

# Modeling and Control of Inversely Connected Rotors of Brushless Doubly Fed Twin Stator Induction Generator

A. Mahdy, S. M. EL-Hakim, M. Abdel Hakim

**Abstract:** This paper presents a steady state and dynamic modeling of inversely connected rotors of Brushless Doubly Fed Twin Stator Induction Generator (BDFTSIG). The BDFTSIG consists of two identical machines, one is called power machine whose stator is connected directly to the grid and the other machine is called control machine whose stator is connected to a pulse width modulation converters. A vector control scheme is applied on the grid side converter (GSC) to control the DC link bus voltage; another vector control scheme is applied on the control machine side converter (CMSC) for synchronization and also to control the active and reactive power delivered to the grid. The system is simulated using MATLAB/Simulink environment. The simulation results show a good performance comparing with conventional doubly fed induction generator, under various loading and rotor speed conditions.

**Keywords:** BDFTSIG, vector control, inversely connected rotors.

Subscripts $sp, sc, rc, rp$	PM & CM stators and rotors
Subscripts $d, q, g$	$d$ -axis & $q$ -axis, grid
$\omega_m$	Mechanical angular speed in rad/s
$p$	Pole pair
$s$	Slip
$L_l, L_m$	Leakage & mutual Inductance in H
$T_e$	Electromagnetic torques in N.m
$P_{wind}$	Mechanical wind power in watt
$V_{wind}$	Wind velocity in m/s
$\rho$	Air density in $kg/m^3$
$C_p$	Wind power coefficient
$\lambda$	Tip speed ratio
$\omega_T$	Turbine angular speed in rad/s
$R_T$	Turbine rotor radius in meter
$E$	DC-link voltage in volt
$\theta$	Orientation angular position in rad
$\varphi$	Magnetic Flux in webers
$P, Q$	Active & reactive power in watt, VAR

## I. INTRODUCTION

Nowadays there is a huge increase in demand and relying on renewable energy resources as an alternative for traditional fossil fuel energy resources.

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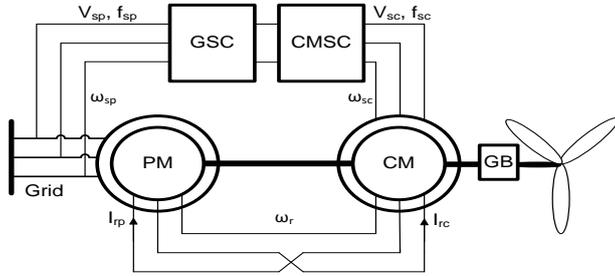
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A Doubly Fed Induction Generator (DFIG) has a significant advantage in high power wind energy conversion systems (WECS), Because of possible independent control of the active and reactive power that can be achieved by using a power electronic converter connected to the rotor side circuit, which handles only a fraction of the total delivered power making them suitable for wind power applications. The main disadvantage of the DFIG is the existence of slip rings and carbon brushes, carbon brushes are known to have limited life span and therefore are replaced every couple of months result in an increases the maintenance costs due to wearing and depreciation, also carbon dust is conducting in nature and when trapped in the generator windings it will increase the risk of sparking between the winding conductors which could result in the weakening of the insulation and therefore a reduction of system reliability [1-3], to overcome this drawback a Brushless DFIG is used instead, the BDFIG are without brushes and slip rings and hence achieving higher reliability and lower maintenance cost. A BDFIG consists of two stator windings with different number of poles located in a single frame, those stators are magnetically coupled and electrically insulated, this type of machines have a common shaft with a special type of rotor “nested-loop” design. The first stator winding (power winding) is directly connected to the grid while the second stator winding (control winding) is connected to the grid through a controlled power converter [4-9]. Another equivalent alternative scheme is used for the BDFIG which is the BDFTSIG which consists of twin slip ring type induction machines. One of the two machines whose stator is connected directly to the grid is called a power machine (PM), while the other machine whose stator is connected to a power converter is called a control machine (CM), in this scheme the two rotors mechanically coupled and direct electrically connected through rotor circuits. The rotor terminals have two possibilities of connections, the first connection is back to back / positive sequence connection, Duro Basic et al. presented a study for the transient performance of BDFTSIG under vector control for this type of rotors connection [10], the second connection is a negative / inverse sequence connection which is more preferable as it ensures that the generated torques act in the same direction and the resultant torques is the summation of both machines’ produced torques, Duro Basic et al. presented a study for steady state performance and modeling of BDFTSIG for this type of rotor connection [11], similarly Marek Adamowicz et al. presented a study for the steady state analysis [12], also Zohier et al. presented a study for the dynamic and steady state performance [13].



A control schemes were presented for back to back rotors connection, Kostyantyn Protsenko et al. presented a control scheme for active and reactive power control by means of a four quadrant power converter under the closed-loop stator flux oriented control scheme for back to back sequence rotors connection [14], also A. Bensadeq et al. presented an indirect vector control for grid operation of a BDFTSIG to control the power flow into the grid with a variable speed prime mover such as a wind turbine [15].

In this paper both steady state and dynamic model is presented for inversely rotor directly connected BDFTSIG system shown in Fig. 1. A vector control scheme is used to control the DC link bus at unity power factor using a PI controlled grid side converter (GSC) which is connected to the grid, another vector control scheme is used for the synchronization with the grid, and for achieving active and reactive power control using a PI controlled machine side converter (CMSC) which achieves a good transient response. Both vector control schemes are validated and the dynamic performance results are represented and analyzed by using MATLAB/Simulink environment. The results show a good performance under various loading and rotor speed conditions.



**Fig. 1 Inverse sequence rotor connected BDFTSIG**

## II. MATHEMATICAL MODELING OF BDFTSIG

### A. Mode of Operation

A BDFTSIG can operate in several ways but the synchronous mode is used for controlled variable-speed operation [5&13]. When the BDFTSIG operates in synchronous mode, there is a single frequency of current in the rotor, which is given by:

$$\omega_{rp} = \omega_{sp} - p_p \omega_m \quad (1)$$

$$\omega_{rc} = \omega_{sc} - p_c \omega_m \quad (2)$$

Accordingly the slips for the two machines are given by:

$$s_p = \frac{\omega_{rp}}{\omega_{sp}} = \frac{\omega_{sp} - p_p \omega_m}{\omega_{sp}} \quad (3)$$

$$s_c = \frac{\omega_{rc}}{\omega_{sc}} = \frac{\omega_{sc} - p_c \omega_m}{\omega_{sc}} \quad (4)$$

To produce constant torque the cross-coupling condition should be satisfied, which is:

$$\omega_{rp} = -\omega_{rc} \quad (5)$$

$$\omega_{sp} - p_p \omega_m = -\omega_{sc} + p_c \omega_m \quad (6)$$

$$\omega_{sc} = (p_p + p_c) \omega_m - \omega_{sp} \quad (7)$$

From (3), (4) and (5) we can deduce the relation between PM and CM slips and frequencies from the following equation:

$$\frac{s_p}{s_c} = -\frac{\omega_{sc}}{\omega_{sp}} \quad (8)$$

The synchronous speed of the BDFTSIG is half the synchronous speed of the both machines, leading to a shaft angular velocity is given by:

$$\omega_m = \frac{\omega_{sp} + \omega_{sc}}{p_p + p_c} \quad (9)$$

### A. Dynamic Modeling of BDFTSIG

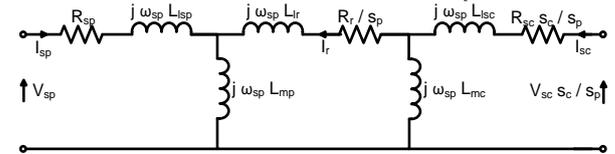
The mathematical dynamic model of the BDFTSIG can be derived from the models of two slip ring induction machines with inversely connected rotors as shown in fig. 1. This dynamic model is represented in two-phase coordinates (dq) rotating at synchronous speed in the matrix form as given in (10), for derivation see appendix.

### B. Steady State Modeling of BDFTSIG

The steady state per-phase equivalent circuit of BDFTSIG after referring to stator winding of PM is shown in Fig. 2. The BDFTSIG steady states per phase modeling equations are as follows:

$$\begin{bmatrix} V_{sp} \\ 0 \\ V_{sc} \end{bmatrix} = \begin{bmatrix} R_{sp} + j\omega_{sp}L_{sp} & j\omega_{sp}L_m & 0 \\ j\omega_{sp}L_m & R_r/s_p + j\omega_{sp}L_r & -j\omega_{sp}L_m \\ 0 & -j\frac{s_p}{s_c}\omega_{sp}L_m & R_{sc} + j\frac{s_p}{s_c}\omega_{sp}L_{sc} \end{bmatrix} * \begin{bmatrix} I_{sp} \\ I_r \\ I_{sc} \end{bmatrix} \quad (11)$$

Neglecting the saturation effect ( $L_{mp} = L_{mc} = L_m$ ),  $L_{sp} = L_{lsp} + L_{mp}$ ,  $L_{sc} = L_{lsc} + L_{mc}$ ,  $L_{rp} = L_{lrp} + L_{mp}$ ,  $L_{rc} = L_{lrc} + L_{mc}$ , because the two machines are assumed to be identical,  $R_r$  is the sum of rotor resistances  $R_r = R_{rp} + R_{rc}$  and  $L_{lr}$  is the sum of rotor leakage inductances  $L_{lr} = L_{lrp} + L_{lrc}$ .



**Fig. 2 Steady state per phase eq. circuit of BDFTSIG**

The electromagnetic torque equation is given by:

$$T_{ep} = \frac{3}{2} p_p L_{mp} \text{im}\{i_{sp} i_{rp}^*\} = \frac{3}{2} p_p L_{mp} \text{im}\{i_{sp} * i_r\} \quad (12)$$

$$T_{ec} = \frac{3}{2} p_c L_{mc} \text{im}\{i_{sc} i_{rc}^*\} = \frac{3}{2} p_c L_{mc} \text{im}\{i_{sc} * i_r\} \quad (13)$$

$$T_e = T_{ep} + T_{ec} \quad (14)$$

$$\begin{bmatrix} V_{qsp} \\ V_{dsp} \\ 0 \\ 0 \\ V_{qsc} \\ V_{dsc} \end{bmatrix} = \begin{bmatrix} (R_{sp} + SL_{sp}) & \omega_{sp}L_{sp} & SL_{mp} & \omega_{sp}L_{mp} & 0 & 0 \\ -\omega_{sp}L_{sp} & (R_{sp} + SL_{sp}) & -\omega_{sp}L_{mp} & SL_{mp} & 0 & 0 \\ SL_{mp} & (\omega_{sp} - p_p\omega_m)L_{mp} & (R_r + SL_r) & (\omega_{sp} - p_p\omega_m)L_r & -SL_{mc} & (\omega_{sp} - p_p\omega_m)L_{mc} \\ -(\omega_{sp} - p_p\omega_m)L_{mp} & SL_{mp} & -(\omega_{sp} - p_p\omega_m)L_r & (R_r + SL_r) & (\omega_{sp} - p_p\omega_m)L_{mc} & SL_{mc} \\ 0 & 0 & -SL_{mc} & ((p_p + p_c)\omega_m - \omega_{sp})L_{mc} & (R_{sc} + SL_{sc}) & ((p_p + p_c)\omega_m - \omega_{sp})L_{sc} \\ 0 & 0 & ((p_p + p_c)\omega_m - \omega_{sp})L_{mc} & SL_{mc} & -((p_p + p_c)\omega_m - \omega_{sp})L_{sc} & (R_{sc} + SL_{sc}) \end{bmatrix} * \begin{bmatrix} i_{qsp} \\ i_{dsp} \\ i_{qr} \\ i_{dr} \\ i_{qsc} \\ i_{dsc} \end{bmatrix} \quad (10)$$



The active and reactive power equations for the PM:

$$P_{sp} = \frac{3}{2} (v_{dsp} i_{dsp} + v_{qsp} i_{qsp}) \quad (23)$$

$$Q_{sp} = \frac{3}{2} (v_{qsp} i_{dsp} - v_{dsp} i_{qsp}) \quad (24)$$

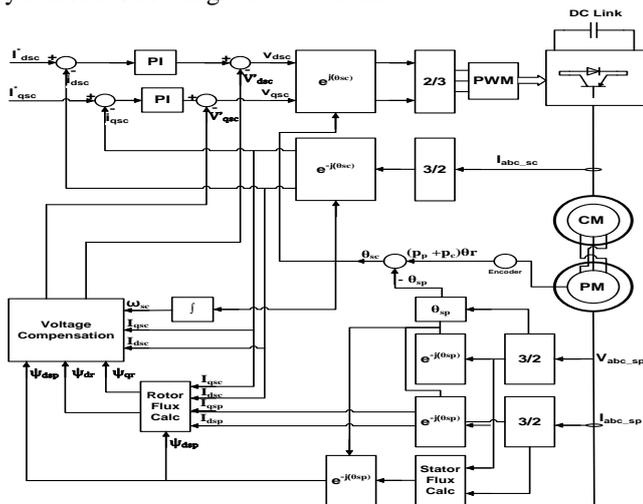
Neglecting the stator resistance and by aligning the synchronously rotating  $d$ -axis reference frame with the stator voltage angle this will result in that the  $v_{qs} = 0$  and since the amplitude of the supply voltage is constant,  $v_{ds}$  will be constant too, so the active and reactive power input to the stator of the PM in (23) & (24) can be rewritten in synchronously rotating  $dq$ -axis reference frame as follows:

$$P_{sp} = \frac{3}{2} v_{dsp} i_{dsp} \quad (25)$$

$$Q_{sp} = -\frac{3}{2} v_{dsp} i_{qsp} \quad (26)$$

Based on (25) & (26), the reactive power ( $Q_{sp}$ ), can be controlled by the q-axis current of the PM ( $i_{qsp}$ ), where the active power ( $P_{sp}$ ), can be controlled by the d-axis current of the PM ( $i_{dsp}$ ).

Fig. 5 shows the vector control of the BDFTSIG control system is implemented using Matlab/Simulink in synchronous rotating reference frame.



**Fig. 5 Block diagram for vector control for CMSC**

Grid synchronization:

The grid voltage component signals  $V_{dg}$  and  $V_{qg}$  are compared to the PM stator voltage component signals  $V_{dsp}$  and  $V_{qsp}$  respectively. The resulting error is processed by the PI controller producing the reference the CM stator current component signals  $i_{dsc\_sync}^*$  and  $i_{qsc\_sync}^*$ . The current control signals control the excitation flux of the CM which is connected to the PM electrically through the rotor circuit controlling the excitation flux of the PM and hence the stator voltage magnitude and frequency.

Active & Reactive power control:

The active power control is achieved by regulating the PM stator d-axis current component  $i_{dsp}$  as shown in (25), this current component is regulated through the CM stator q-axis current component  $i_{qsc}$ . The available mechanical wind power is given by:

$$P_{wind} = \frac{1}{2} \rho A_T C_p(\lambda) V_{wind}^3 \quad (27)$$

$$\lambda = \frac{\omega_T R_T}{V_{wind}} \quad (28)$$

$$C_p(\lambda) = 0.22 \left( \frac{116}{\lambda_i} - 5 \right) e^{-12.5/\lambda_i} \quad (29)$$

$$\frac{1}{\lambda_i} = \frac{1}{\lambda} - 0.03 \quad (61)$$

$$P_{Wind opt} = K_{opt} \omega_T^3 \quad (30)$$

For a gear ratio of 1:5, the power input to the generator at the generator speed is given by:

$$P_{Wind opt} = K'_{opt} \omega_g^3 \quad (31)$$

$$K'_{opt} = \frac{1}{5^3} K_{opt} \quad (32)$$

The constant  $K'_{opt}$  can be selected to extract the desired maximum wind power from the wind and this is done by controlling the active power of the system to achieve the optimum tip speed ratio ( $\lambda$ ) and the coefficient of performance  $C_p(\lambda)$  for a preset generator speed, for an optimum performance, the reactive power control is also controlled by regulating the PM stator q-axis current component  $i_{qsp}$  as shown in (26) this current component is regulated through the CM stator d-axis current component  $i_{dsc}$  to achieve the desired power factor.

## IV. SIMULATION RESULTS

The system shown in fig. 1 is simulated and validated in Matlab/Simulink environment. The parameters shown in table 1 are for BDFTSIG, each machine is of rated power 4kW, 380V, 9.5A and 1500rpm, the parameters are referred to stator side, and the BDFTSIG system is driven at different reference speeds of 650 rpm sub-synchronous speed and at 850 rpm hyper-synchronous speed for different loading conditions. The wind turbine rated power is 5.5kW and the parameters are shown in table 2. The GSC was designed to regulate the DC link voltage at 150V with a 1000 $\mu$ F DC link capacitor, and a supply impedance of 0.1 $\Omega$  resistance and 15mH inductance.

Table 1 Parameters of BDFTSIG

$R_s$ [ $\Omega$ ]	$R_r$ [ $\Omega$ ]	$L_{ls}$ [H]	$L_{lr}$ [H]	$L_m$ [H]	$p_p$	$p_c$
1.405	1.395	0.006	0.006	0.172	2	2

Table 2 Parameters of wind turbine

$R_T$ [m]	$A_T$ [m <sup>2</sup> ]	Cut in speed[m/s]	Cut out speed[m/s]
3.1	30.2	2.5	11

A. GSC Simulation Results

Fig. 6 shows the simulation result with  $i_{qg}^*$  set to zero (unity pf). The phase voltage and current are in phase.

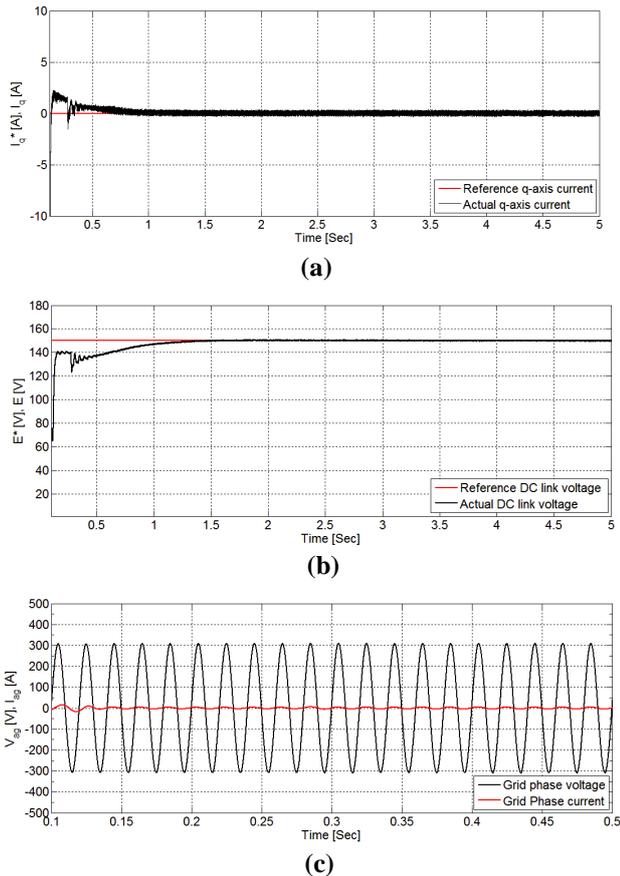


Fig. 6 GSC simulation results at unity pf: (a) q-axis current component, (b) DC-link voltage, (c) Phase voltage and current

B. Synchronization Simulation Results

Fig. 7 show the simulation results of the phase voltages for both the grid and PM stator wave forms, also the controlled grid & PM stator voltages in dq synchronous rotating reference frame during the synchronization process which takes 0-0.1 sec till the synchronization ends.

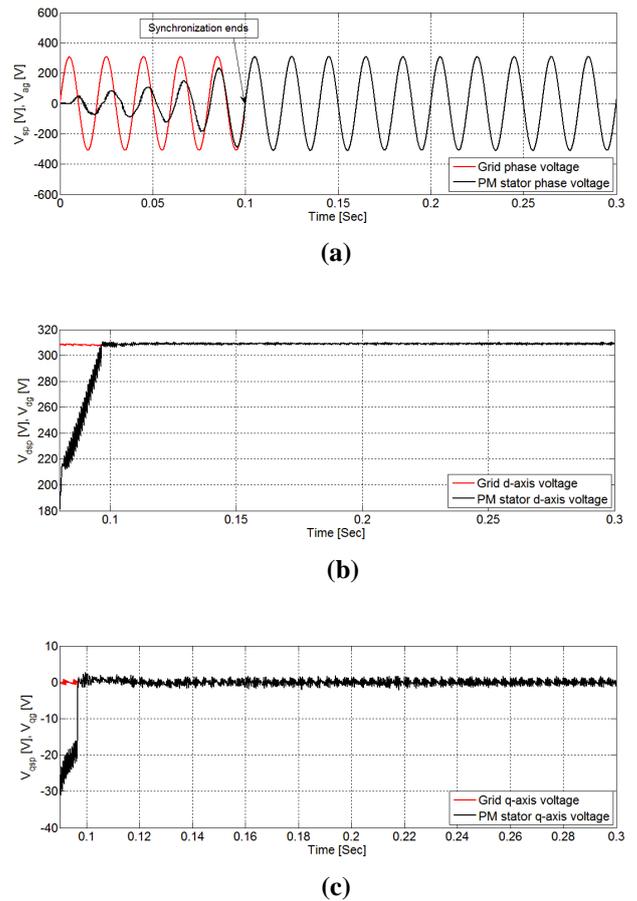
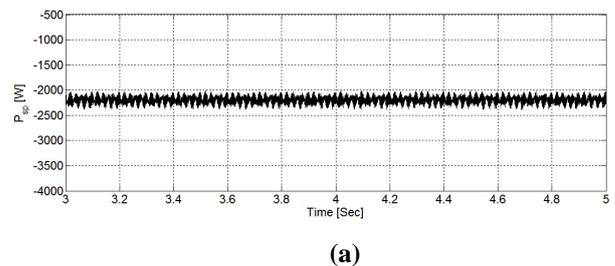


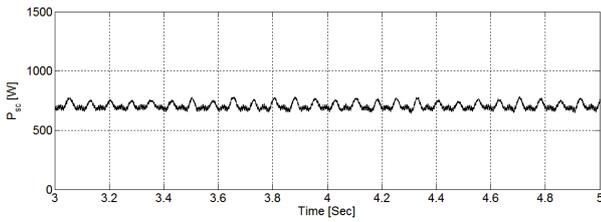
Fig. 7 PM Stator dq-axis components during synchronization: (a) Grid and PM phase voltages during synchronization, (b) d-axis voltage, & (c) q-axis voltage.

C. Active & Reactive Power Control Simulation Results

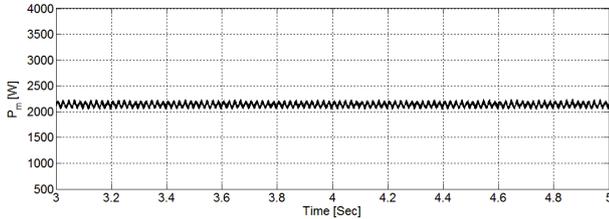
Fig. 8 (a) shows that the PM stator active power at 650rpm, which is regulated at 2200W; Fig. 8 (b) shows that the CM stator absorbs 670W from the grid. Fig. 8 (c) shows the system mechanical power input of 2150W. The efficiency of the system is almost 70%. Fig. 9 (a) shows that the PM stator active power at 850rpm, which is regulated at 3800W; Fig. 9 (b) shows that the CM stator power supplies 170W to the grid. Fig. 9 (c) shows the system mechanical power input of 5050W. The efficiency of the system is almost 78%. In all the cases the gear box losses are neglected.



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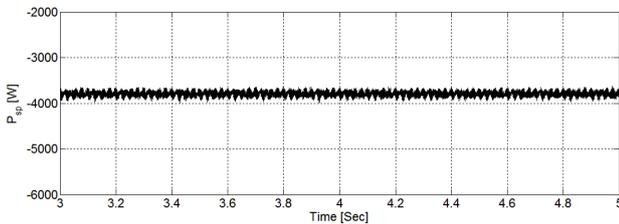


(b)

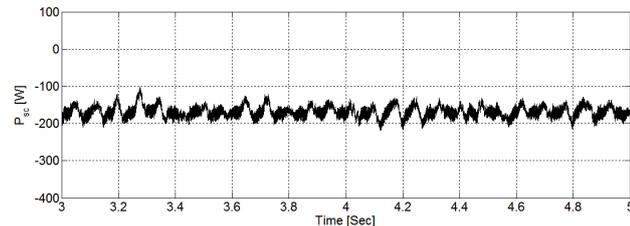


(c)

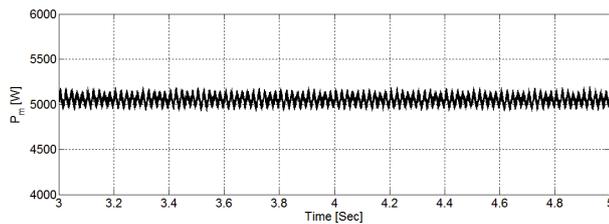
**Fig. 8 Simulation results at 650rpm: (a) PM Stator active power, (b) CM Stator active power, (c) Mechanical power input**



(a)



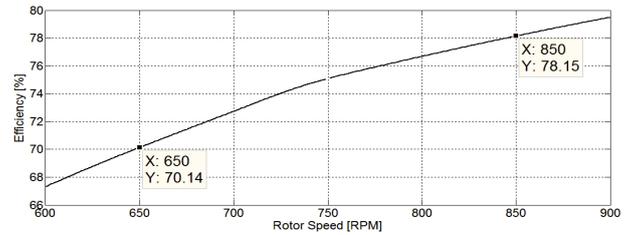
(b)



(c)

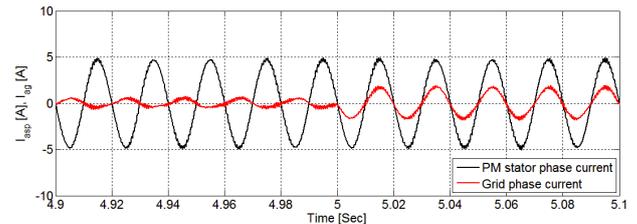
**Fig. 9 Simulation results at 850rpm: (a) PM Stator active power, (b) CM Stator active power, (c) Mechanical power input**

Fig. 10 shows the simulation results of the steady state BDFTSIG efficiency at different rotor speeds of 650 and 850rpm for varying loading conditions of 2200 and 3800W respectively extracted at PM stator terminals.

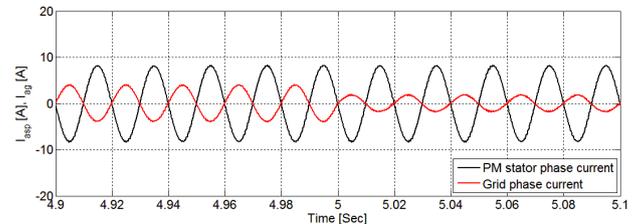


**Fig. 10 BDFTSIG efficiency**

Fig. 11 shows the simulation results of the BDFTSIG at different rotor speeds of 650 and 850rpm for a varying loading with additional load connected at in the instant 5s.



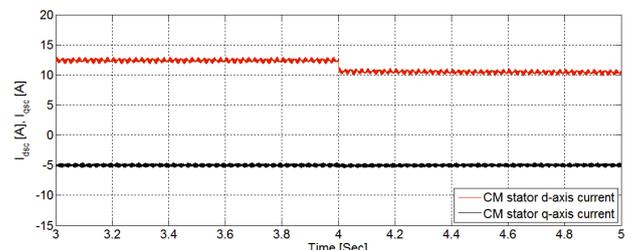
(a)



(b)

**Fig. 11 PM Stator and grid phase currents: (a) at 650 rpm, (b) at 850 rpm.**

Figs. 12-13 show the simulation results for CM stator dq current components, PM stator dq current components, and PM stator active and reactive power, due to a step d-axis current component reference input at rotor speeds of 650 and 850 rpm, these figures show the decoupled control of both active and reactive power components, there is a slightly coupling effect, causing a slight change on the active power component.



(a)

V. CONCLUSION

A steady state and dynamic modeling for inversely connected rotor of a BDFTSIG with vector control for both grid side and control machine side converters have been presented in this paper. The vector control scheme of GSC for controlling the DC link voltage was achieved regardless the power flow through the CM stator terminals, also the synchronization process was achieved in a fast and smooth way, matching the amplitude, frequency and phase of the PM stator voltage with the grid voltages, it assured the conditions for the synchronization through controlling the CM stator current components, also the vector control scheme of CMSC for controlling both the active and reactive power of the PM stator was validated, the power factor at the PM stator was improved. The vector control scheme of the inversely connected rotor of BDFTSIG system was robust and the simulation results showed a good transient performance under variable loading, power factor settings and rotor speed conditions. The only disadvantage is the efficiency of the BDFTSIG which can be compensated by the lower maintenance cost by the elimination of slip rings and carbon brushes and also by increasing the reliability and availability of the wind turbine system. Finally The inversely connected rotor of a BDFTSIG proves to be a good alternative for DFIG.

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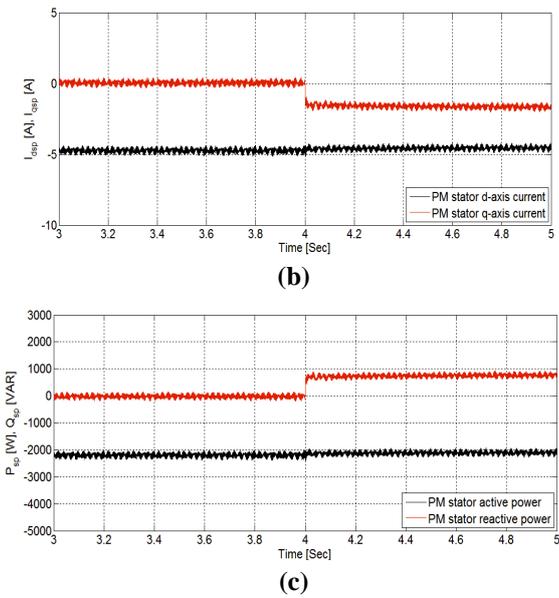


Fig. 12 Simulation results @ 650 rpm. (a) CM Stator dq currents, (b) PM Stator dq currents, (c) PM Active and reactive power

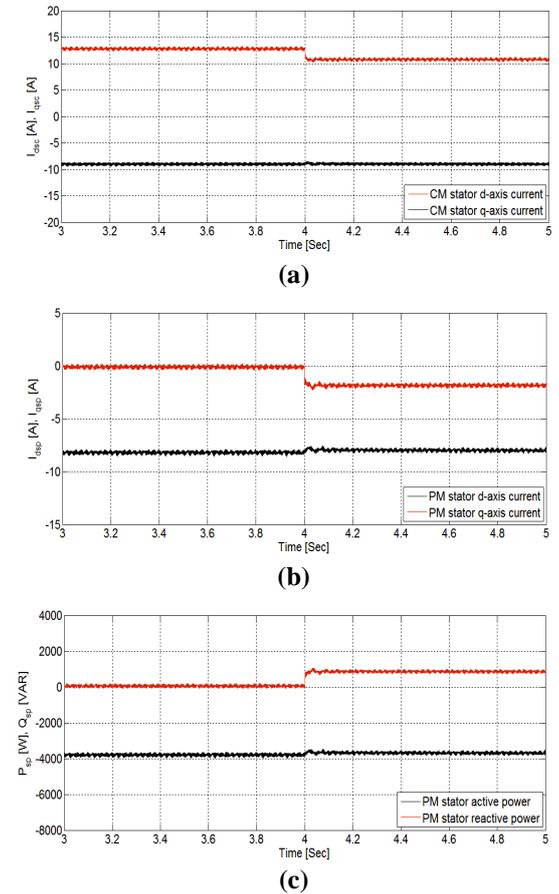


Fig. 13 Simulation results @ 850 rpm. CM Stator dq currents, (b) PM Stator dq currents, (c) PM Active and reactive power

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## APPENDIX

### A. Dynamic model of BDGTSIG derivation:

#### The stator voltage equations for PM:

$$V_{qsp} = R_{sp} i_{qsp} + \frac{d}{dt} \psi_{qsp} + \omega_{sp} \psi_{dsp} \quad (A.1)$$

$$V_{dsp} = R_{sp} i_{dsp} + \frac{d}{dt} \psi_{dsp} - \omega_{sp} \psi_{qsp} \quad (A.2)$$

#### The rotor voltage equations for PM:

$$V_{qrp} = R_{rp} i_{qrp} + \frac{d}{dt} \psi_{qrp} + (\omega_{sp} - p_p \omega_m) \psi_{drp} \quad (A.3)$$

$$V_{drp} = R_{rp} i_{drp} + \frac{d}{dt} \psi_{drp} - (\omega_{sp} - p_p \omega_m) \psi_{qrp} \quad (A.4)$$

#### Flux linkage equations:

$$\psi_{qsp} = L_{lsp} i_{qsp} + L_{mp} (i_{qsp} + i_{qrp}) \quad (A.5)$$

$$\psi_{dsp} = L_{lsp} i_{dsp} + L_{mp} (i_{dsp} + i_{drp}) \quad (A.6)$$

$$\psi_{qrp} = L_{lrp} i_{qrp} + L_{mp} (i_{qrp} + i_{qsp}) \quad (A.7)$$

$$\psi_{drp} = L_{lrp} i_{drp} + L_{mp} (i_{drp} + i_{dsp}) \quad (A.8)$$

Now, rewriting the stator and rotor voltage equations (A1 – A3) after substituting with the flux linkage (A.5 – A.8)

$$V_{qsp} = (R_{sp} + sL_{sp}) i_{qsp} + \omega_{sp} L_{sp} i_{dsp} + sL_{mp} i_{qrp} + \omega_{sp} L_{mp} i_{drp} \quad (A.9)$$

$$V_{dsp} = -\omega_{sp} L_{sp} i_{qsp} + (R_{sp} + sL_{sp}) i_{dsp} - \omega_{sp} L_{mp} i_{qrp} + sL_{mp} i_{drp} \quad (A.10)$$

Where  $s = \frac{d}{dt}$ ,

$$V_{qrp} = sL_{mp} i_{qsp} + (\omega_{sp} - p_p \omega_m) L_{mp} i_{dsp} + (R_{rp} + sL_{rp}) i_{qrp} + (\omega_{sp} - p_p \omega_m) L_{rp} i_{drp} \quad (A.11)$$

$$V_{drp} = -(\omega_{sp} - p_p \omega_m) L_{mp} i_{qsp} + sL_{mp} i_{dsp} - (\omega_{sp} - p_p \omega_m) L_{rp} i_{qrp} + (R_{rp} + sL_{rp}) i_{drp} \quad (A.12)$$

#### The stator and rotor voltage equations for CM:

$$V_{qsc} = (R_{sc} + sL_{sc}) i_{qsc} + \omega_{sc} L_{sc} i_{dsc} + sL_{mc} i_{qrc} + \omega_{sc} L_{mc} i_{drc} \quad (A.13)$$

$$V_{dsc} = -\omega_{sc} L_{sc} i_{qsc} + (R_{sc} + sL_{sc}) i_{dsc} - \omega_{sc} L_{mc} i_{qrc} + sL_{mc} i_{drc} \quad (A.14)$$

$$V_{qrc} = sL_{mc} i_{qsc} + (\omega_{sc} - p_c \omega_m) L_{mc} i_{dsc} + (R_{rc} + sL_{rc}) i_{qrc} + (\omega_{sc} - p_c \omega_m) L_{rc} i_{drc} \quad (A.15)$$

$$V_{drc} = -(\omega_{sc} - p_c \omega_m) L_{mc} i_{qsc} + sL_{mc} i_{dsc} - (\omega_{sc} - p_c \omega_m) L_{rc} i_{qrc} + (R_{rc} + sL_{rc}) i_{drc} \quad (A.16)$$

According to the *inversely sequence* rotor connection between the PM & CM, the electrical quantities has the following relationships:

#### Rotor voltages and currents relations:

$$V_{qrp} = V_{qrc} \quad (A.17)$$

$$V_{drp} = -V_{drc} \quad (A.18)$$

$$i_{qrp} = -i_{qrc} = i_{qr} \quad (A.19)$$

$$i_{drp} = i_{drc} = i_{dr} \quad (A.20)$$

From (A.17), (A.11), (A.15), (7), (A.19) and (A.20)

$$sL_{mp} i_{qsp} + (\omega_{sp} - p_p \omega_m) L_{mp} i_{dsp} + (R_r + sL_r) i_{qrp} + (\omega_{sp} - p_p \omega_m) L_r i_{drp} - sL_{mc} i_{qsc} + (\omega_{sp} - p_p \omega_m) L_{mc} i_{dsc} = 0 \quad (A.21)$$

From (A.18), (A.12), (A.16), (7), (A.19) and (A.20)

$$-(\omega_{sp} - p_p \omega_m) L_{mp} i_{qsp} + sL_{mp} i_{dsp} - (\omega_{sp} - p_p \omega_m) L_r i_{qrp} + (R_r + sL_r) i_{drp} + (\omega_{sp} - p_p \omega_m) L_{mc} i_{qsc} + sL_{mc} i_{dsc} = 0 \quad (A.22)$$

From (A.13), (A.14), (7), (A.19) and (A.20)

$$V_{qsc} = -sL_{mc} i_{qrp} + ((p_p + p_p) \omega_m - \omega_{sp}) L_{mc} i_{drp} + (R_{sc} + sL_{sc}) i_{qsc} + ((p_p + p_p) \omega_m - \omega_{sp}) L_{sc} i_{dsc} \quad (A.23)$$

$$V_{dsc} = ((p_p + p_p) \omega_m - \omega_{sp}) L_{mc} i_{qrp} + sL_{mc} i_{drp} - ((p_p + p_p) \omega_m - \omega_{sp}) L_{sc} i_{qsc} + (R_{sc} + sL_{sc}) i_{dsc} \quad (A.24)$$