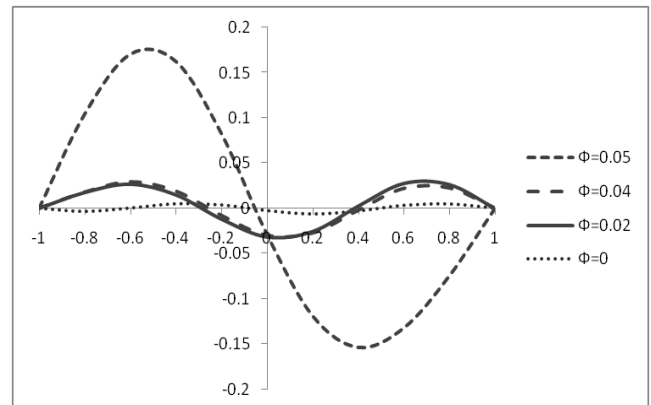


Fig.6 Variation of v with ϕ

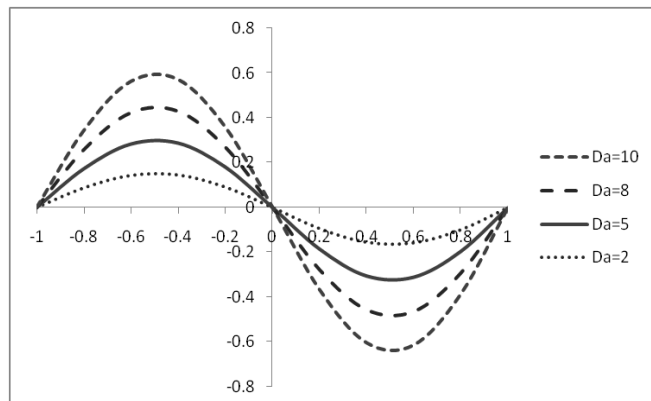


Fig.7 Variation of v with Da

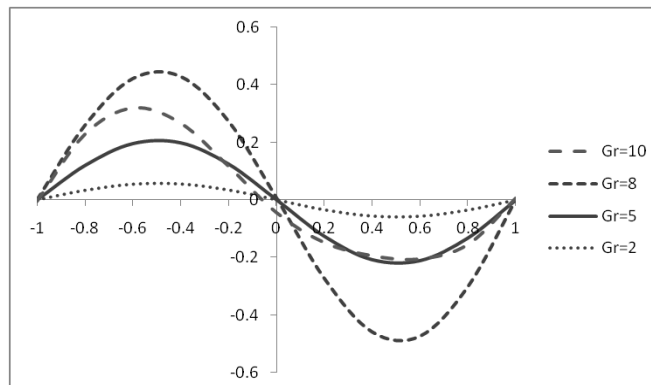


Fig.8 Variation of V with Gr

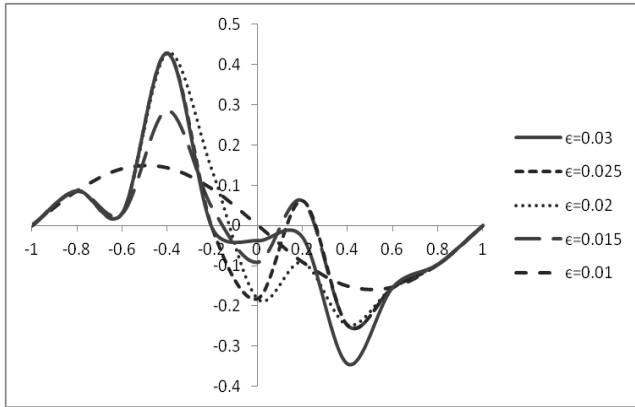


Fig.9 Variation of v with ϵ

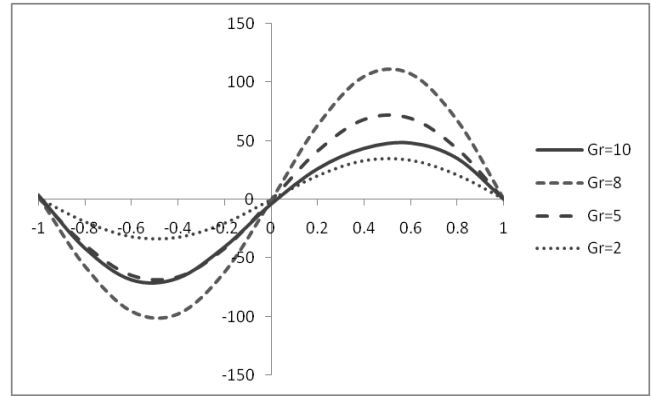


Fig.13 Variation of Θ with Gr

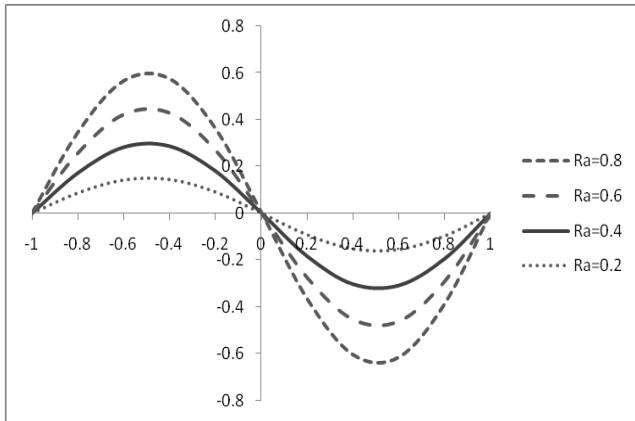


Fig.10 Variation of v with Ra

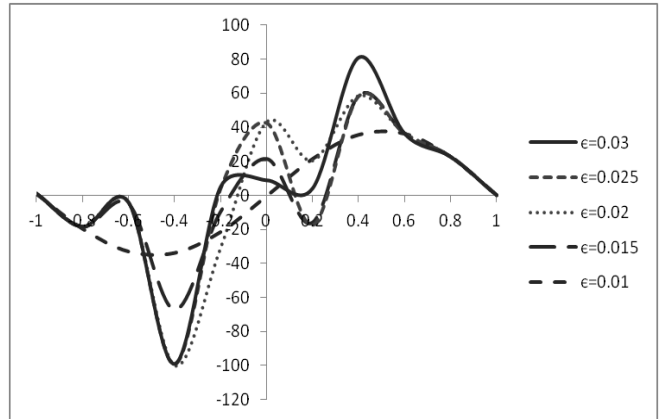


Fig.14 Variation of Θ with ϵ

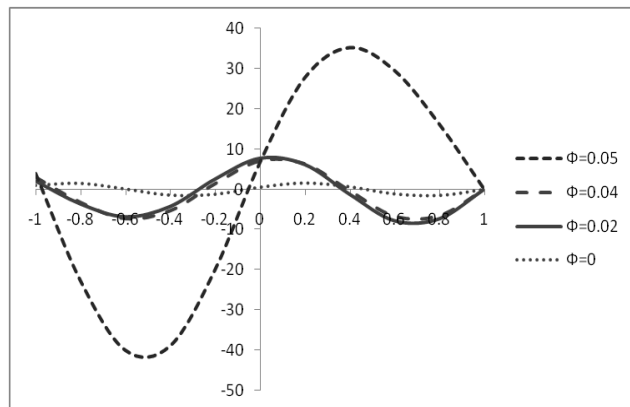


Fig.11 Variation of Θ with ϕ

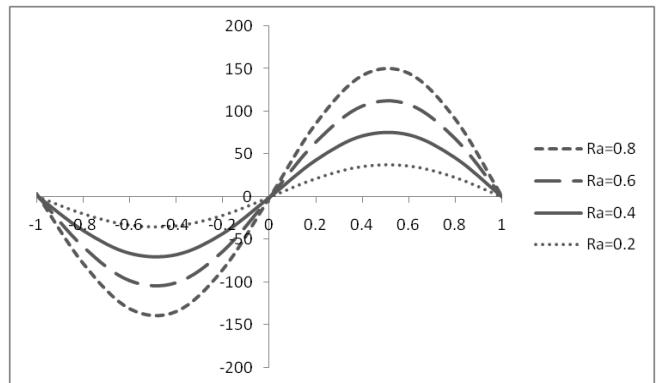


Fig.15 Variation of Θ with Ra

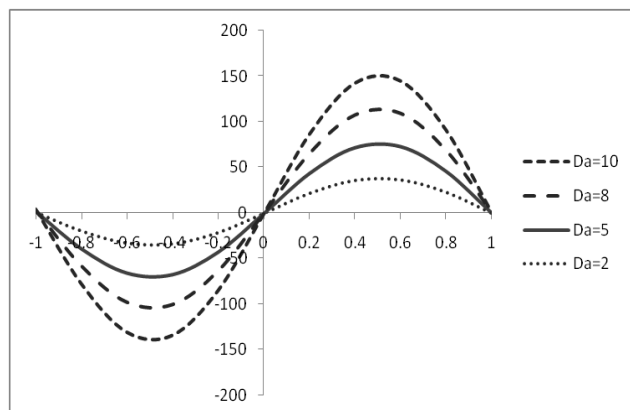


Fig.12 Variation of Θ with Da

TABLES FOR NUSSELT NUMBERS

Table-1

Φ	0	0.02	0.04	0.05
Nu-1	-1.2365 $\times 10^{15}$	-1.1190 $\times 10^{17}$	-2.1291 $\times 10^{15}$	-1.2729 $\times 10^{19}$
Nu-2	-17.1596	-60.9359	7.3538	208.383

Table-2

Da	2	5	8	10
Nu-1	-1.285 $\times 10^{19}$	-1.2729 $\times 10^{19}$	-1.2635 $\times 10^{19}$	-1.2429 $\times 10^{19}$
Nu-2	210.541	208.383	207.012	205.899

Table-3

Gr	2	5	8	10
Nu-1	-6.6414 $\times 10^{18}$	-1.2729 $\times 10^{19}$	-1.2709 $\times 10^{19}$	-3.1539 $\times 10^{18}$
Nu-2	189.561	208.383	220.246	-225.747

Table-4

€	0.01	0.015	0.02	0.025	0.03
Nu-1	-2.03 $\times 10^{16}$	-1.03 $\times 10^{17}$	-3.25 $\times 10^{17}$	-7.95 $\times 10^{17}$	-1.64 $\times 10^{18}$
Nu-2	-1262.21	-1165.26	-1033.64	-871.29	-683.05

Table-5

Ra	0.2	0.4	0.6	0.8
Nu-1	-1.2729 $\times 10^{19}$	-1.2729 $\times 10^{19}$	-1.2729 $\times 10^{19}$	-1.2729 $\times 10^{19}$
Nu-2	208.383	208.383	208.383	208.383

The cooling effect is found more with metal particle proportion on the left wall and heating is found more on right wall of the channel. This is biggest advantage of nano fluid. The porous medium is showing the behavior as that of solid volume fraction. The rate of heat transfer enhances with buoyancy and frequency parameters. The lesser the frequency of the wave the more the heat transfer rate. Radiation is giving the constant heat transfer on both walls.

Nomenclature:

- d – Half of the distance between the wavy walls
- g – Acceleration right wall of the channel
- p – Pressure
- β – Molecular Diffusivity
- α – Thermal Diffusivity
- ω – non dimensional due to gravity
- a – Constant
- λ – Non dimensional wave number
- ρ – Density
- μ – Viscosity
- u – Velocity in x - direction
- v – Velocity in y - direction
- T – Temperature
- T_0' – Temperature on the left wall of the channel
- T_1' – Temperature on the frequency parameter
- P_r – Prandtl number
- R_a – Radiation parameter
- T_∞ – free Stream temperature

IV. CONCLUSIONS

1. The temperature is maximum in the right half of the channel.
2. The wavy boundary reverses the flow and temperature in either part of the channel.
3. Cooling effect is observed with increase in solid volume fraction and wave frequency.

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