

BER Performance of Free-Space Optical System Over Gamma Gamma Turbulence with Pointing Error

Narendra Kumar Verma, Hemant Narayan

Abstract: This paper investigates BER performance of free space optical (FSO) communication over gamma-gamma turbulence channel. Which is widely accepted model for moderate to strong atmospheric turbulence condition. By considering Atmospheric turbulence, Pointing error and Atmospheric attenuation a combined Statistical model for intensity fluctuation at the receiver is described for given weather and pointing error condition, a closed form expression is derived for BER performance of FSO communication System.

Keywords: Free-space optical communication, BER, Pointing error, Atmospheric turbulence., atmospheric loss.

I. INTRODUCTION

Free Space Optical Communication is a line of sight (LOS) Technology. That transmit laser beam of light through atmosphere for a broad band communication [1] [2]. FSO system have its own advantage over existing RF system, FSO System is license free with high bandwidth communication technology and FSO is prominent of last mile connectivity. It unique properties make it also appearing for a number of different application, including wide area network fiber backup, backhaul for wireless cellular network, Redundant link and disaster recovery[3]. Despite of this point to point laser signal is very secure for transmission. Having the lot of advantage of FSO Communication, various drawback should be taken into account in the desire of FSO link in the atmospheric turbulence ,which is result of variation refractive index to in homogeneity of air particle [4]. This degrade the performance of FSO system particular the FSO link which is greater than 1 km in length [5]. Another factor which degrade the performance of FSO link in building swag , which causes pointing error all a result of misalignment between the transmitter and receiver over FSO link[6]. Various statistical model have been proposed to describe atmospheric turbulence over different degree of atmospheric turbulence [77]. In particularly log normal is suitable model for only weak atmospheric condition and gamma-gamma have found to be suitable for moderate to strong turbulence condition respectively. Meanwhile as a standard performance metric adopted by most FSO system, there have been several study which described the performance parameter of the FSO communication system

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II. SYSTEM AND CHANNEL MODEL

Consider a FSO link, the transmitter modulates data onto the instantaneous intensity of optical beam. The laser beam propagate through gamma-gamma turbulence channel and the receive integrates the photocurrent signal which is related to incident power by the detector responsively for each bit period at the receiver the signal suffer from a fluctuation in signal intensity due to atmospheric turbulence and a misalignment as well as noise an can be modeled as

$$y = hRx + n \tag{1}$$

Where $x \in (0,1)$ and R is Responsivity of Photodetector ,h is normalized fading coefficient. Consider to be constant over a large no. of transmitted bits and n is AWGN with mean is zero and variance is σ_n^2 .

Consider the channel fading coefficient h is modeled as using three phenomena, distance dependent atmospheric attenuation h_e ,Pointing error h_p and atmospheric turbulence h_a .the channel state is described as

$$h = h_e h_p h_a \tag{2}$$

In this Paper, we consider intensity modulated Direct detection (IM/DD) channel using on-off keying(OOK) modulation. The transmitted signal is equally probably from an OOK constellation such that $x \in (0,2p_t)$, and p_t is average transmitted power.

A. Atmospheric Attenuation:-

Atmospheric attenuation is a deterministic phenomenon which is best described by Beers-Lambert Law as[13]

$$h_e(z) = \exp(-\alpha z) \tag{3}$$

$h_e(z)$ is the atmospheric attenuation over a propagation distance in z and α is attenuation constant[3].

B. Atmospheric Turbulence

There has been significant research after finding accurate and efficient model for atmospheric turbulence. For weak turbulence log-normal is widely accepted model while for moderate to strong atmospheric turbulence Gamma-Gamma is a perfect distribution. And intensity Distribution is Gamma-Gamma Turbulence is given by[14]

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h_a^{\frac{\alpha+\beta}{2}} K_{\alpha-\beta}(2\sqrt{\alpha\beta}h_a) \tag{4}$$



where $\Gamma(\cdot)$ denotes Gamma function [15, eq.(8.310.1)], $K_\nu(\cdot)$ is the ν th-order modified Bessel function of the kind [15, eq.(8.432.2)], and α, β are the effective number of small-scale and large-scale eddies of scattering environment, respectively. According to [16], α and β can be obtained as

$$\alpha = \left[\exp\left(\frac{-49\sigma_r^2}{(1 + 1.11\sigma_r^{12/5})^{7/6}}\right) - 1 \right]^{-1} \quad (5)$$

$$\beta = \left[\exp\left(\frac{-51\sigma_r^2}{(1 + 69\sigma_r^{12/5})^{5/6}}\right) - 1 \right]^{-1} \quad (6)$$

where σ_R^2 is the Rytov variance defined as [15]

$$\sigma_R^2 = 1.23 C_n^2 k^7 L^{11/6} \quad (7)$$

where $k = 2\pi/\lambda$ is the optical number, λ is the wavelength, d is the propagation distance and $C_n^2(L)$ is the index of refraction structure parameter at altitude L [17].

C. Pointing Error Model

By considering a circular detection aperture of radius ρ and a Gaussian beam, the PDF of h_p can be derived using the assumptions and methodology described in [17] as

$$f_{h_p}(h_p) = \frac{\rho^2}{A_0^{\rho^2}} h_p^{\rho^2-1} \quad 0 \leq h_p \leq A_0 \quad (8)$$

The ratio of corresponding beam radius of the receiver to the pointing error displacement is denoted as $\rho = w_{zeq}/2\sigma_s$ and w_{zeq} is corresponding beam width with $w_{zeq}^2 = w_z^2 \sqrt{\pi} \operatorname{erf}(v)/2 v \exp(-v^2)$ and $A_0 = [\operatorname{erf}(v)]^2$ is the power received at $r=0$. error function is denoted as $\operatorname{erf}(\cdot)$ [5] (beam radius is calculated at e^{-2}) is referred as w_z beam waist and $v = \sqrt{\pi}r/\sqrt{2} w_z$.

D. Combined fading Model

Combined PDF for due to all these three fading parameter is given by following equation

$$f_h(h) = \int f_{h/h_a h_a}(h/h_a) f_{h_a}(h_a) dh_a \quad (9)$$

Where $f_{h/h_a h_a}(h/h_a)$ is the conditional probability given h_a state and given by

$$f_{h/h_a h_a}(h/h_a) = \frac{1}{h_a^{\rho^2}} f_{h_p}(h/h_a) = \frac{\rho^2}{A_0^{\rho^2} h_a^{\rho^2}} \left(\frac{h}{h_a}\right)^{\rho^2-1} \quad (10)$$

By substituting (4) and (10) into (9), the PDF of h is given

By

$$f_h(h) = \int_{h/A_0}^{\infty} \frac{\rho^2}{A_0^{\rho^2} h_a^{\rho^2}} \left(\frac{h}{h_a}\right)^{\rho^2-1} \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h_a^{\frac{\alpha+\beta}{2}} K_{\alpha-\beta}(2\sqrt{\alpha\beta}I) dh_a \quad (11)$$

According to [18, eq.(14)] and [18, eq.(26)], (11) can be rewritten as

$$f_h(h) = \frac{\alpha\beta\rho^2}{A_0 h_a \Gamma(\alpha)\Gamma(\beta)} \times G_1^3 \left[\begin{matrix} 0 \\ 3 \end{matrix} \left[\begin{matrix} 1 - \frac{\alpha+\beta}{2} + \rho^2 \\ -\frac{\alpha+\beta}{2} + \rho^2 \\ \frac{\alpha-\beta}{2} \\ \frac{\beta-\alpha}{2} \end{matrix} \right] \right] \quad (12)$$

Where $G_p^m \left[\begin{matrix} n \\ q \end{matrix} \right]$ is the Meijer's G-function [15, eq.(9.301)].

Using [19, eq.(9.31.5)], (12) can further simplified as

$$f_h(h) = \frac{\alpha\beta\rho^2}{A_0 h_a \Gamma(\alpha)\Gamma(\beta)} \times G_1^3 \left[\begin{matrix} 0 \\ 3 \end{matrix} \left[\begin{matrix} \frac{\alpha\beta}{A_0 h_a} \cdot h \\ \rho^2 - 1 \\ \alpha - 1 \\ \beta - 1 \end{matrix} \right] \right] \quad (13)$$

III. AVERAGE BER ANALYSIS

BER analysis plays a crucial role in FSO communication system. Mathematically, the BER of IM/DD with OOK can be given by $P(e) = p(1)p(e/1) + p(0)p(e/0)$, where $p(1)$ and $p(0)$ are the probabilities of sending 1 and 0 bits, respectively, and $p(e/1)$ and $p(e/0)$ denote the conditional bit error probabilities when the transmitted bit is 1 and 0, respectively. Assume that $p(0) = p(1) = 1/2$ and $p(e/1) = p(e/0)$, it is easy to show that conditioned on h , the bit error probabilities can be derived as follow

$$P(e/h) = \frac{1}{2} \operatorname{erfc}\left(\frac{P_t R h}{\sqrt{2}\sigma_n}\right) \quad (14)$$

$$P_b(e) = \int_0^{\infty} P_b(e/h) \cdot f_h(h) dh \quad (15)$$

Substituting (13) and (14) into (15), and using [20, eq.(06.27.26.0006.01)], [18, eq. (21)], [15, eq.(9.31.1)], a closed form expression for average BER is derived as

$$P(e) = \frac{2^{\alpha+\beta-4} \rho^2}{\sqrt{\pi}^3 \Gamma(\alpha)\Gamma(\beta)} \times G_6^2 \left[\begin{matrix} 5 \\ 6 \end{matrix} \left(\frac{8A_0^2 h_a^2 P_t^2 R^2}{\alpha^2 \beta^2 \sigma_n^2} \right) \left[\begin{matrix} \frac{2-\rho^2}{2} \\ \frac{1-\alpha}{2} \\ \frac{2-\alpha}{2} \\ 0 \\ \frac{1}{2} \\ \frac{2-\beta}{2} \end{matrix} \right] \right] \quad (16)$$

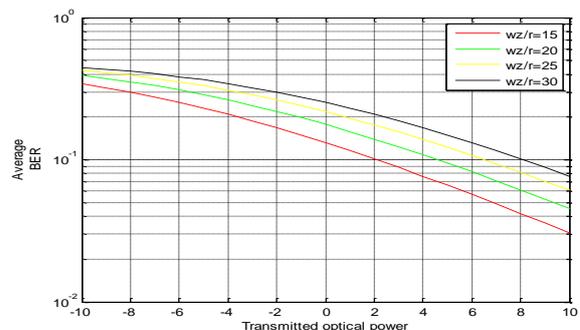


Fig 1 Average BER versus transmitted power for several value of normalized beamwidth.



IV. NUMERICAL RESULT

This paper considers FSO communication link in clear weather conditions. In this section, the analytical results are compared with the simulation results by Monte-Carlo simulations and the parameters of the FSO communication system are presented in Table I. Here, the derived closed-form expression of FSO communication system will be verified, and the impacts of transmitted power, transmitted distance and pointing errors on the system average BER will be discussed. Fig.1 illustrates the average BER versus transmitted power for various values of the normalized beam width. It can be seen from Fig.1 that the average BER decreases with the increase of the transmitted signal power. Meanwhile, BER performance will be better if a narrow beam width is used. It is because the received signal power is increased when a narrow beam width is used. Moreover, it can be found that simulation results show close agreement with analytical results.

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