Dominator Coloring Of Certain Graphs

T.Manjula, R.Rajeswari , Anumita Dey , Krishna Deepika

Abstract: A proper vertex or node coloring of a graph where every vertex of the graph dominates all vertices of some color class is called the dominator coloring of the graph. The least number of colors used in the dominator coloring of a graph is called the dominator coloring number denoted by χ_d (G). The dominator chromatic number and domination number of closed sun graph, closed helm graph, generalized Flower snark, Double star snark and Watkins snark graph are derived and the relation between them are expressed in this paper.

Keywords: Coloring, Domination, Dominator Coloring

I. INTRODUCTION

A dominating set is a subset DS of the vertex or node set of graph *G* which is such that each node in the graph either belongs to D*S* or has a neighbour in D*S*[1]. The domination number $\gamma(G)$ is the cardinality of a smallest dominating set of *G*[1]. A proper coloring of a graph G is a function $f: V \to Z_+$ such that for $u, v \in V$, $f(u) \neq f(v)$ whenever u and v adjacent nodes in G. A dominator coloring of a graph G is a proper coloring of graph such that every node or vertex of G dominates all nodes of at least one color class. The minimum cardinality of colors used in the graph for dominator coloring is called the dominator coloring number denoted by $\chi_d(G)$. [2].

The concept of dominator coloring was introduced by Ralucca Michelle Gera in 2006 [2]. The relation between dominator coloring, proper coloring and domination number of different classes of graphs were shown in [3], [5]. The dominator coloring of prism graph, quadrilateral snake, triangle snake and barbell graph and M-Splitting graph and M-Shadow graph of Path graph were also studied in various papers [6], [7], [8]. The algorithmic aspects of dominator coloring in graphs have been discussed by Arumugam S et.al. in [4].

The closed sun graph [9] denoted by CS_n with $2n$ nodes is constructed as follows: A complete graph K_n with the nodes $\{v_1, v_2, \dots, v_n\}$, is surrounded by a cycle C_n with nodes $\{u_1, u_2, \dots, u_n\}$. Then the edges $u_i v_i$ and $u_i v_{i+1}$ (with $v_{n+1} = v_1$) are added.

A closed Helm graph or a belt graph [11] is constructed from the Helm graph by joining its outer vertices. It has 2n+1 vertices 4n edges. It is denoted by CH_n.

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Snarks [10] play a central role for several well known conjectures in graph theory.

The Flower snark [10] was introduced by Rufus Isaacs in 1975 and they form an infinite family of snarks. The Double Star Snark [10] discovered by Rufus Isaacs is a 30 vertex graph with 45 edges. The Watkins Snark [10] discovered by John J.Watkin in 1989 is a 50 vertex graph with 75 edges. They are connected, bridgeless cubic graphs with chromatic index equal to 4 and are non-planar and non-Hamiltonian.

The flower snark J_n [10] has $4n$ nodes and $6n$ edges. It is constructed as follows: The n copy of the star graph on 4 nodes is taken. The central node of each star is denoted by a_i and the outer nodes by b_i, c_i and d_i. This results in a disconnected graph on $4n$ nodes with $3n$ edges (a_ib_i, a_ic_i and a_id_i for $1 \leq i \leq n$).

- A cycle of length n is constructed by connecting the n nodes $b_1 \dots bn$.
- Finally a cycle of length $2n$ is constructed by connecting the $2n$ nodes $c_1, c_2 \dots, c_n, d_1, d_2 \dots d_n$.

For a flower snark J_n to have the required properties, n should be odd. The flower snark J_3 is also known as the Tietze's graph named after Heinrich Franz Tietze.

In this paper the domination number and dominator chromatic number of closed Sun graph, closed Helm graph, Flower snark, Double star snark and Watkins snark is obtained and a relation between the dominator chromatic number, chromatic number and domination number is expressed.

II. Dominator Chromatic Number of Closed Sun Graph

A. Proposition :

The chromatic number of a closed sun graph CS_n where $n \in \mathbb{Z}_+, n \geq 3$, is given by $\chi(CS_n) = n$

B. Theorem:

Every closed sun graph CS_n where $n \in \square_+, n \geq 3$, has dominator chromatic number $\chi_d(CS_n) = n$

Proof:

The closed sun graph CS_n with $2n$ nodes is constructed as follows: A complete graph K_n with the nodes $\{v_1, v_2, \dots v_n\}$, is surrounded by a cycle with nodes $\{u_1, u_2, \dots u_n\}$. Then the edges $u_i v_i$ and $u_i v_{i+1}$ (with $v_{n+1} = v_1$) are added.

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Let the node set and edge set of the closed Sun graph CS_n be

$$V = \{v_i, u_i \,/\, 1 \le i \le n\}$$
$$E = \{v_i v_j / \ 1 \le i \le n-1,\ i+1 \le j \le n\} \cup \{u_i v_i,\ u_i v_{i+1}, \frac{u_i u_{i+1}}{}\ 1 \le i \le n-1\} \cup \{\, u_n v_n,\ u_n v_1,\ u_1 u_n \,\}.$$

The procedure below explains the dominator coloring of nodes.

For $1 \le i \le n$, the nodes v_i are painted with color i. The node u_1 is allotted color n. And for $2 \le i \le n$ the nodes u_i are painted with color $i-1$ respectively.

Then for $1 \le i \le n$, the nodes u_i are dominated by color class i respectively. And for $3 \le i \le n$, the nodes v_i are dominated by color class $i-2$ respectively. The nodes v_1, v_2 are dominated by color class $n-1, n-2$ respectively.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of closed Sun graph CS_n where $n \in \square_+$, $n > 3$ is given by $\chi_d(CS_n) = n$.

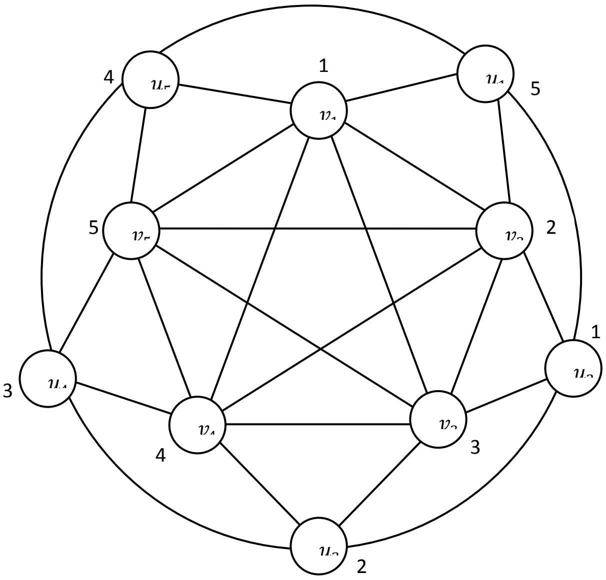

Figure 1: Dominator chromatic number of closed Sun graph CS_5 is 5. i.e, $\chi_d(CS_5) = 5$

C. Lemma:

The domination number of closed Sun graph CS_n where $n \in \square_+$, $n \ge 3$ is given by $\gamma(CS_n) = \left\lceil \frac{n}{2} \right\rceil$

Proof:

The node set of the closed Sun graph CS_n is
$V = \{v_i, u_i \,/\, 1 \le i \le n\}$.

Let the dominating set of the closed Sun graph be $DS = \left\{v_{2i-1} / 1 \le i \le \left\lceil \frac{n}{2} \right\rceil\right\}$. Clearly every node in *V-DS* is adjacent to atleast one node of *DS* and the dominating set *DS* has the minimum cardinality. Hence the domination number of the closed Sun graph is given by $\gamma(CS_n) = \left\lceil \frac{n}{2} \right\rceil$.

D.Corollary:

Every closed Sun graph CS_n where $n \in \square_+, n \ge 3$ satisfies the relation

$$\chi_d = \gamma + \left\lfloor \frac{n}{2} \right\rfloor.$$

Proof: By applying theorem 2.2 and lemma 2.3 we get the result.

III. Dominator Chromatic Number of Closed Helm Graph

A. Proposition:

The chromatic number of a closed Helm graph denoted by CH_n where $n \in \mathbb{Z}_+, n \ge 3$, is given by

$$\chi(CH_n) = \begin{cases} 4 & when\ n\ is\ odd \\ 3 & when\ n\ is\ even \end{cases}.$$

B. Theorem:

Every closed Helm graph denoted by CH_n where $n \ge 4$ and n is even, has dominator chromatic number

$$\chi_d(CS_n) = \left\lceil \frac{n}{3} \right\rceil + 3.$$

Proof:

A closed Helm graph or a belt graph [11] is constructed from the Helm graph by joining its outer vertices. It has 2n+1 vertices and 4n edges. It is denoted by CH_n.

Let the node set and edge set of the closed Sun graph CS_n be

$$V = \{w\} \cup \{v_i, u_i \,/\, 1 \le i \le n\}$$
$$E = \{v_1 v_n, u_1 u_n\} \cup \left\{v_i v_j, \frac{u_i u_j}{}\ 1 \le i \le n-1,\right\} \cup \{u_i v_i, w v_i / \ 1 \le i \le n\}.$$

The procedure below explains the dominator coloring of nodes.

Case 1: When n is even and $n(mod\ 3) \equiv 0$

For $1 \le i \le n$, the nodes v_i are allotted color 2 when i is odd and color 3 when i is even. The node w is painted with color 1. For $1 \le i \le \left\lceil \frac{n}{3} \right\rceil$ the nodes u_{3i-2}, u_{3i} are allotted color 3 when i is odd and color 2 when i is even and the nodes u_{3i-1} are painted with color $i+3$ respectively.

The node w and for $1 \le i \le n$, the nodes v_i are dominated by color class 1. Then for $1 \le i \le \left\lceil \frac{n}{3} \right\rceil$, the nodes $u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+3$ respectively.

Every neighboring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of closed Helm graph CH_n where $n \ge 4\ and\ n\ is\ even$ is given by

$$\chi_d(CH_n) = \left\lceil \frac{n}{3} \right\rceil + 3.$$

Case 2: When n is even and $n(mod\ 3) \equiv 1$

For $1 \le i \le n$, the nodes v_i are allotted color 2 when i is odd and color 3 when i is even. The node w is painted with color 1. For $1 \le i \le \left\lceil \frac{n}{3} \right\rceil$ the nodes u_{3i-2}, u_{3i} are pinted with color 3 when i is odd and color 2 when i is even and the

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nodes u_{3i-1} are allotted color $i+3$ respectively. The node u_n is painted with color $\left\lceil\frac{n}{3}\right\rceil + 3$.

The node w and for $1 \le i \le n$, the nodes v_i are dominated by color class 1. Then for $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes $u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+3$ respectively. The node u_n is dominated by color class $\left\lceil\frac{n}{3}\right\rceil + 3$.

Every neighboring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of closed Helm graph CH_n where $n \ge 4 \; and \; n \; is \; even$ is given by

$$\chi_d(CH_n) = \left\lceil\frac{n}{3}\right\rceil + 3.$$

Case 3: When n is even and $n(mod\ 3) \equiv 2$

For $1 \le i \le n$, the nodes v_i are allotted color 2 when i is odd and color 3 when i is even. The node w is painted with color 1. For $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$ the nodes u_{3i-2}, u_{3i} are allotted color 3 when i is odd and color 2 when i is even and the nodes u_{3i-1} are painted with color $i+3$ respectively. The nodes u_{n-1} and u_n are allotted color 3 and $\left\lceil\frac{n}{3}\right\rceil + 3$ respectively.

The node w and for $1 \le i \le n$, the nodes v_i are dominated by color class 1. Then for $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes $u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+3$ respectively. The nodes u_{n-1} and u_n are dominated by color class $\left\lceil\frac{n}{3}\right\rceil + 3$.

Every neighboring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of closed Helm graph CH_n where $n \ge 4 \; and \; n \; is \; even$ is given by

$$\chi_d(CH_n) = \left\lceil\frac{n}{3}\right\rceil + 3.$$

Hence the dominator coloring number of closed Helm graph CH_n where $n \ge 4 \; and \; n \; is \; even$ is given by $\chi_d(CH_n) = \left\lceil\frac{n}{3}\right\rceil + 3$.

C.Theorem :

Every closed Helm graph denoted by CH_n where $n \ge 5$ and n is odd, has dominator chromatic number

$$\chi_d(CS_n) = \begin{cases} \left\lceil\frac{n}{3}\right\rceil + 3 & w\square en\ n(mod3) \equiv 1 \\ \left\lceil\frac{n}{3}\right\rceil + 4 & w\square en\ n(mod3) \equiv 0 \; or \; 2 \end{cases}.$$

Proof:

Let the node set and edge set of the closed Sun graph CS_n be

$$V = \{w\} \cup \{v_i, u_i \;/\; 1 \le i \le n\}$$
$$E = \{v_1 v_n, u_1 u_n\} \cup \{v_i v_j,\; u_i u_j /\; 1 \le i \le n-1,\} \cup \{u_i v_i, w v_i /\; 1 \le i \le n\}.$$

The procedure below explains the dominator coloring of nodes.

Case 1: When n is even and $n(mod\ 3) \equiv 1$

For $1 \le i \le n$, the nodes v_i are allotted color 2 when i is odd and color 3 when i is even. The node w is painted with color 1. For $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$ the nodes u_{3i-2}, u_{3i} are allotted color 3 when i is odd and color 2 when i is even and the nodes u_{3i-1} are painted with color $i+4$ respectively. The nodes u_{n-3} and u_n are allotted color 3, the nodes u_{n-1}, u_{n-2} are painted with color 4 and color $\left\lceil\frac{n}{3}\right\rceil + 3$ respectively.

The node w and for $1 \le i \le n$, the nodes v_i are dominated by color class 1. Then for $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes $u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+4$ respectively. The node u_n is dominated by color class 4. Every neighboring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of closed Helm graph CH_n where $n \ge 4 \; and \; n \; is \; odd$ is given by

$$\chi_d(CH_n) = \left\lceil\frac{n}{3}\right\rceil + 3.$$

Case 2: When n is odd and $n(mod\ 3) \equiv 0$

For $1 \le i \le n-1$, the nodes v_i are allotted color 2 when i is odd and color 3 when i is even. The nodes v_n and w is painted with color 4 and 1 respectively. For $1 \le i \le \left\lceil\frac{n}{3}\right\rceil - 1$ the nodes u_{3i-2}, u_{3i} are allotted color 3 when i is odd and color 2 when i is even and the nodes u_{3i-1} are painted with color $i+4$ respectively. The nodes u_{n-2}, u_{n-1} and u_n are allotted color 3, $\left\lceil\frac{n}{3}\right\rceil + 4$ and 2 respectively.

The node w and for $1 \le i \le n$, the nodes v_i are dominated by color class 1. Then for $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$, the nodes $u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+4$ respectively.

Every neighboring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of closed Helm graph CH_n where $n \ge 4 \; and \; n \; is \; odd$ is given by

$$\chi_d(CH_n) = \left\lceil\frac{n}{3}\right\rceil + 4.$$

Case 3: When n is odd and $n(mod\ 3) \equiv 2$

For $1 \le i \le n-1$, the nodes v_i are allotted color 2 when i is odd and color 3 when i is even. The nodes v_n and w is painted with color 4 and 1 respectively. For $1 \le i \le \left\lceil\frac{n}{3}\right\rceil$ the nodes u_{3i-2}, u_{3i} are allotted color 3 when i is odd and color 2 when i is even and the nodes u_{3i-1} are allotted color $i+4$ respectively. The nodes u_{n-1} and u_n are painted with color 4 and $\left\lceil\frac{n}{3}\right\rceil + 4$ respectively.

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The node w and for $1 \le i \le n$, the nodes v_i are dominated by color class 1. Then for $1 \le i \le \left\lceil \frac{n}{3} \right\rceil$, the nodes $u_{3i-2}, u_{3i-1}, u_{3i}$ are dominated by color class $i+4$ respectively. The nodes u_{n-1} and u_n are dominated by color class $\left\lceil \frac{n}{3} \right\rceil + 4$.

Every neighboring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of closed Helm graph CH_n where $n \ge 4 \text{ and } n \text{ is odd}$ is given by

$$\chi_d(CH_n) = \left\lceil \frac{n}{3} \right\rceil + 4.$$

Hence the dominator coloring number of closed Helm graph CH_n where $n \ge 4 \text{ and } n \text{ is odd}$ is given by

$$\chi_d(CH_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 3 & \text{w□en } n(mod3) \equiv 1 \\ \left\lceil \frac{n}{3} \right\rceil + 4 & \text{w□en } n(mod3) \equiv 0 \text{ or } 2 \end{cases}.$$

*D. Lemma***:**

The domination number of closed helm graph denoted by CH_n where $n \in \square_+$, $n \ge 3$ is given by

$$\gamma(CH_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 1 & \text{w□en } n \ne 4 \\ \left\lceil \frac{n}{3} \right\rceil & \text{w□en } n = 4 \end{cases}$$

Proof:

The closed helm graph denoted by CH_n has its node set

$$V = \{w\} \cup \{v_i, u_i \,/\, 1 \le i \le n\}.$$

Case 1: When $n \ge 3 \text{ and } n(mod\ 3) \not\equiv 1$

Let the dominating set of the closed helm graph be

$$DS = \left\{w, u_{3i-1} / 1 \le i \le \left\lceil \frac{n}{3} \right\rceil\right\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and the dominating set DS has the minimum cardinality. Hence the domination number of the closed helm graph denoted by $CH_n$is given by

$$\gamma(CH_n) = \left\lceil \frac{n}{3} \right\rceil + 1.$$

Case 2: When $n(mod\ 3) \equiv 1 \text{ and } n \ne 4$

Let the dominating set of the closed helm graph be

$$DS = \left\{u_{3i-1} / 1 \le i \le \left\lfloor \frac{n}{3} \right\rfloor\right\} \cup \{w, v_n\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the closed helm graph denoted by CH_n is given by

$$\gamma(CH_n) = \left\lceil \frac{n}{3} \right\rceil + 1.$$

Case 3: When $n = 4$

Let the dominating set of the closed helm graph be

$$DS = \{u_2, v_4\}.$$

Clearly every vertex in $V - DS$ is adjacent to atleast one node of DS and and the dominating set DS has the minimum cardinality. Hence the domination number of the closed helm graph denoted by CH_n is given by

$$\gamma(CH_n) = \left\lceil \frac{n}{3} \right\rceil.$$

The domination number of closed helm graph denoted by CH_n where $n \in \mathbb{Z}_+$, $n \ge 3$ is given by

$$\gamma(CH_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil + 1 & \text{w□en } n \ge 3 \text{ and } n \ne 4 \\ \left\lceil \frac{n}{3} \right\rceil & \text{w□en } n = 4 \end{cases}$$

E. Corollary

Every closed helm graph denoted by CH_n where $n \in \mathbb{Z}_+$, $n \ge 5$ satisfies the relation

$$\chi_d = \begin{cases} \gamma + \chi - 2 & \text{w□en } n(mod\ 6) \equiv 1 \\ \gamma + \chi - 1 & \text{w□en } n(mod\ 6) \not\equiv 1 \end{cases}$$

Proof: By applying proposition 3.1, theorem 3.2, 3.3 and lemma 3.4 we get the result.

IV. DOMINATOR CHROMATIC NUMBER OF FLOWER SNARK

*A. Proposition***:**

The chromatic number of Flower Snark J_n where n is odd and $n \ge 3$, is

$$\chi(J_n) = 3.$$

B.Theorem:

Every Flower snark J_n when n is odd has dominator chromatic number

$$\chi_d(J_n) = n + 3.$$

Proof:

The n copies of a star graph on 4 nodes are taken. The central node of each star is denoted by a_i and the outer nodes by b_i, c_i and d_i. Thus the node set of a flower snark is given by

$$V = \{a_i, b_i, c, d_i /\ 1 \le i \le n\}.$$

The 6n edges of the flower snark are as follows: There are n copies of 3 edges $\{a_i b_i, a_i c, a_i d_i /\ 1 \le i \le n\}$ which gives 3n edges. Then a cycle of length n is constructed by connecting the n nodes $b_1, \ldots b_n$ which adds n edges. Finally a cycle of length 2n is constructed by connecting the 2n nodes $c_1, c_2 \ldots, c_n, d_1, d_2 \ldots d_n$ which adds another 2n edges.

The following procedure gives dominator coloring of nodes

For $1 \le i \le n$, the nodes a_i are allotted color i+3. For $1 \le i \le n-1$, the nodes b_i are painted with color 1 when i is odd and color 2 when i is even. The node b_n is allotted color 3. For $1 \le i \le n$, the nodes c_i are assigned color 1 when i is odd and color 2 when i is even. For $1 \le i \le n$, the nodes d_i are allotted color 2 when i is odd and color 1 when i is even.

Then the nodes a_i, b_i, c_i, d_i for $1 \le i \le n$ are dominated by color class $i+3$ respectively.

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Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of flower snark J_n where n is odd and $n \geq 3$, is given by

$$\chi_d(G) = n + 3.$$

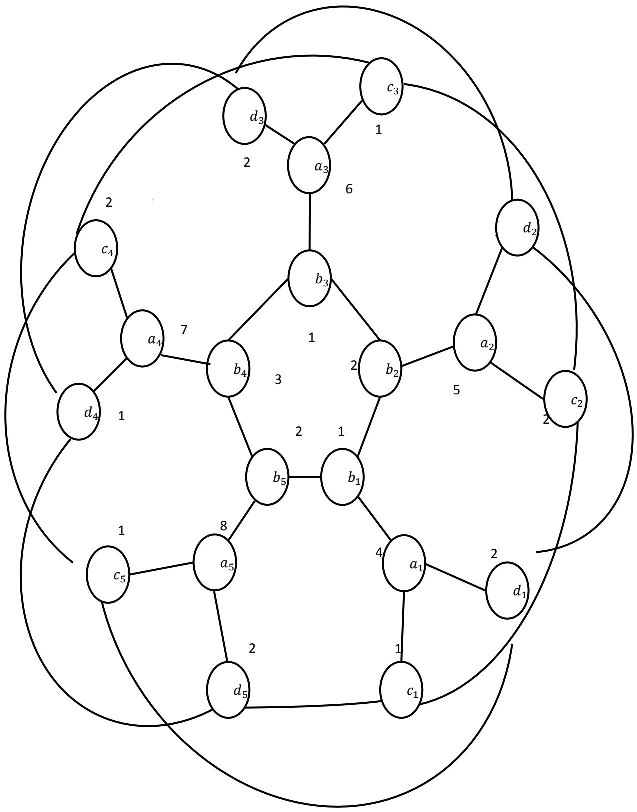

Figure 2: Dominator chromatic number of Flower snark J_5 is 8. i.e, $\chi_d(J_5) = 8$

C.Lemma:

Every Flower snark J_n when n is odd has domination number

$$\gamma(J_n) = n.$$

Proof:

The node set of a flower snark is given by

$$V = \{a_i, b_i, c, d_i / \ 1 \leq i \leq n\}.$$

Let $DS = \{a_i \ / \ 1 \leq i \leq n\}$. Clearly every node in $V - DS$ has atleast a neighbor in DS and the dominating set DS has the minimum cardinality.

Hence the domination number of Flower Snark J_n when n is odd is given by

$$\gamma(J_n) = n.$$

D. Corollary:

The Flower Snark J_n when n is odd satisfies the relation

$$\chi_d(J_n) = \gamma(J_n) + \chi(J_n).$$

Proof:

The result follows from proposition 4.1, theorem 4.2 and lemma 4.3.

V. DOMINATOR CHROMATIC NUMBER OF WATKINS SNARK

A.Proposition:

The chromatic number of Watkins snark is $\chi(G) = 3$.

B.Theorem :

The dominator chromatic number of Watkins Snark is given by $\chi_d(G) = \left\lceil \frac{n}{3} \right\rceil$.

Proof:

The number of nodes of Watkins Snark is 50 and the number of edges is 75. i.e., $n = 50$ *and* $e = 75$. The node set and the edge set of Watkins snark is given by

$$V = \{v_i / 1 \leq i \leq n\}.$$

$E(G) = \{v_i v_{i+1} / 1 \leq i \leq n, \ i \neq 10,20,30,40\} \cup \{v_{10i-9}v_{10i-5}, v_{10i-8}v_{10i-3}, v_{10i-6}v_{10i-2}, v_{10i-4}v_{10i} / 1 \leq i \leq 5\}$
$\cup \{v_{10i-1}v_{10i+1} / 1 \leq i \leq 4\} \cup \{v_{10i-7}v_{10i+20} / 1 \leq i \leq 3\} \cup \{v_{10i}v_{10i+23} / 1 \leq i \leq 2\} \cup \{v_1 v_{n-1}\}$

The following procedure gives dominator coloring of nodes

For $1 \leq i \leq \left\lfloor \frac{n}{12} \right\rfloor$, the nodes v_{3i-2} are allotted color 1. For $1 \leq i \leq \left\lceil \frac{n}{8} \right\rceil$, the nodes v_{3i+9}, v_{3i+29} are painted with color 1, the nodes v_{3i+19} are allotted color 2 respectively. For $1 \leq i \leq \frac{n}{10}$, the nodes v_{3i+5} are painted with color 2. For $1 \leq i \leq \frac{n}{25}$, the nodes v_{3i} are painted with color 2. For $1 \leq i \leq \left\lfloor \frac{n}{20} \right\rfloor$, the nodes $v_{3i-1+\left\lfloor \frac{i-1}{2} \right\rfloor}, v_{3i+10}, v_{3i+20}, v_{3i+30}, v_{3i+39}, v_{3i+40}$ are allotted color $i+2, i+5, i+8, i+11, \ 2, \ i+14$ respectively.

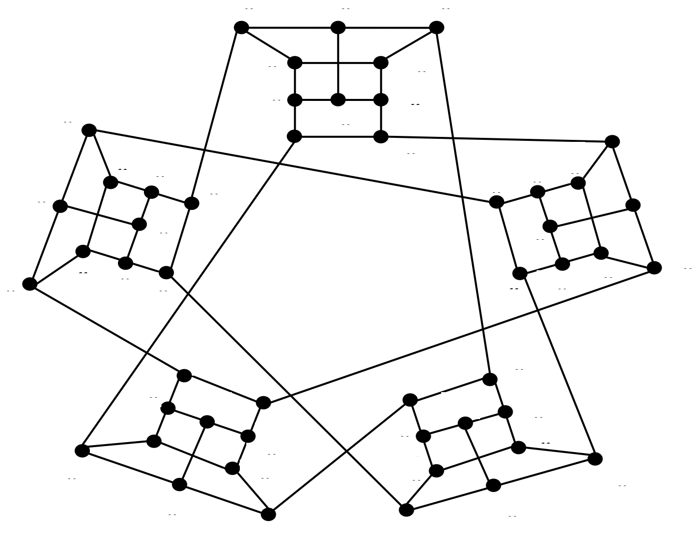

Figure 3: Dominator chromatic number of Watkins snark is $\chi_d(G) = 17$

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For $1 \le i \le \frac{n}{25}$, the nodes $v_{3i-2}, v_{3i-1}, v_{3i}$ are dominated by color class $i+2$ and the nodes v_{i+6} are dominated by color class $2i+1$ respectively. For $1 \le i \le \left\lceil \frac{n}{12} \right\rceil$, the nodes $v_{3i+6}, v_{3i+7}, v_{3i+8}$ are dominated by color class $i+4$ respectively. For $1 \le i \le \left\lceil \frac{n}{20} \right\rceil$, the nodes $v_{3i+19}, v_{3i+20}, v_{3i+21}$ are dominated by color class $i+8$, the nodes $v_{3i+29}, v_{3i+30}, v_{3i+31}$ are dominated by color class $i+11$, the nodes $v_{3i+39}, v_{3i+40}, v_{3i+41}$ are dominated by color class $i+14$, the nodes v_{10i+11} are dominated by color class $3i+5$ respectively.

Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of Watkins snark is given by

$$\chi_d(G) = \left\lceil \frac{n}{3} \right\rceil$$

C.Lemma :

The domination number of Watkins snark is given by

$$\gamma(G) = \left\lceil \frac{n}{3} \right\rceil - 2.$$

Proof:

The node set of a Watkins snark is given by

$$V = \{v_i / \ 1 \le i \le n\}.$$

Let $DS = \left\{v_{10i-9}, v_{10i-7}, v_{10i-3} \ / \ 1 \le i \le \left\lceil \frac{n}{10} \right\rceil\right\}$.

Clearly every node in $V - DS$ has atleast a neighbor in DS and the dominating set DS has the minimum cardinality.

Hence the domination number of Watkins snark is given by

$$\gamma(G) = \left\lceil \frac{n}{3} \right\rceil - 2.$$

D. Corollary:

The Watkins snark satisfies the relation

$$\chi_d(G) = \gamma(G) + \chi(G) - 1.$$

Proof:

By applying proposition 5.1, theorem 5.2 and lemma 5.3, the result follows.

VI. DOMINATOR CHROMATIC NUMBER OF DOUBLE STAR SNARK

A.Proposition:

The chromatic number of double star snark is

$$\chi(G) = 3.$$

B.Proposition :

The domination number of double star snark is

$$\gamma(G) = 8.$$

C.Theorem:

The dominator chromatic number of double star snark is given by

$$\chi_d(G) = \frac{n}{3} + 1.$$

Proof:

The number of nodes of the double star snark is 30 and the number of edges is 45. i.e., $n = 30$ *and* $e = 45$.

The node set is given by $V = \{v_i / 1 \le i \le n\}$. And the edge set is given by

$$E(G) = \left\{v_i v_{i+1} / \ 1 \le i \le n, i \ne \frac{n}{2}\right\} \cup \left\{\frac{v_{3i-2}v_{3i}}{} 1 \le i \le 4\right\} \cup \left\{v_{3i-4}v_{3i+14}, \frac{v_{3i+13}v_{3i+18}}{} 1 \le i \le 3\right\} \cup \{v_{15i-14}v_{15i}, v_{2i+1}v_{13i}, v_{7i+4}v_{n-i} / \ 1 \le i \le 2\} \cup \{v_{n-5}v_n\}$$

The following procedure gives dominator coloring of nodes

For $1 \le i \le \left\lceil \frac{n}{6} \right\rceil$, the nodes v_{3i-2}, v_{2i+18} are alloted color 1 and the nodes v_{3i-1} are alloted $i+2$ respectively. For $1 \le i \le \left\lceil \frac{n}{4} \right\rceil$, the nodes v_{3i} are painted with color 2 . For $1 \le i \le \frac{n}{15}$ the nodes $v_{3i+13}, v_{3i+24}, v_{2i+21}$ are alloted color $i+7, i+9, 2$ respectively. The nodes $v_{\frac{n}{2}+2}, v_{n-1}$ are painted with color $1, 2$ respectively.

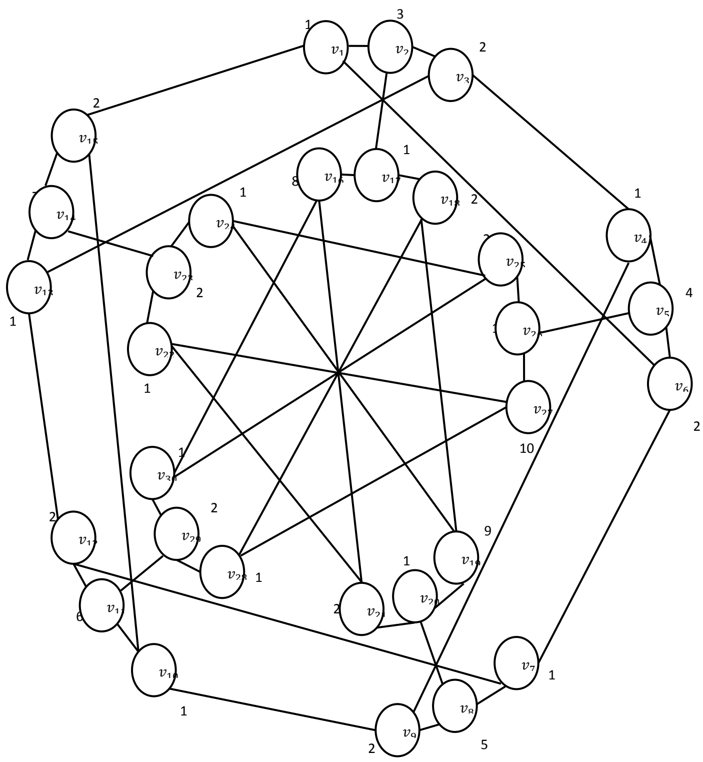

Figure 4: Dominator chromatic number of Double star snark is, $\chi_d(G) = 11$

Then for $1 \le i \le \left\lceil \frac{n}{6} \right\rceil$, the nodes $v_{3i-2}, v_{3i-1}, v_{3i}$ are dominated by color class $i+2$ respectively. For $1 \le i \le \frac{n}{10}$, the nodes v_{3i+15} are dominated by color class $\frac{n}{3} - 1$ w□en $i (mod\ 2) \equiv 1$ and color class $\frac{n}{3} - 2$ w□en $i (mod\ 2) \equiv 0$ and the nodes v_{3i+13}, v_{3i+14} are dominated by color class $i+7, 2i+1$ respectively. For $1 \le i \le \frac{n}{15}$ the nodes v_{3i+23}, v_{3i+24} are dominated by color class $\frac{n}{3} + i - 1$ and the nodes v_{3i+22} are dominated by color class $\frac{n}{3} - i + 2$.

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Every neighbouring node is given different color and also it is observed that every node of the graph dominates all the nodes of atleast one color class. Thus it is a dominator coloring of nodes and the dominator coloring number of Double star snark graph is

$$\chi_d(G) = \frac{n}{3} + 1.$$

D. Result :

The Double star Snark G satisfies the relation

$$\chi_d(G) = \gamma(G) + \chi(G).$$

Proof:

By applying proposition 6.1, 6.2 and theorem 6.3 we get the result.

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