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Dominator Coloring Number of Some Class of Graphs

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Abstract: A proper node coloring of a graph where every node of the graph dominates all nodes of some color class is called the dominator coloring of the graph. The least number of colors used in the dominator coloring of a graph is called the dominator chromatic number denoted by χd (G). The dominator chromatic number and domination number of rose graph, triangular belt graph and alternate triangular belt graph is obtained and a relation between dominator chromatic number, domination number and chromatic number is expressed in this paper.

Keywords: Coloring, Domination, Dominator Coloring

I. INTRODUCTION

A dominating set is a subset DS of the vertex or node set of graph G which is such that each node in the graph either belongs to DS or has a neighbor in DS [1]. The domination number γ(G) is the cardinality of a smallest dominating set of G[1]. A proper coloring of a graph G is a function $f: V \to Z_+$such that for $u, v \in V,\ f(u) \neq f(v)$ whenever u and v adjacent nodes in G. A dominator coloring of a graph G is a proper coloring of a graph such that every node of G dominates all nodes of at least one color class. The minimum cardinality of colors used in the graph for dominator coloring is called the dominator coloring number denoted by χ_d *(G).* [2]. The concept of dominator coloring was introduced by Ralucca Michelle Gera in 2006 [2]. The relation between dominator coloring, proper coloring and domination number of different classes of graphs were shown in [3], [5]. The dominator coloring of prism graph, quadrilateral snake, triangle snake and barbell graph and M-Splitting graph and M-Shadow graph of Path graph were also studied in various papers [6], [7], [8]. The algorithmic aspects of dominator coloring in graphs have been discussed by Arumugam S et.al. in[4]. A rose graph [9] denoted by R_n is constructed from a wheel graph $W_{1,n}$ with nodes $\{w, v_1, v_2, .., v_n\}$ by joining the adjacent nodes $\{\, v_i /1 \le i \le n\}$ by new nodes $\{\, u_i /1 \le i \le n\}$. It has $2n + 1$ nodes and $4n$ edges.

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A triangular belt denoted by T_n [10] is constructed from the ladder graph L_n by adding the edges $u_i v_{i+1}\ \forall\, 1 \le i \le n-1$. The alternate triangular belt AT_n [10]

is constructed from the ladder graph L_n by adding the edges $u_{2i+1} v_{2i+2}\ \ \forall\, 0 \le i \le n-1$ and $u_{2i+1} v_{2i}\ \ \forall\, 1 \le i \le n-1$.

In this paper the domination number and dominator chromatic number of rose graph , triangular belt graph and alternate triangular belt graph is obtained and the relation between dominator chromatic number, domination number and chromatic number is expressed.

II.DOMINATOR CHROMATIC NUMBER OF ROSE GRAPH

A. *Proposition* **:**

The chromatic number of rose graph R_n where $n \in \mathbb{Z}_+$, $n \ge 3$, is given by

$$\chi(R_n) = \begin{cases} 4 & when\ n(mod\ 2) \equiv 1 \\ 3 & when\ n(mod\ 2) \equiv 0 \end{cases}$$

B.Theorem:

Every rose graph R_n where $n \in \mathbb{Z}_+$, $n \ge 3$, has dominator chromatic number

$$\chi_d(R_n) = \left\lceil \frac{n}{2} \right\rceil + 2.$$

Proof:

The rose graph denoted by R_n is constructed from a wheel graph $W_{1,n}$ with nodes $\{w, v_1, v_2, .., v_n\}$ by joining the adjacent nodes of $\{\, v_i /1 \le i \le n\}$ by new nodes $\{\, u_i / 1 \le i \le n\}$. It has $2n + 1$ nodes and $4n$ edges which are defined as

$$V(G) = \left\{w, \frac{v_i}{1} \le i \le n\right\} \cup \left\{\frac{u_i}{1} \le i \le n\right\} \text{ and } E(G) = \left\{\frac{v_i v_{i+1}}{1} \le i \le n-1\right\} \cup \{\, v_1 v_n,\ v_1 u_n\} \cup \left\{\frac{w v_i}{1} \le i \le n\right\} \cup \left\{\frac{u_i v_i}{1} \le i \le n\right\} \cup \{\, u_i v_{i+1}/1 \le i \le n-1\}.$$

For dominator coloring of R_n, the nodes are assigned colors as explained below

Case 1: When $n(mod\ 2) \equiv 1$

The node w and the nodes u_i for $1 \le i \le n$ are painted with color 1. For $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$ the nodes v_{2i-1} and v_{2i} are allocated color 2 and $i + 2$ respectively. The node v_n is painted with color $\left\lceil \frac{n}{2} \right\rceil + 2$.

Then for $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$ the nodes $u_{2i-1}, u_{2i}\ v_{2i-1}$ and v_{2i} are dominated by color class $\left\lceil \frac{i}{2} \right\rceil + 2$. The nodes $w, u_n,\ v_n$ are dominated by color class $\left\lceil \frac{n}{2} \right\rceil + 2$.

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Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of Rose graph R_n where $n \in \mathbb{Z}_+, n \geq 3$ is

$$\chi_d(R_n) = \left\lceil \frac{n}{2} \right\rceil + 2.$$

Case 2: When $n(mod\ 2) \equiv 0$

The node w and the nodes u_i for $1 \leq i \leq n$ are painted with color 1. For $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$ the nodes v_{2i-1} and v_{2i} are allocated color 2 and $i+2$ respectively.

Then for $1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$ the nodes u_{2i-1}, u_{2i} v_{2i-1} and v_{2i} are dominated by color class $\left\lceil \frac{i}{2} \right\rceil + 2$. The node w is dominated by color class $\left\lceil \frac{n}{2} \right\rceil + 2$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of Rose graph R_n where $n \in \mathbb{Z}_+, n \geq 3$ is

$$\chi_d(R_n) = \left\lceil \frac{n}{2} \right\rceil + 2.$$

B. Lemma:

The domination number of rose graph R_n where $n \in \mathbb{Z}_+$, $n \geq 3$, is

$$\gamma(R_n) = \left\lceil \frac{n}{2} \right\rceil$$

Proof:

The node set of R_n is

$$V(G) = \{w, v_i /1 \leq i \leq n\} \cup \{u_i /1 \leq i \leq n\}.$$

Case 1: When $n(mod\ 2) \equiv 1$

Let the dominating set of the rose graph R_n be

$$DS = \left\{v_{2i}, v_n /1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\right\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the rose graph R_n is given by

$$\gamma(R_n) = \left\lceil \frac{n}{2} \right\rceil.$$

Case 2: When $n(mod\ 2) \equiv 0$

Let the dominating set of the rose graph R_n be

$$DS = \left\{v_{2i} /1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil\right\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the rose graph R_n is given by

$$\gamma(R_n) = \left\lceil \frac{n}{2} \right\rceil.$$

C.Corollary :

Every rose graph R_n where $n \in \mathbb{Z}_+, n \geq 3$ satisfies the relation

$$\chi_d = \begin{cases} \gamma + \chi - 2 & when\ n(mod\ 2) \equiv 1 \\ \gamma + \chi - 1 & when\ n(mod\ 2) \equiv 0 \end{cases}$$

Proof: By applying proposition 2.1, theorem 2.2and lemma 2.3 we get the result.

III. Dominator Chromatic Number of Triangular belt Graph

A.Proposition **:**

The chromatic number of a triangular belt graph T_n where $n \in \mathbb{Z}_+, n \geq 2$, is given by

$$\chi(T_n) = 3$$

*B.Theorem***:**

Every triangular belt graph T_n where $n \in \mathbb{Z}_+$, $2 \leq n \leq 4$, has dominator chromatic number

$$\chi_d(T_n) = \begin{cases} 2\left\lceil \frac{n}{5} \right\rceil + 1 & when\ n(mod\ 5) \equiv 2 \\ 2\left\lceil \frac{n}{5} \right\rceil + 2 & when\ n(mod\ 5) \equiv 3\ or\ 4 \end{cases}$$

Proof:

A triangular belt is denoted by T_n [10] and is obtained from the ladder graph L_n by adding the edges

$$\{u_i v_{i+1} / 1 \leq i \leq n-1\}.$$

For dominator coloring of T_n, the nodes are assigned colors as explained below

For $1 \leq i \leq n$, the nodes v_i are alloted color 1 when $i(mod\ 5) \equiv 1\ or\ 4$, color 2 when $i(mod\ 5) \equiv 3$ and color $2\left\lceil \frac{i}{5} \right\rceil + 1$ when $i(mod\ 5) \equiv 2$. The nodes u_i for $1 \leq i \leq n$ are painted with color 2 when $i(mod\ 5) \equiv 1\ or\ 4$, color 1 when $i(mod\ 5) \equiv 2$ and color $2\left\lceil \frac{i}{5} \right\rceil + 2$ when $i(mod\ 5) \equiv 3$.

For $1 \leq i \leq n$ and $i(mod\ 5) \not\equiv 4$ the nodes v_i are dominated by color class $2\left\lceil \frac{i}{5} \right\rceil + 1$. For $1 \leq i \leq n$ and $i(mod\ 5) \equiv 4$ the node v_i is dominated by color class $2\left\lceil \frac{i}{5} \right\rceil + 2$. For $1 \leq i \leq n$, when $i(mod\ 5) \not\equiv 3\ or\ 4$ the nodes u_i are dominated by color class $2\left\lceil \frac{i}{5} \right\rceil + 1$ and when $i(mod\ 5) \equiv 3\ or\ 4$, the nodes u_i are dominated by color class $2\left\lceil \frac{i}{5} \right\rceil + 2$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of triangular belt graph T_n where $n \in \mathbb{Z}_+, 2 \leq n \leq 4$ is

$$\chi_d(T_n) = \begin{cases} 2\left\lceil \frac{n}{5} \right\rceil + 1 & when\ n = 2 \\ 2\left\lceil \frac{n}{5} \right\rceil + 2 & when\ n = 3, 4 \end{cases}$$

C.Theorem **:**

Every triangular belt graph T_n where$n \in \mathbb{Z}_+$, $n \geq 5$, has dominator chromatic number

$$\chi_d(T_n) = \begin{cases} 2\left\lceil \frac{n}{5} \right\rceil + 2 & when\ n(mod\ 5) \equiv 1\ or\ 2 \\ 2\left\lceil \frac{n}{5} \right\rceil + 3 & when\ n(mod\ 5) \equiv 0\ or\ 3\ or\ 4 \end{cases}$$

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Proof:

A triangular belt denoted by T_n [10] is obtained from the ladder graph L_n by adding the edges $\{ u_i v_{i+1} / 1 \le i \le n-1\}$.

For dominator coloring of T_n , the nodes are assigned colors as explained below

Case: 1 When $n(mod\ 5) \equiv 0$

For $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-1}, u_{5i-3}$ are painted with color 1, the nodes $v_{5i-2}, v_{5i}, u_{5i-4}$ are alloted color 2, the nodes u_{5i-2}, u_{5i} are painted with color 3, the nodes v_{5i-3} are alloted color $2(i+1)$ and the nodes u_{5i-1} are painted with color $2i+3$ respectively.

Then for $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-3}, v_{5i-2}, u_{5i-4}, u_{5i-3}$ are dominated by color class $2(i+1)$ and the nodes $v_{5i-1}, v_{5i}, u_{5i-2}, u_{5i-1}, u_{5i}$ are dominated by color class $2i+3$ respectively.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of triangular belt graph T_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 0$, is given by

$$\chi_d(T_n) = 2\left\lceil \frac{n}{5} \right\rceil + 3.$$

Case 2: When $n(mod\ 5) \equiv 1$

For $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-1}, u_{5i-3}$ are alloted color 1, the nodes $v_{5i-2}, v_{5i}, u_{5i-4}$ are painted with color 2, the nodes u_{5i-2}, u_{5i} are painted with color 3, the nodes v_{5i-3} are alloted color $2(i+1)$ and the nodes u_{5i-1} are painted with color $2i+3$ respectively. The nodes u_n and v_n are painted with color 2 and color $2\left\lceil \frac{n}{5} \right\rceil + 2$ respectively.

Then for $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-3}, v_{5i-2}, u_{5i-4}, u_{5i-3}$ are dominated by color class $2(i+1)$ and the nodes $v_{5i-1}, v_{5i}, u_{5i-2}, u_{5i-1}, u_{5i}$ are dominated by color class $2i+3$ respectively. The nodes v_n and u_n are dominated by color class $2\left\lceil \frac{n}{5} \right\rceil + 2$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of triangular belt graph T_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 0$, is given by

$$\chi_d(T_n) = 2\left\lceil \frac{n}{5} \right\rceil + 2.$$

Case 3: When $n(mod\ 5) \equiv 2$

For $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-1}, u_{5i-3}$ are alloted color 1, the nodes $v_{5i-2}, v_{5i}, u_{5i-4}$ are painted with color 2, the nodes u_{5i-2}, u_{5i} are painted with color 3, the nodes v_{5i-3} are alloted color $2(i+1)$ and the nodes u_{5i-1} are painted with color $2i+3$ respectively. The nodes u_{n-1}, u_n are given color 2 and 1 respectively. And the nodes v_{n-1}, v_n are alloted color 1 and color $2\left\lceil \frac{n}{5} \right\rceil + 2$ respectively.

Then for $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-3}, v_{5i-2}, u_{5i-4}, u_{5i-3}$ are dominated color class $2(i+1)$ and the nodes $v_{5i-1}, v_{5i}, u_{5i-2}, u_{5i-1}, u_{5i}$ are dominated by color class $2i+3$ respectively. The nodes v_{n-1}, u_{n-1}, v_n and u_n are dominated by color class $2\left\lceil \frac{n}{5} \right\rceil + 2$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of triangular belt graph T_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 0$, is given by $\chi_d(T_n) = 2\left\lceil \frac{n}{5} \right\rceil + 2$.

Case 4: When $n(mod\ 5) \equiv 3$

For $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-1}, u_{5i-3}$ are alloted color 1, the nodes $v_{5i-2}, v_{5i}, u_{5i-4}$ are alloted color 2, the nodes u_{5i-2}, u_{5i} are painted with color 3, the nodes v_{5i-3} are painted with color $2(i+1)$ and the nodes u_{5i-1} are alloted color $2i+3$ respectively. The nodes v_{n-2}, u_{n-1} are painted with color 1 and the nodes v_n, u_{n-2} are alloted color 2 respectively. The nodes v_{n-1}, u_n are given color $2\left\lceil \frac{n}{5} \right\rceil + 2$ and color $2\left\lceil \frac{n}{5} \right\rceil + 3$ respectively.

Then for $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-3}, v_{5i-2}, u_{5i-4}, u_{5i-3}$ are dominated by color class $2(i+1)$ and the nodes $v_{5i-1}, v_{5i}, u_{5i-2}, u_{5i-1}, u_{5i}$ are dominated by color class $2i+3$ respectively. The nodes $v_{n-2}, u_{n-2}, v_{n-1}, u_{n-1}$ and v_n are dominated by color class $2\left\lceil \frac{n}{5} \right\rceil + 2$. The node u_n is dominated by color class $2\left\lceil \frac{n}{5} \right\rceil + 3$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of triangular belt graph T_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 0$, is given by

$$\chi_d(T_n) = 2\left\lceil \frac{n}{5} \right\rceil + 3.$$

Case 5: When $n(mod\ 5) \equiv 4$

For $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-1}, u_{5i-3}$ are alloted color 1, the nodes $v_{5i-2}, v_{5i}, u_{5i-4}$ are painted with color 2, the nodes u_{5i-2}, u_{5i} are alloted color 3, the nodes v_{5i-3} are painted with color $2(i+1)$ and the nodes u_{5i-1} are alloted color $2i+3$ respectively. The nodes v_{n-3}, v_n, u_{n-2} are painted with color 1 and the nodes v_{n-1}, u_{n-3} are alloted color 2 respectively. The node v_{n-2} is given color $2\left\lceil \frac{n}{5} \right\rceil + 2$. The nodes u_{n-1}, u_n are given color 3 and color $2\left\lceil \frac{n}{5} \right\rceil + 3$ respectively.

Then for $1 \le i \le \left\lceil \frac{n}{5} \right\rceil$, the nodes $v_{5i-4}, v_{5i-3}, v_{5i-2}, u_{5i-4}, u_{5i-3}$ are dominated by color class $2(i+1)$ and the nodes $v_{5i-1}, v_{5i}, u_{5i-2}, u_{5i-1}, u_{5i}$ are dominated by color class $2i+3$ respectively. The nodes $v_{n-3}, u_{n-3}, v_{n-2}, u_{n-2}$ and v_{n-1} are dominated by color

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class $2\left\lceil\frac{n}{5}\right\rceil+2$. The nodes u_{n-2}, u_{n-1} and v_n are dominated by color class $2\left\lceil\frac{n}{5}\right\rceil+3$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of triangular belt graph T_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 0$, is given by $\chi_d(T_n) = 2\left\lceil\frac{n}{5}\right\rceil+3$

Hence the dominator chromatic number of triangular belt graph $TB(n)$ where $n \in \mathbb{Z}_+$, $n \geq 5$, is given by

$$\chi_d(T_n) = \begin{cases} 2\left\lceil\frac{n}{5}\right\rceil+2 & when\ n(mod\ 5) \equiv 1\ or\ 2 \\ 2\left\lceil\frac{n}{5}\right\rceil+3 & when\ n(mod\ 5) \equiv 0\ or\ 3\ or\ 4 \end{cases}$$

D.Lemma:

The domination number of triangular belt graph T_n where $n \in \mathbb{Z}_+$, $n \geq 2$, is

$$\gamma(T_n) = \begin{cases} 2\left\lceil\frac{n}{5}\right\rceil & when\ n(mod\ 5) \equiv 0\ or\ 3\ or\ 4 \\ 2\left\lceil\frac{n}{5}\right\rceil-1 & when\ n(mod\ 5) \equiv 1\ or\ 2 \end{cases}$$

Proof:

The triangular belt graph T_n has its node set $V = \{v_i, u_i\ /\ 1 \leq i \leq n\}$.

Case 1: When $n(mod\ 5) \equiv 0\ or\ 4$

Let the dominating set of the triangular belt graph T_n be

$$DS = \left\{v_{5i-3}, u_{5i-1}/1 \leq i \leq \left\lceil\frac{n}{5}\right\rceil\right\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the triangular belt graph T_n is given by

$$\gamma(T_n) = 2\left\lceil\frac{n}{5}\right\rceil.$$

Case 2: When $n(mod\ 5) \equiv 1$

Let the dominating set of the triangular belt graph T_n be

$$DS = \left\{v_{5i-3}, u_{5i-1}/1 \leq i \leq \left\lceil\frac{n}{5}\right\rceil-1\right\} \cup \{v_n\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the triangular belt graph T_n is given by

$$\gamma(T_n) = 2\left\lceil\frac{n}{5}\right\rceil-1.$$

Case 3: When $n(mod\ 5) \equiv 2$

Let the dominating set of the triangular belt graph T_n be

$$DS = \left\{v_{5i-3}\ /1 \leq i \leq \left\lceil\frac{n}{5}\right\rceil\right\} \cup \left\{u_{5i-1}/1 \leq i \leq \left\lceil\frac{n}{5}\right\rceil-1\right\}.$$

Clearly every vertesx in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the triangular belt graph T_n is given by

$$\gamma(T_n) = 2\left\lceil\frac{n}{5}\right\rceil-1.$$

Case 4: When $n(mod\ 5) \equiv 3$

Let the dominating set of the triangular belt graph $TB(n)$ be

$$DS = \left\{v_{5i-3}, u_{5i-1}/1 \leq i \leq \left\lceil\frac{n-3}{5}\right\rceil\right\} \cup \{v_{n-1}, u_n\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the triangular belt graph T_n is given by

$$\gamma(T_n) = 2\left\lceil\frac{n}{5}\right\rceil.$$

Hence the domination number of triangular belt graph $TB(n)$ where $n \in \mathbb{Z}_+$, $n \geq 2$, is

$$\gamma(T_n) = \begin{cases} 2\left\lceil\frac{n}{5}\right\rceil & when\ n(mod\ 5) \equiv 0\ or\ 3\ or\ 4 \\ 2\left\lceil\frac{n}{5}\right\rceil-1 & when\ n(mod\ 5) \equiv 1\ or\ 2 \end{cases}$$

E.Corollary:

Every triangular belt graph T_n where $n \in \mathbb{Z}_+, n \geq 2$ satisfies the relation

$$\chi_d = \begin{cases} \gamma+\chi-1 & when\ 2 \leq n \leq 4 \\ \gamma+\chi & when\ n \geq 5 \end{cases}$$

Proof: By applying proposition 3.1, theorem 3.2, 3.3 and lemma 3.4 we get the result.

IV. Dominator Chromatic Number of Alternate Triangular Belt Graph

A. Proposition:

The chromatic number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+, n \geq 2$, is given by

$$\chi(AT_n) = 3$$

B.Theorem:

Every triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $2 \leq n \leq 7, n \neq 6$, has dominator chromatic number

$$\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil+2.$$

Proof:

The alternate triangular belt AT_n [10] is obtained from the ladder graph L_n by adding the edges $u_{2i+1}v_{2i+2}$ $\forall\ 0 \leq i \leq n-1$ and $u_{2i+1}v_{2i}$ $\forall\ 1 \leq i \leq n-1$.

For dominator coloring of AT_n, the nodes are assigned colors as explained below

The nodes v_i for $1 \leq i \leq n$ are alloted color 1 when i is odd and color $\left\lceil\frac{i}{3}\right\rceil+2$ when i is even. The nodes u_i for $1 \leq i \leq n$ are painted with color 2 when i is odd and color 1 when i is even.

Case 1: when $n(mod\ 3) \equiv 0$

Then for $1 \leq i \leq \left\lceil\frac{n}{3}\right\rceil$, the nodes $v_{3i-2}, v_{3i-1}, v_{3i}, u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+2$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 0$, is given by $\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil+2$

Case 2: when $n(mod\ 3) \equiv 1$

Then for $1 \leq i \leq \left\lceil\frac{n}{2}\right\rceil-1$, the nodes $v_{2i-1}, v_{2i}, u_{2i-1}, u_{2i}$ are dominated by color class $i+2$. When n is odd, the

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Nodes v_n and u_n are dominated by color class $\left\lceil\frac{n}{3}\right\rceil + 2$ and when n is even, the nodes v_{n-1}, v_n, u_{n-1} and u_n are dominated by color class $\left\lceil\frac{n}{3}\right\rceil + 2$.

Thus by giving different color to every adjacent node it is observed that every node of the graph dominates all nodes of atleast one color class which is a dominator coloring of nodes. Hence the dominator chromatic number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 1$, is given by

$$\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil + 2$$

Case 3: when $n(mod\ 3) \equiv 2$

Then for $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes $v_{3i-2}, v_{3i-1}, v_{3i}, u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+2$. The nodes v_{n-1}, v_n, u_{n-1} and u_n are dominated by color class $\left\lceil\frac{n}{3}\right\rceil + 2$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 2$, is given by

$$\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil + 2.$$

Hence the dominator chromatic number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+, 2 \le n \le 7, n \ne 6$ is given by

$$\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil + 2.$$

C. Theorem:

Every alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $n = 6$ and $n \ge 8$, has dominator chromatic number

$$\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil + 3$$

Proof:

The alternate triangular belt AT_n [10] is obtained from the ladder graph L_n by adding the edges $u_{2i+1}v_{2i+2}$ $\forall\ 0 \le i \le n-1$ and $u_{2i+1}v_{2i}$ $\forall\ 1 \le i \le n-1$.

For dominator coloring of AT_n, the nodes are assigned colors as explained below

Case 1: when $n(mod\ 3) \equiv 0$

For $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes u_{3i-2}, u_{3i} are alloted color 2 when i is odd and color 1 when i is even, , the nodes u_{3i-1} are painted with color 1 when i is odd and color $i+3$ when i is even, the nodes v_{3i-2}, v_{3i} are alloted color 1 when i is odd and color 3 when i is even, and the nodes v_{3i-1} are painted with color $i+3$ when i is odd and color 1 when i is even.

Then for $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes $v_{3i-2}, v_{3i-1}, v_{3i}, u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+3$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 0$, is given by

$$\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil + 3.$$

Case 2: when $n(mod\ 3) \equiv 1$

For $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes u_{3i-2}, u_{3i} are allotted color 2 when i is odd and color 1 when i is even, , the nodes u_{3i-1} are painted with color 1 when i is odd and color $i+3$ when i is even, the nodes v_{3i-2}, v_{3i} are allotted color 1 when i is odd and color 3 when i is even, and the nodes v_{3i-1} are painted color $i+3$ when i is odd and color 1 when i is even. The node u_n is alloted color 2 when n is odd and color $\left\lceil\frac{n}{3}\right\rceil + 3$ when n is even. The node v_n is painted with color $\left\lceil\frac{n}{3}\right\rceil + 3$ when n is odd and color 3 when n is even.

Then for $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes $v_{3i-2}, v_{3i-1}, v_{3i}, u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+3$. The nodes v_n and u_n are dominated by color class $\left\lceil\frac{n}{3}\right\rceil + 3$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 1$, is given by

$$\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil + 3.$$

Case 3: when $n(mod\ 3) \equiv 2$

For $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes u_{3i-2}, u_{3i} are alloted color 2 when i is odd and color 1 when i is even, , the nodes u_{3i-1} are painted with color 1 when i is odd and color $i+3$ when i is even, the nodes v_{3i-2}, v_{3i} are alloted color 1 when i is odd and color 3 when i is even, and the nodes v_{3i-1} are painted with color $i+3$ when i is odd and color 1 when i is even. The node v_n is allotted color 1 when n is odd and color $\left\lceil\frac{n}{3}\right\rceil + 3$ when n is even. The node v_{n-1} is painted with color 3 when n is odd and color 1 when n is even. The node u_n is alloted color $\left\lceil\frac{n}{3}\right\rceil + 3$ when n is odd and color 1 when n is even. The node u_{n-1} is painted with color 1 when n is odd and color 2 when n is even.

Then for $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes $v_{3i-2}, v_{3i-1}, v_{3i}, u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+3$. The nodes v_{n-1}, v_n, u_{n-1} and u_n are dominated by color class $\left\lceil\frac{n}{3}\right\rceil + 3$.

Every neighboring node is given different colors and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $n(mod\ 3) \equiv 2$, is given by

$$\chi_d(AT_n) = \left\lceil\frac{n}{3}\right\rceil + 3.$$

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Hence the dominator chromatic number of alternate triangular belt graph $ATB(n)$ where $n \in \mathbb{Z}_+$, $n \geq 8$, is given by

$$\chi_d(AT_n) = \left\lceil \frac{n}{3} \right\rceil + 3.$$

D.Lemma:

The domination number of alternate triangular belt graph AT_n where $n \in \mathbb{Z}_+$, $n \geq 2$, is

$$\gamma(AT_n) = \left\lceil \frac{n}{3} \right\rceil$$

Proof:

The alternate triangular belt graph AT_n has its node set

$$V = \{v_i, u_i \ / 1 \leq i \leq n\}.$$

Case 1: when $n(mod\ 3) \not\equiv 1$

Let the dominating set of the alternate triangular belt graph AT_n be

$$DS = \left\{v_{3i-1}/1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil and\ i\ is\ odd\right\} \cup \left\{u_{3i-1}/1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil and\ i\ is\ even\right\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and the dominating set DS has the minimum cardinality. Hence the domination number of the alternate triangular belt graph AT_n is given by

$$\gamma(AT_n) = \left\lceil \frac{n}{3} \right\rceil.$$

Case 2: when $n(mod\ 3) \equiv 1$

Let the dominating set of the alternate triangular belt graph AT_n be

$$DS = \left\{v_{3i-1}/1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil and\ i\ is\ odd\right\} \cup \left\{u_{3i-1}/1 \leq i \leq \left\lceil \frac{n}{3} \right\rceil and\ i\ is\ even\right\} \cup \left\{ u_n / \left\lceil \frac{n}{3} \right\rceil\ is\ even\right\} \cup \left\{ v_n / \left\lceil \frac{n}{3} \right\rceil\ is\ odd\right\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the alternate triangular belt graph AT_n is given by $\gamma(AT_n) = \left\lceil \frac{n}{3} \right\rceil$.

E. Result:

Every alternate triangular belt graph $ATB(n)$ where $n \in \mathbb{Z}_+$, satisfies the relation

$$\chi_d = \begin{cases} \gamma + \chi - 1 & when\ 2 \leq n \leq 7\ and\ n \neq 6 \\ \gamma + \chi & when\ n = 6\ and\ n \geq 8 \end{cases}$$

Proof: By applying proposition 4.1, theorem 4.2, 4.3 and lemma 4.4 we get the result.

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