

Optimization of Regular Lattice Structure for Maximum Shear Capacity

Arunraj E, Hemalatha G, Ramya M, Arun Solomon A, Elizabeth Amudhini Stephen

Abstract: The present study was undertaken to optimize the shear strength of the regular hexagonal, triangle and square lattice structure which can be used in the exterior beam column joint. Here, shear strength and shear stress values are compared to the normal exterior beam column joint which is detailed as per IS13920:2016 and IS456:2000. Optimization of the shape of the unit cell was carried out to obtain maximum shear stress. The optimum shear stress of lattice unit cell is found by varying the thickness and length of lower limit and upper limit. The unit cell of 10mm is taken as a maximum length and the thickness is varied for various shapes. Genetic Algorithm which is a non – traditional optimization is used for optimizing the shear stress.

Keywords: Regular lattice; Genetic algorithm; Shear strength

I. INTRODUCTION

Strength to weight ratio is the most important factor for enhancing the efficiency of the structure subjected to external forces. Metal foams, honey comb sandwiches and lattice structures are lightweight structure configurations that have been popularly used in engineering application (Gibson and Ashby, 1997), due to heat insulation, high relative stiffness, strength and energy absorbability of their superior properties.

Usually there is no filling between the lattice cell frame of 2D or 3D, because lattice is a molecule lattice (Evans et al., 2001). This reduces large volume of interface and sufficient quantity of mass can be reduced. Due to this the relative strength and relative stiffness of the repetitive lattice increases compared to the regular metal foam, based on the bending and shear deformation of cell wall analytical model is formulated to prevent the equivalent transverse shear modulus and strength of honeycomb cores (Shi-Dong Pan et al., 2007) and not only the core wall length, thickness and angle affects the equivalent transverse shear modulus and strength of honeycomb but also the height of the core. In honeycombs, the theoretical calculation of the direct stiffness

from the elastic modulus of the lattice material and the geometry of the honeycomb is taken and it is directly proportional to the average density of core. In practical side of test on honeycomb core, based on the faces and measure, directly or indirectly, the relative shear displacement of the faces the shear stress is applied. The direction of the resultant shear displacement, Δ , will not coincide with the direction of the applied shear force due to the orthotropic nature of the honeycomb core (S. Kelsey et al., 1958). For the deformation of the honeycomb cell by flexure, stretching and hinging the theoretical model developed from the elastic constants of the honeycombs (Masters et al., 1996) for different shape like hexagonal and re-entrant. In three deformation mechanism regular hexagonal cell are truly isotropic in the 1, 2- plane and re-entrant call are highly anisotropic. (Soroohan et al., 2016) found that the equivalent properties of the honeycomb core by analytical and numerical relationship for the bending and torsion loads.

When the beam-column joint are subjected to seismic load the beams are subjected to the reversal moment. Due to this the moment can produce high shear force, breaking of joint and cracking. If the stiffness of the joint decreases it causes the story drift and damage of the structure. By considering the above problem the joint of the structure should be light in weight and enforcing to load carrying capacity.

II. LATTICE STRUCTURE

Pattern of repeat is called lattice structure and it is two types; namely regular lattice structure and irregular or stochastic lattice structure as shown in Figure 2.1. In regular lattice structures there will be uniform distribution, uniform shape and geometry of cell. In stochastic lattice structure there will be random distribution, different shape and geometry of cell. Balance of strength, stiffness, cost, durability and relative static and dynamic properties requires the regular shape and choice of material for a given problem. Regular lattice structure can offer desirable properties with multifunctional material. The lattice are commonly constructed by 3D printing or duplicating three dimensional mesoscale unit cell at the scale of few mm. Based on the relative density, stud aspect ratio (radius/length), unit cell geometrical configuration, unit cell size, properties of parent material and rate of loading the stiffness and strength of the lattice will be obtained. Relative density will be considered to follow uniform distribution in the structure, hence relative density is the important feature in cellular structure.

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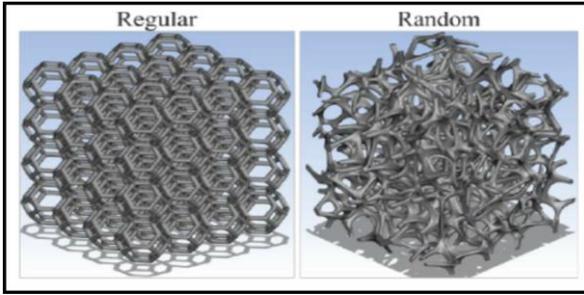


Fig. 1 Lattice Structure

III. ANALYTICAL INVESTIGATION

A. Identification of Parameters

The parameters like shape, thickness and length of the regular lattice structure are optimized for maximum shear capacity. Here, the chosen shapes are square, hexagonal and triangular regular lattice structure as shown in figure 2, 3 and 4. The shear strength of a regular lattice structure is found by varying the dimensions, namely the length and thickness analytically, optimizing by using genetic algorithm in MATLAB.

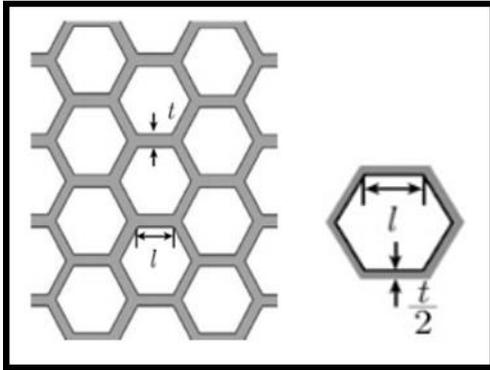


Fig. 2 Hexagonal Regular Lattice

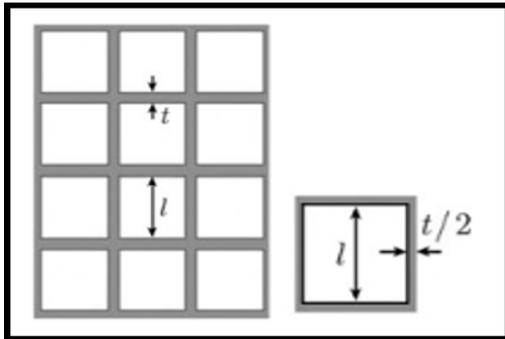


Fig. 3 Square Regular Lattice

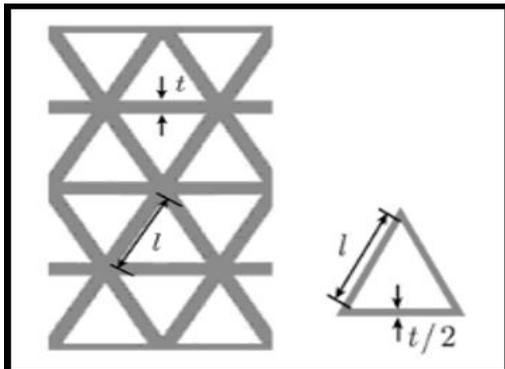


Fig. 4 Triangular Regular Lattice

B. Formulation of Objective Function and Constraints

To find the shear strength of the regular lattice structure as per code, first the objective function is to be generated and constrains for the normal exterior beam column joint which gives the shear strength of the joint are determined from the literature. The objective function for different shapes like square, hexagonal and triangular are derived. By comparing the exterior beam column joint and regular lattice structure, the shear strength can be arrived for the chosen parameters of regular lattice, which gives the exact shear strength as the normal exterior beam column joint shear strength.

C. Objective Function and Constrains for Normal Joint

As per the Indian Standard (13920:1993 and IS 456:2000) the objective function for normal joint is formulated, shear strength of joint is taken as sum of shear strength of concrete and shear strength of reinforcement. (Equation 1)

Shear strength of joint = shear strength of concrete + shear strength of reinforcement

$$\tau_{je} = \tau_c + V_{us} \tag{1}$$

For exterior beam column joints:

(As per 13920:2016- clause: 9.1.1; As per 456:2000- clause:40.4)

$$\tau_c = 1.2A_{ej}\sqrt{f_{ck} + 0.87 f_y A_{sv} d / s_v} \tag{2}$$

For interior beam column joints:

$$\tau_c = 1.5A_{ej}\sqrt{f_{ck} + 0.87 f_y A_{sv} d / s_v} \tag{3}$$

For other joint:

$$\tau_c = 1.0A_{ej}\sqrt{f_{ck} + 0.87 f_y A_{sv} d / s_v} \tag{4}$$

Shear Strength:

$$V_{joint(shear\ joint)} = T_1 + T_2 - V_{column} \tag{5}$$

$$T_1 = 1.25f_y A_{st}; \quad T_2 = 1.25f_y A_{st};$$

$$V_{column} = 1.4((M_s + M_h)/A_{st})$$

Where,

A_{ej} = shear area of joint;

$A_{ej} = b_j h_j$

f_{ck} = characteristic strength of concrete;

f_y = characteristic strength of stirrups

A_{sv} = total cross-sectional area of stirrups;

d = effective depth;

S_v = spacing of stirrups

M_s = Sagging moment; M_h = Hogging moment; A_{st} = Area of steel

Therefore, the Mathematical formulation is

$$\text{Maximize } \tau_{je} = \tau_c + V_{us}$$

Subjected to the constrains for the optimization problem is derived based on the shear strength consideration as given in the code.



$$\tau_{je} < V_{joint}$$

$$\tau > \tau_v$$

Where τ_{je} = shear strength of joint; V_{joint} =shear in joint; τ = shear stress

D. Design of Normal Exterior Beam Column Joint

Design data were taken from Explanatory Examples for Ductile Detailing of RC Buildings”

IITK-GSDMA-EQ22-V3.0] (Pg. No: 39) [7] (figure 5)

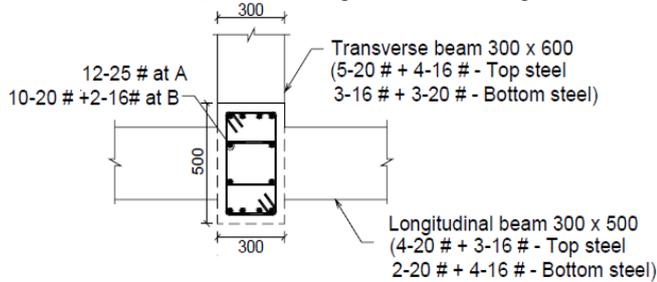


Fig. 5 Reinforcement Details of Longitudinal and Transverse Beam

$f_{ck} = 20 \text{ N/mm}^2$; $f_y = 415 \text{ N/mm}^2$; $A_{sv} = 78.54 \text{ mm}^2$; $S_v = 90\text{mm}$; $d = 447.5 \text{ mm}^2$

$A_{st} = 1185\text{mm}^2$; $M_s = 180\text{kNm}$; $M_h = 238\text{kNm}$

From the data the shear strength of joint was found using the relation given in equation (2). The condition for the safety of the joint in shear was tested as per the relation given in constrain and it was observed that it is us safe.

E. Objective Function of Regular Lattice

The regular honeycomb structure is a periodic one, so that analysis is made for the unit-element cell which is isolated from the entire honeycomb core. The model used in the study to derive the equation of honeycomb core is unit-element of Kelsey and Gibson model as shown in figure 6 & 7. Honeycomb cores built up by repeating the unit element of “Gibson” model are seldom used in engineering, while honeycomb cores with double thickness horizontal walls are more familiar which can be built up by repeating the unit-element of “Kelsey” model. The stress distribution of cell walls of honeycomb core under out-of-plane shear loads is not simple which is proposed unit element shown in figure 8, each cell wall suffers a non-uniform deformation. By using the numerical method only exact calculation is possible. By using the theorems of minimum potential energy and minimum complementary energy, the upper and lower bounds of the out-of-plane shear moduli can be obtained. It states that the strain energy calculated from any specific set of displacements which are compatible with the external boundary conditions and with themselves will be minimum for the exact displacement distribution.

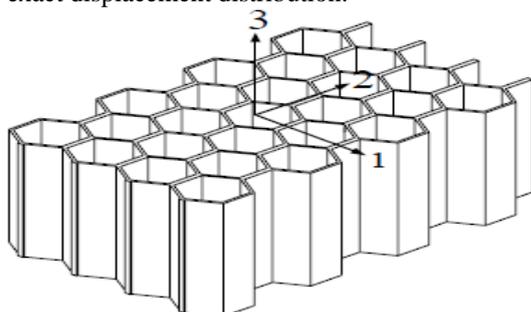


Fig. 6 Typical Honeycomb Core

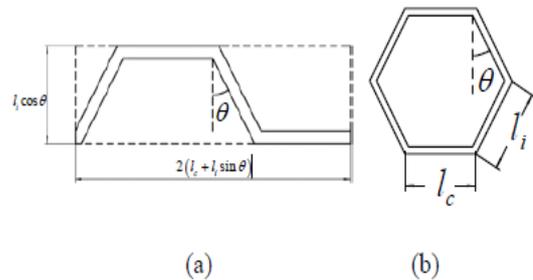


Fig. 7 The Unit-Element of “Kelsey” model (b) The unit Element of “Gibson” Model

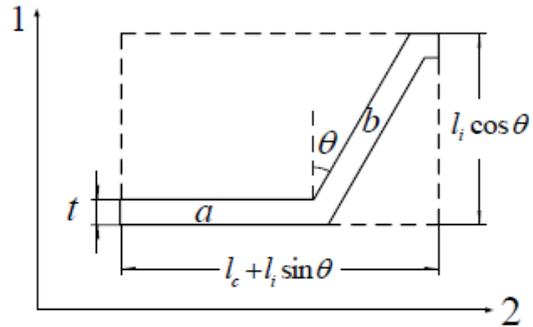


Fig. 8 The Unit-Element Proposed

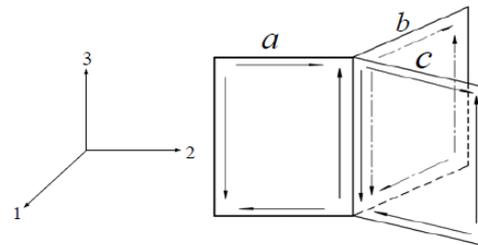


Fig. 9 The Stress Distribution of the Cell Walls

On the honeycomb in the 13 direction a uniform strain, γ_{13} , is acting. The shear strains in walls a and b (as shown in figure 9) is:

$$\gamma_a = 0$$

$$\gamma_b = \gamma_{13} \cos \theta$$

The unit-element volume is:

$$V^* = (l_c + l_i \sin \theta) l_i \cos \theta b \quad (6)$$

Where, height of honeycomb core is b. Therefore, the shear displacement of all the strain energy is stored in the wall of honeycomb core. The bending stiffness and energies are much smaller. Shear in the 13 direction, ignoring the energies with respect to bending can be expressed as an inequality which has the form:

$$\frac{1}{2} G^*_{13} \gamma^2_{13} V^* \leq \frac{1}{2} \sum_i (G_i \gamma^2_i V_i) \quad (7)$$

Where, the shear modulus of the cell wall material is G_s , the shear strain in the two cell wall is γ_i .

Evaluation of the sum given

$$G^*_{13} \leq t \cos \theta G_s / (l_c + l_i \sin \theta) \quad (8)$$

Calculation for shear γ_{23} in 23 direction, the strains in a and b cell wall are $\gamma_b = \gamma_{23} \sin \theta$ and

$$G^*_{23} \leq \frac{t G_s (l_c + l_i \sin^2 \theta)}{(l_c + l_i \sin \theta) l_i \cos \theta} \quad (9)$$



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Lower bound for the moduli is obtained from the minimum complementary theorem. It states, that, at each point the stress distributions that satisfy equilibrium and are in equilibrium with the applied loads, for the exact stress distribution the strain energy is a minimum. Expressed as an inequality, for shear in the 13 direction:

$$\frac{1}{2} \frac{\tau_{13}^2}{G_{13}^*} V^* \leq \frac{1}{2} \sum \frac{\tau_i^2}{G_s} V_i \quad (10)$$

In the 13 direction, considering first loading, it is again accepted that an external stress in the two cell wall τ_{13} induces a set of shear stresses τ_a and τ_b . As the wall a is loaded in simple bending, it carries no specific load, so τ_a equal to zero.

$$\tau_{13}(l_c + l_i \sin \theta) l_i \cos \theta = \tau_b t l_i \cos \theta \quad (11)$$

Then,

$$\tau_b = \tau_{13}(l_c + l_i \sin \theta) / t \quad (12)$$

Then the lower bound given by the inequality equation (7):

$$G_{13}^* \geq t \cos \theta G_s (l_c + l_i \sin \theta) \quad (13)$$

The postulation of external shear stress τ_{23} induces a set of shear stresses τ_a , τ_b and τ_c in the three cell walls as shown in fig.3.8 for loading in the 23 direction, equilibrium with the external stress gives:

$$\tau_{23}(l_c + l_i \sin \theta) 2 l_i \cos \theta = \tau_a l_c 2t + 2 \tau_b l_i t \sin \theta \quad (14)$$

The stress at the nodes requires the equilibrium:

$$\tau_a(2t) + \tau_b t + \tau_c t \quad (15)$$

require $\tau_b = \tau_c$, so $\tau_a = \tau_b = \tau_c$, and then

$$G_{23}^* \geq \frac{t G_s (l_c + l_i \sin^2 \theta)}{(l_c + l_i \sin \theta) l_i \cos \theta} \quad (16)$$

Combining the equations (3), (4), (7) and (9) gives:

$$G_{13}^* = t \cos \theta G_s (l_c + C \sin \theta) t \cos \theta G_s (l_c + l_i \sin \theta) / (l_c + l_i) l_i \cos \theta \leq G_{23}^* \leq t G_s (l_c + l_i \sin^2 \theta) / (l_c + l_i \sin \theta) l_i \cos \theta \quad (17)$$

For the regular hexagon honeycomb cores, $l_c = l_i = l$, $\theta = 30^\circ$, the equation is reduced to:

$$G_{13}^* = \frac{\sqrt{3} t}{3 l} G_s \quad (18)$$

$$\frac{\sqrt{3} t}{2 l} G_s \leq G_{13}^* \leq \frac{5\sqrt{3} t}{9 l} G_s \quad (19)$$

By using the second moment of inertia of the cell wall and by the width l the buckling load for a cell wall (9) is determined as

$$\tau_{crit} = \frac{C E_s}{(1 - \gamma_s^2)} \left(\frac{t}{l} \right)^2 \quad (20)$$

From Roark and Young, $C=7.78$ was chosen by Zhang and Ashby, which is the value intermediate to that for $b/l=1$ and that for $b/l=\infty$ for these specific boundary conditions. The stress distribution of cell walls derived from the minimum complementary theorem was used in the calculation of the out-of-plane shear elastic buckling strength of honeycomb cores. The equilibrium of the stress requires τ_a equal to zero and

$$\tau = \frac{C E_s}{(1 - \gamma_s^2)} \frac{1}{\frac{l_c}{l_i} + \sin \theta} \left(\frac{t}{l} \right)^3 \quad (21)$$

For Hexagonal:

$$\tau l_i^2 = \tau_b l_i t \cos \theta \quad (22)$$

For the hexagon honeycomb cores,

$$l_c = l_i = l, \quad \tau_b = \tau_{crit}$$

$$\tau = \frac{1}{\cos \theta} \frac{C E_s}{(1 - \gamma_s^2)} \left(\frac{t}{l} \right)^3 \quad (23)$$

For Triangle:

$$\tau l_i^2 \cos \theta = 2 \tau_b l_i t \cos \theta \quad (24)$$

For the triangle honeycomb cores,

$$l_c = l_i = l, \quad \tau_b = \tau_{crit}$$

$$\tau = 2 * \frac{C E_s}{(1 - \gamma_s^2)} \left(\frac{t}{l} \right)^3 \quad (25)$$

For Square:

$$\tau l_i^2 = \tau_b l_i t \quad (26)$$

For the square honeycomb cores,

$$l_c = l_i = l, \quad \tau_b = \tau_{crit}$$

$$\tau = \frac{C E_s}{(1 - \gamma_s^2)} \left(\frac{t}{l} \right)^3 \quad (27)$$

Where, C values are taken from Formulas for Stress and Strain [9] and E_s values are taken from Cellular Solids Structure and Properties [1].

F. Optimization of Regular Lattice

By using the above genetic algorithm, optimizing the shape is done with bounds for thickness (upper bound $\leq t \leq$ lower bound) and length (upper bound $\leq l \leq$ lower bound). In MATLAB the objectives function were entered in form of function [f] for different shapes and the bound were entered in the form of matrix.



IV. RESULT AND DISCUSSION

A. General

The shear stress behaviors of the regular lattice structure for three different shapes were found by using genetic algorithm in MATLAB. The objective function of lattice unit cell was run in MATLAB software using genetic algorithm (GA) solver. From this, the result of shear stress for a lattice unit cell is found by changing the thickness and length which is tabulated in table 1. The optimum value of lattice was compared with the design shear stress of exterior beam column joint.

B. Shear Stress Behavior

The amount of shear force per unit area perpendicular to the axis of the member is the shear stress in the normal joint. Shear stress will vary by changing the effective area of joint. But in lattice by changing the thickness and length of unit cell shear stress will vary. Hence, thickness and length influencing the shear stress of lattice structure.

C. Shear Stress for Regular Lattice Structure

Optimum value of shear stress with varying thickness and length bound for three different Regular Lattice Structure is shown in table 1.

Table 1. Optimized Shear Stress value for three different Regular Lattice Structure

Sl. No	Thickness limit (mm)	Length limit (mm)	Shear Stress (N/mm ²)	Optimum Shear Stress (N/mm ²)
For Regular Hexagonal Lattice Structure				
1	0.1 ≤ t ≤ 0.6	5 ≤ l ≤ 10	4.908	4.909
		6 ≤ l ≤ 10	1.642	
		7 ≤ l ≤ 10	0.651	
2	0.1 ≤ t ≤ 0.7	6 ≤ l ≤ 10	4.415	4.415
		7 ≤ l ≤ 10	1.643	
		8 ≤ l ≤ 10	0.737	
3	0.1 ≤ t ≤ 1.2	9 ≤ l ≤ 10	9.235	4.909
		10 ≤ l ≤ 10	4.909	
For Regular Triangle Lattice Structure				
1	0.1 ≤ t ≤ 0.8	7 ≤ l ≤ 10	4.766	4.766
		8 ≤ l ≤ 10	2.1399	
		9 ≤ l ≤ 10	1.0546	
2	0.1 ≤ t ≤ 0.9	7 ≤ l ≤ 10	9.658	4.321
		8 ≤ l ≤ 10	4.321	
		9 ≤ l ≤ 10	2.139	
3	0.1 ≤ t ≤ 1.0	7 ≤ l ≤ 10	4.025	4.025
		8 ≤ l ≤ 10	8.14	
		10 ≤ l ≤ 10	2.139	
4	0.1 ≤ t ≤ 1.1	9 ≤ l ≤ 10	7.121	3.789
		10 ≤ l ≤ 10	3.789	

For Regular Square Lattice Structure

1	0.1 ≤ t ≤ 0.2	7 ≤ l ≤ 10	4.319	4.319
		8 ≤ l ≤ 10	1.938	
2	0.1 ≤ t ≤ 0.4	3 ≤ l ≤ 10	4.461	4.461
		4 ≤ l ≤ 10	0.793	
3	0.1 ≤ t ≤ 0.8	6 ≤ l ≤ 10	4.459	4.459
		7 ≤ l ≤ 10	1.769	
		8 ≤ l ≤ 10	0.7937	
4	0.1 ≤ t ≤ 0.9	7 ≤ l ≤ 10	3.586	3.586
		8 ≤ l ≤ 10	1.608	
		9 ≤ l ≤ 10	0.7937	
5	0.1 ≤ t ≤ 1.2	8 ≤ l ≤ 10	9.0186	4.459
		9 ≤ l ≤ 10	4.459	
		10 ≤ l ≤ 10	2.37	
6	0.1 ≤ t ≤ 1.3	9 ≤ l ≤ 10	7.196	3.83
		10 ≤ l ≤ 10	3.83	

The minimum and maximum shear stress of an exterior beam column joint from the above study is 0.54 N/mm² and 6.3 N/mm² respectively. Here, the maximum and minimum shear stress is taken as a reference for optimization of the regular lattice structure. To obtain the optimum shear stress for a hexagonal unit cell function, iterations are carried out by varying the upper and lower limits of thickness and length. The optimum shear stress is chosen in between the minimum and maximum shear stress of the normal beam column joint. The shear strength value is based on the average value of the iteration process. Optimum shear stress for a hexagonal lattice structure was found to be 4.909, 4.415, 3.663 and 4.909 N/mm² based on different thickness and length bounds 0.1 ≤ t ≤ 0.6; 5 ≤ l ≤ 10, 0.1 ≤ t ≤ 0.7; 6 ≤ l ≤ 10, 0.1 ≤ t ≤ 0.8; 7 ≤ l ≤ 10 and 0.1 ≤ t ≤ 1.2; 10 ≤ l ≤ 10 respectively.

Optimum shear stress for a triangle lattice structure are 4.025, 4.321 and 4.766 N/mm² based on different thickness and length bounds 0.1 ≤ t ≤ 0.7; 6 ≤ l ≤ 10, 0.1 ≤ t ≤ 0.8; 7 ≤ l ≤ 10 and 0.1 ≤ t ≤ 0.9; 8 ≤ l ≤ 10 respectively.

Optimum shear stress for a square lattice structure are 4.319, 4.456, 4.459, 4.461, 3.586, and 3.83 N/mm² based on different thickness and length bounds 0.1 ≤ t ≤ 0.2, 7 ≤ l ≤ 10; 0.1 ≤ t ≤ 0.8, 6 ≤ l ≤ 10; 0.1 ≤ t ≤ 1.2, 9 ≤ l ≤ 10; 0.1 ≤ t ≤ 0.4, 3 ≤ l ≤ 10; 0.1 ≤ t ≤ 0.9, 7 ≤ l ≤ 10; and 0.1 ≤ t ≤ 1.3; 10 ≤ l ≤ 10 respectively.

V. CONCLUSION

Based on the results obtained in this study, the following conclusions can be drawn

In this study the optimum shear stress capacity for regular hexagonal, triangle and square lattice structure which can be used in the beam column joint instead of reinforcement stirrups to resist the seismic force was analyzed.

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The optimized value of different shapes of lattice structure was found by using genetic algorithm in MATLAB. The optimized shear stress values are as follows,

Optimum shear stresses of Hexagonal unit cell is 4.904 N/mm² with thickness and length bound $0.1 \leq t \leq 0.6$; $5 \leq l \leq 10$.

Optimum shear stresses of triangle unit cell 4.119 N/mm² with thickness and length bounds $0.1 \leq t \leq 0.9$; $8 \leq l \leq 10$.

Optimum shear stresses of square unit cell is 3.972 N/mm² with thickness and length bound $0.1 \leq t \leq 0.4$; $3 \leq l \leq 10$.

Obtained shear strength value of regular lattice structure is more than the shear strength of normal beam column joint, hence this lattice structure can be used to resist the shear forces.

REFERENCES

1. Gibson, L.J, Ashby.M.F, "Cellular Solids: Structure and properties", second ed., Cambridge University Press, Cambridge, 1997.
2. Evans, A.G, "Lightweight materials and structures", Materials Research Bulletin 26(2001), 790-797.
3. Pan, Shi-Dong, Lin-Zhi Wu, and Yu-Guo Sun. "Transverse shear modulus and strength of honeycomb cores." Composite Structures 84.4 (2008), 369-374.
4. Kelsey, S, R. A. Gellatly, B. W. Clark, "The shear modulus of foil honeycomb cores: A theoretical and experimental investigation on cores used in sandwich construction", Aircraft Engineering and Aerospace Technology 30.10 (1958): 294-302.
5. Masters, I. G., and K. E. Evans. "Models for the elastic deformation of honeycombs." Composite structures 35.4 (1996), 403-422.
6. Sorohan, Ștefan, et al, "Estimation of Out of Plane Shear Moduli For Honeycomb Cores With Modal Finite Element Analyses", Ro.J.Techn Sci,Appl Mechanics, Vol. 61(2016), 71-88.
7. Ingle, R. K., and Sudhir K. Jain. "Explanatory examples for ductile detailing of RC buildings." IITK-GSDMA Project Report on Building Codes, IIT, Kanpur, India, 2005.
8. Young, Warren Clarence, and Richard Gordon Budynas. Roark's formulas for stress and strain. Vol. 7, McGraw-Hill, New York, 2002.
9. Wang, A-J, and D. L. McDowell, "In-plane stiffness and yield strength of periodic metal honeycombs", Journal of engineering materials and technology 126.2 (2004): 137-156.
10. Cote, François, Vikram Deshpande, and Norman Fleck, "The shear response of metallic square honeycombs." Journal of Mechanics of Materials and Structures 1.7 (2006), 1281-1299.
11. Xie, H., et al, "Study on the out of plane shear properties of super alloy honeycomb core", 18th Int. Conf. on Composite Materials.
12. Andrews, E.W, et al, "Size effects in ductile cellular solids. Part II: experimental results", International Journal of Mechanical Sciences 43.3 (2001), 701-713.
13. Ashby, Michael F, and RF Mehl Medalist. "The mechanical properties of cellular solids", Metallurgical Transactions 14.9 (1983), 1755-1769.
14. R. Akbari Alashti, S. A. Latifi Rostami and G. H. Rahimi, "Buckling Analysis of Composite Lattice Cylindrical Shells With Ribs Defects", IJE Transactions A: Basics Vol. 26, No. 4 (April 2013) 411-420.
15. Indian Standard, IS 456: 2000, Plain and Reinforced Concrete Code of Practice (2000).
16. IS 13920: 2016, Ductile Design and detailing of reinforced concrete structures subjected to seismic forces Code of practice.
17. Amiri GG, Massah SR, Boostan A. "Seismic response of 4-legged self-supporting telecommunication towers". International Journal of Engineering. 2007 Aug;20(2).
18. J.Zhang and M.F.Ashby, "The out-of-plane properties of honeycombs", International Journal of Mechanical Sciences, 34.6(1992), 475-489.
19. S.D.Pan, L.Z.Wu, Y.G.Sun. "Transverse shear modulus and strength of honeycomb cores", Composites Structures, 84.4(2008): 369-374.
20. S.D.Pan, L.Z.Wu, Y.G.Sun, Z.G.Zhou and J.L.Qu, "Longitudinal shear strength and failure process of honeycomb cores", Composites Structures, 72.1(2006): 42-46.
21. Rastgar, M., and H. Showkati. "Field Study and Evaluation of Buckling Behavior of Cylindrical Steel Tanks with Geometric Imperfections under Uniform External Pressure." International Journal Of Engineering 30, no. 9 (2017): 1309-1318.