

Variance Based Method for Signal Separation in Ultrasonic Non-Destructive Testing

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Abstract— This paper proposes a variance based method for ultrasonic defect detection for non-destructive testing of maraging steel. Maraging steel is a carbon free iron-nickel alloy which has superior strength and toughness. It also has a high malleability making possible for it to be easily machined and welded. Maraging steels are used extensively in the space industry for the construction of rocket motor casings, owing to its greater strength and fracture toughness. During its fabrication, defects like cracks may develop in the maraging steel. The cracks have a tendency to grow and spread, eventually leading to the fracture of the material. Non-destructive testing methods like ultrasound testing are used for the regular inspection of maraging steel rocket motor cases. Improving the probability of detection is a demanding task since the space industry has a very rigorous acceptance criteria and the permissibility of defects is very small. The sensitivity and resolution of ultrasonic systems is greatly reduced by the noise in the acquired ultrasound signals produced due to the coarse and textured microstructure of maraging steel. The main goal here is to successfully detect the defect signal hidden in the noise. Defects of a large size may be easier to detect, but the difficulty arises in the case of smaller defects which produces ultrasonic echoes whose amplitude is similar to that of the material noise. Successful detection of these smaller defects is essential for the space vehicle to achieve its designed payload capacity. The method presented here calculates and compares the variance of the acquired ultrasound signals, for separating the defect signal from noise. Further improvement in the detection can be achieved by comparing variance of Fourier transform coefficients of the acquired signals.

Index Terms— Maraging steel, non-destructive testing, ultrasound, Fast Fourier transform, QUT 2003, Variance.

I. INTRODUCTION

Various defects arise in the fabrication and use of materials. Non-destructive testing is the process of testing the materials or assemblies for defects in such a way that the serviceability of the part is not affected. That is, the part is still usable after the inspection. It is an improvement over other tests which are destructive in nature and are done on a limited number of samples, rather than on the actual parts being used. Ultrasonic testing is a family of non-destructive testing based on the propagation of ultrasonic waves through the material being tested. Ultrasonic testing uses high frequency sound waves in the range from 0.5 MHz to 15 MHz to conduct examinations. Ultrasonic testing consists of the capture and analysis of reflected waves, in case of *pulse-echo method*, or the transmitted waves in case of *through-transmission technique*.

Manuscript published on 30 June 2017.

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Pulse-echo method is more widely used since they require only one sided access to the material being inspected. High frequency sound waves are very directional in nature and they travel through a medium like plastic or steel, until they encounter a boundary with another medium, like air. At this boundary they are reflected back to the source. By analyzing this reflected signals, it is possible to detect the presence of any defects or cracks in the material. An ultrasonic testing system consists of a pulser/receiver, a transducer and a display device. Driven by the pulser, the transducer produces high frequency ultrasonic energy which is then introduced into the material being tested. The sound energy travels through the material and if it encounters a discontinuity such as a crack, part of the energy is reflected back from it. The reflected signal reaches the transducer where it is converted to an electrical signal and is displayed on a screen.

In ultrasonic testing, the voltage amplitude is proportional to the amount of energy echoed by the discontinuity or flaw. The front and back surfaces of the block generally reflect most of the energy, while flaws reflect much less. Thus the signal from both these surfaces appear as spikes of the highest amplitude. Signals from any flaws or discontinuities, if present, appear as spikes of smaller amplitude between the front and back-wall echoes. We are looking for these spikes which indicate the presence of cracks or faults in the maraging steel blocks. The whole material of the maraging steel blocks are examined by moving the angle probe along its surface, and examining the signal waveforms obtained. If a crack signal is obtained, the ultrasonic signal is paused and the signal values are saved. These numerical values can be used to re-create the signal waveform and also to do Signal processing operations.

Now from the signal values saved, variance of the signals are calculated. By analyzing the values we find that, for the fault signal, the values of variance lie around 0.04. Whereas for the noise signals the values lie around 0.1. So the noise signal has a high variance when compared to a defect signal. Following this, we take the analysis a little further and calculate the variance of the Fast Fourier Transform (FFT) of the signals. The signals are normalized and the Fast Fourier transform is taken. Now the variance of the FFT coefficients are calculated. The values of variance for FFT of defect signals fall in and around 0.1. While the variance of FFT of noise are much higher ranging from 2 to 45. This provides a much more conspicuous and better distinction than when taking the variance of the signal only. Thus the proposed method that makes use of variance, proves to be very efficient in distinguishing the defect signal from noise. MATLAB software from MathWorks is used for calculating the variance and for obtaining the Fast Fourier transform coefficients. QUT 2003 software is used for controlling the ultrasound parameters and for saving the signals of interest, as text files.

Microsoft Excel is used for properly tabulating the signals so that it can be imported into Matlab. Organization of this paper is as follows: Section II gives an overview of the basic theory. Section III provides the experimental setup. Section IV presents the proposed method. Section V and Section VI presents the result and a discussion of the result. Section VII concludes the paper.

II. THEORY

A. Maraging Steel

The term maraging steel is derived from the word "martensitic" and "aging". They are low-carbon ultra-high-strength steels that are known for possessing superior strength and toughness without losing malleability. *Ageing* refers to the extended heat treatment involved in its fabrication. Strengthening mechanism involves transforming the alloy to martensite with subsequent age hardening. They derive their strength from the precipitation of intermetallic compounds and the main alloying element is nickel. Tempering at 480° C to 500° C results in strong hardening due to the precipitation of a number of intermetallic phases. It has moderate corrosion resistance, good machinability and good weldability. They are widely used in the space industry for the creation of rocket motor casings. During fabrication, defects like cracks may develop in the maraging steel. These cracks may eventually grow in size or spread through the material leading to imminent fracture. To prevent this, parts fabricated from maraging steel are subjected to regular inspections using non-destructive testing methods such as ultrasound. Maraging steel has a coarse and textured micro-structure which introduces a lot of noise to the ultrasound waves which are used for inspection. This can negatively impact the successful detection of smaller defects present. Successful detection of these smaller defects is essential for the space vehicle to achieve its designed payload capacity.

B. Pulse-echo method

Ultrasonic testing involves the capture and analysis of reflected waves in case of *pulse-echo method*, or the transmitted waves in case of *through-transmission technique*. Pulse-echo method is more widely used since they require only one sided access to the material being inspected. Flaws are detected and their sizes estimated by comparing the intensity of sound reflected from a reference interface of known size or the back surface of a test specimen having no flaws. The reflection from the back surface serves as a reference point for the time level measurements, to locate the depth of some internal flaws. It is essential that the internal flaw reflect at least part of the energy, on to the receiving transducer, for making depth measurements. Short bursts of ultrasonic energy pulses or wave packets are introduced into a specimen at regular intervals of time. If the pulses encounter a reflecting surface, some or all the energy is reflected back. The proportion of the energy that is reflected is dependent on the area of the reflecting surface, the size of the incident beam, the orientation of the reflecting surface with respect to the incident beam and the nature of the reflecting surface.

Normally a single transducer acts as a sending and receiving transducer. The clock and signal generators are housed in a single electronic unit. At regular intervals the electronic

clock triggers the signal generator. The signal generator in turn imposes a short burst of high frequency alternating voltage on the transducer. Simultaneously the clock activates a time-measuring circuit connected to the display device. A constant interval between pulses is maintained by means of a "Pulse-repetition Rate" control on the instrument. The pulser circuit controls the duration of the pulse, called Pulse-layer and the timer controls the pulse repetition rate. Pulses are usually repeated 50 to 2000 times per second. The transducer converts the pulse of alternating voltage into a pulse of mechanical vibration. The mechanical vibration is introduced into the test specimen through a couplant, and travels by wave-motion through the test specimen at the speed of sound.

A large percentage of sound is reflected from the front surface of the test specimen back to the transducer and the remainder travels through the material. When the pulse of ultrasound encounters a reflecting surface of discontinuities within the material, that is perpendicular to the direction of the propagation of ultrasonic energy, it is reflected back and returns to the transducer. The returning pulse travels along the same path and at the same speed as the initial pulse, but in the opposite direction, reaches the transducer through the couplant and causes the transducer element to vibrate. This vibration induces an alternating voltage across the transducer. This induced voltage is amplified and sometimes demodulated and is fed into the display device. This process of alternately sending and receiving of pulses of ultrasonic energy is repeated for each successive pulse, with the display device recording any echoes each time.

C. Variance

Variance is the expectation of the squared deviation of a random variable from its mean. It informally measures how far a set of random numbers are spread out from their mean. Variance is an absolute measure of dispersion. It gives a very general idea of the spread of the obtained data. It is defined as the average of the squared difference between each of the observations in a set of data and the mean. The greater the actual variation in the values, the greater the different between the values used to calculate variance. A variance value of zero indicates that all values within a set of numbers are identical. All variances that are non-zero will be positive numbers. A large variance indicates that numbers in the set are far from the mean and each other, while a small variance indicates the opposite. The advantage of variance is that it treats all deviations from the mean the same, regardless of direction.

The general formula for variance is:

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

Instantaneous power of each sample is given by:

$$P = x.^2$$

From this *average power* of signal is calculated using:

$$P_{avg} = \frac{(\sum P)}{\text{length}(x)}$$

From these Variance of the signal is obtained using:

$$\text{Var} = \frac{(\sum P)}{\text{length}(x)} - \left(\frac{(\sum x)}{\text{length}(x)} \right)^2$$

D. Fourier Transform

The Fourier transform decomposes the signal which is a function of time into the frequencies that make it up. It gives a *frequency domain representation* of the original signal. It is a tool that breaks a waveform (a function or signal) into an alternate representation, characterized by sines and cosines. The Fourier Transform shows that any waveform can be re-written as the sum of sinusoidal functions. It gives us a unique way of viewing any function - as the sum of simple sinusoids. The Fourier transform of the function F is calculated by:

$$F_k = \int_{-\infty}^{\infty} f(x)e^{-2\pi ikx} dx$$

A Fast Fourier transform (FFT) algorithm computes the Discrete Fourier transform (DFT) of a sequence. It reduces the number of computations needed for N points from $2N^2$ to $2N \log_2 N$.

E. Signal Normalization

Normalization refers to more sophisticated adjustments where the intention is to bring the entire probability distributions of adjusted values into alignment. Normalization is a basic statistical operation. It's used to scale heterogeneous sets of data, so that they could be compared relevantly. Also normalization facilitates defining thresholds in different threshold algorithms. The data range decreases. Here a function *normalised_diff* is created in Matlab to normalize the signals and confine them to the range [-1:1].

III. EXPERIMENTAL SETUP

A. PCUS 11 Pulser/Receiver Board

The PCUS 11 PC integrated ultrasonic board (PCB), is special ultrasonic PCB, designed to meet the increasing requirements with respect to high data acquisition rates and high inspection speeds. In combination with custom software modules, the user can inspect components effectively with high quality and confidence levels. All components required for ultrasonic testing are contained on a single PCB, including transmitter, receiver, A/D converter, signal processor and A-scan memory. The efficient, powerful BUS-interface (PCI/Master DMA) allows a high data acquisition and high data transfer rates. It provides a selectable display of RF or rectified A-scans through DSP signal processing. Integration with PC provide comfortable data analysis and data management. It has a standard frequency in range 0.5 MHz to 20 MHz, with custom frequency-filter configurations. It has a dynamic range of 110 dB adjustable in 0.1 dB increments. There is a built-in 256 point DAC (40 dB range) adjustable in 0.1 dB increment. It has A-scan memory of 64k samples. It has connectors for P/E, transmitter and receiver, external trigger-in/out. It has dimensions 105 mm x 312 mm.

B. Angle-beam Inspection

Angle beam inspection uses the same type of transducer but it is mounted on an angled wedge that is designed to transmit the sound beam into the part at a known angle. The most commonly used inspection angles are 45°, 60° and 70°, with the angle being calculated up from a line drawn through the thickness of the part to be inspected. In angle beam inspections, the transducer and wedge combination (also referred to as a "probe") is moved back and forth towards the weld so that the sound beam passes through the full volume of the weld. As with straight beam inspections, reflectors aligned more or less perpendicular to the sound beam will send sound back to the transducer and are displayed on the screen. Angle beam assemblies, consisting of a transducer and a wedge, are extremely important to the field of ultrasonic nondestructive testing, commonly used in a wide variety of weld inspection applications and also for detection of cracks oriented perpendicular to the surface in metal plates, pipes, billets and forgings, as well as machined and structural components. Cracks or other discontinuities perpendicular to the surface of a test piece, or tilted with respect to that surface, are usually invisible with straight beam test techniques because of their orientation with respect to the sound beam. Perpendicular cracks do not reflect any significant amount of sound energy from a straight beam because the beam is looking at a thin edge that is much smaller than the wavelength, and tilted cracks may not reflect any energy back in the direction of the transducer. This situation can occur in many types of welds, in structural metal parts, and in many other critical components. An angle beam assembly directs sound energy into the test piece at a selected angle. A perpendicular crack will reflect angled sound energy along a path that is commonly referred to as a corner trap.

Sound energy at ultrasonic frequencies is highly directional and the sound beams used for flaw detection are well defined. In situations where sound reflects off a boundary, the angle of reflection equals the angle of incidence. A sound beam that hits a surface at perpendicular incidence will reflect straight back. A sound beam that hits a surface at an angle will reflect forward at the same angle. Sound energy that is transmitted from one material to another bends in accordance with Snell's Law of refraction. Refraction is the bending of a sound beam (or any other wave) when it passes through a boundary between two materials of different velocities. A beam that is traveling straight will continue in a straight direction, but a beam that strikes a boundary at an angle will be bent according to the formula:

$$\sin \theta_i / c_i = \sin \theta_{rl} / c_{rl} = \sin \theta_{rs} / c_{rs}$$

where:

- θ_i = Incident angle of the wedge
- θ_{rl} = Angle of the refracted longitudinal wave
- θ_{rs} = Angle of the refracted shear wave
- c_i = Velocity of the incident material (longitudinal)
- c_{rl} = Material sound velocity (longitudinal)
- c_{rs} = Velocity of the test material (Shear)

This is called Snell's law.

Typical angle beam assemblies make use of mode conversion and Snell's Law to generate a shear wave at a selected angle (most commonly 30, 45, 60, or 70 degrees) in the test piece. As the angle of an incident longitudinal wave with respect to a surface increases, an increasing portion of the sound energy is converted to a shear wave in the second material, and if the angle is high enough, all of the energy in the second material will be in the form of shear waves. There are two advantages to designing common angle beams to take advantage of this mode conversion phenomenon. First, energy transfer is more efficient at the incident angles that generate shear waves in steel and similar materials. Second, minimum flaw size resolution is improved through the use of shear waves, since at a given frequency, the wavelength of a shear wave is approximately 60 % the wavelength of a comparable longitudinal wave, and minimum flaw size resolution increases as the wavelength of a sound beam gets smaller.

C. QUT2003

The application software used for non-destructive testing is QUT2003. It is a data acquisition software for the use with our PCUS 11 hardware. QUT2003 is conceived for manual operation in the field as well in a laboratory and offers multiple windows, pull-down menus, toolbar commands, and many other features. QUT is conceived for manual operation in the field as well as in the laboratory. All controls can be operated either from the keyboard or the mouse. The mouse operation even allows for alphanumeric input through a pop-up keyboard that is displayed on the screen. This feature is important when a file name is to be entered and the PC can only be operated by a touch screen or mouse. QUT has all the functions and facilities to setup, calibrate, test and document manual ultrasonic tests. There are special routines for thickness measurement. Within a project, all relevant data are stored. These include UT measurement (traces), testing conditions, probes and personnel.

D. Data acquisition from maraging steel specimen

Inspection is done using angle probes. Angle probes of both 45° and 60° are used. The maraging steel blocks are placed on bubble-wrap sheet so as to insulate it and prevent the dissipation of ultrasonic energy on to the table surface. Both the surfaces of the blocks are properly cleaned using cotton-waste so remove any oil, as it may hinder proper inspection. Inspection is done using angle probes. Oil is used as couplant for the proper conduction of ultrasonic energy between the transducer and the maraging steel block.

The following settings need to be ensured before the commencement of ultrasonic inspection of the specimen.

Calibration

From Options menu select Calibration.

Set Probe delay to 0.1.

Set Velocity to 2900 m/s

The Settings dialog box has three tabs namely: Set1, Set2 and Probes.

The following values have to be set for Set1, Set2 and Probes.

Set1

Gain: 42.4 dB

Range: 100 mm

Shift: 0.2 mm

Rejection: 0

43

5.5. Data Acquisition from Maraging steel specimen

Avg: 64

Set2

Display: RF

X-axis: distance

Pulser imp: 300

Rec imp: 300

Sample rate: 40

Pulser Energy: 550

Trace hold: No

Probe

Filter: 4.5 (2.25 - 9) MHz

Freq: 0 MHz

Probe: Single/PE

Wave mode: Shear

Angle decrease: 45

The maraging steel blocks and contact area of angle probe are thoroughly cleaned with cotton waste to remove any oil or dirt and placed on the bubble-wrap sheet. Oil is applied to the area to be inspected and at the base of the probe which acts as the couplant. Ultrasound is turned *ON* from the QUT software. The angle probe is pressed on the inspecting area of maraging steel block and the waveforms which appear on the QUT software are examined.

In ultrasonic testing, the voltage amplitude is proportional to the amount of energy echoed by the discontinuity or flaw. The front and back surfaces of the block generally reflect the most energy, while flaws reflect much less. Thus the signal from both these surfaces appear as spikes of the highest amplitude. Signals from any flaws or discontinuities, if present, appear as spikes of smaller amplitude between the front and back-wall echoes. We are looking for these spikes which indicate the presence of cracks or faults in the maraging steel blocks.

The whole material of the maraging steel blocks are examined by moving the angle probe along its surface and examining the signal waveforms obtained. If a crack signal is obtained, the ultrasonic signal is turned *OFF*. Now the signals are saved using 'Save as ASCII' option on the QUT interface. Using this option, the signal can be saved as a text file with the numerical values pertaining to the current signal waveform. These numerical values can be used to recreate the signal waveform and also to do a variety of various Signal processing operations.

After the defect signals are saved, again the ultrasound signal is turned *on* from the QUT, and the remaining portions of maraging steel block is examined. After the block has been completely examined for defects, and the signals have been saved the block is now inverted and placed on the bubble-wrap sheet. Now the probe is placed on a region devoid of any defect.

The signal in this region is only due to the noise in the material. Maraging steel a highly attenuative material owing to its grainy micro-structure. The gain of the obtained noise signal is increased from the QUT interface to a point when the amplitude of this noise signal becomes equal to the amplitude of spike due to the defect signal obtained previously. At this point, the signal is stopped and the values are saved as before. Both the fault and the noise signals are saved for further processing using various signal processing algorithms.

IV. PROPOSED METHOD

A. Using Variance of the signals

To analyze the signal we estimate how much the signal varies around the average. This property of a signal is called signal variance. Real-world signals have noise and other short-term features that obscure the underlying behavior of the signal. Taking an average of the signal removes some of that noise, but it may still be desirable to know how much the signal was changing to get a sense of the volatility of the signal. One way to get an idea of how quickly a signal is changing is by looking at how quickly the averaged signal changes, but this can be misleading if there is a lot of noise but the underlying signal isn't changing. A better way to calculate how much a signal is changing is to find the difference between the original signal and its average. The average of the squared differences is called the signal variance.

This is done in Matlab software. The signals are saved as text files from the QUT software. The values are then imported to Microsoft Excel and tabulated so that they can be imported to Matlab. The signals are normalized and then variance is calculated.

B. By taking variance of FFT coefficients of the signals

Here we take the analysis a little further and calculate the variance of the Fast Fourier transform of the signals. The signals are normalized and the Fast Fourier transform is taken. Now the variance of the FFT coefficients are calculated. Again Matlab software is used to perform the calculations. The signals which were saved from QUT software are tabulated using Microsoft Excel and imported to Matlab where the FFT coefficients are generated and their variances are calculated.

V. RESULT

A. Variance of the signals

The following values of variance are obtained for the defect signals.

Table 1: Variance values of Defect Signals

Fault Signals	Variance
F1	0.0406
F2	0.0414
F3	0.0409

Fault Signals	Variance
F4	0.0408
F5	0.0399
F6	0.0398
F7	0.0398
F8	0.0394
F9	0.0397
F1	0.0401
F1	0.0397
F1	0.0394
F1	0.0392
F1	0.0395
F1	0.0397
F1	0.0396
F1	0.04
F1	0.04
F1	0.04
F2	0.0401

For noise signals the values of variance obtained are:

Table 2: Variance values of Noise Signals

Noise Signals	Variance
N1	0.0904
N2	0.1369
N3	0.1371
N4	0.1667
N5	0.1678
N6	0.1233
N7	0.1436
N8	0.0917
N9	0.1273
N10	0.1787
N11	0.1792
N12	0.0899
N13	0.0932
N14	0.0918
N15	0.0908
N16	0.0909
N17	0.0987
N18	0.1695
N19	0.1924
N20	0.1071

Variance Based Method for Signal Separation in Ultrasonic Non-Destructive Testing

B. Variance of FFT coefficients of signal

For fault signal the values obtained for variance of FFT coefficients are:

Table 3: Variance values of FFT coefficients Defect Signals

Fault Signals	Variance of FFT
F1	0.023983
F2	0.096769
F3	0.12887
Fault Signals	Variance of FFT
F4	0.095984
F5	0.11994
F6	0.11947
F7	0.0048566
F8	0.17047
F9	0.022261
F10	0.011483
F11	0.11553
F12	0.13045
F13	0.15614
F14	0.15614
F15	0.086566
F16	0.056862
F17	0.2198
F18	0.18282
F19	0.062652
F20	0.14827

For noise signals the values obtained for variance of FFT coefficients are:

Table 4: Variance values of FFT coefficients Noise Signals

Noise Signals	Variance of FFT
N1	29.5674
N2	24.9697
N3	24.7145
N4	4.5516
N5	8.7015
N6	40.4937
N7	6.7117
N8	8.3759
N9	2.6222
N10	4.1009
N11	3.0571
N12	4.89
N13	13.7833
N14	2.0963
N15	12.7653
N16	5.8854

N17	45.03
N18	6.2848
N19	2.4103
N20	17.6012

VI. DISCUSSION

By analyzing the values we find that, for the fault signal, the values of variance of signals lie around 0.04. Whereas for the noise signals the values lie around 0.1. So the noise signal has a high variance when compared to a defect signal. Thus by calculating the variance of the signal, we may be able to ascertain whether it is a noise or defect signal. Hence, variance method proves to be a good technique for differentiating between defect and noise signal.

By analyzing the values of variance of FFT coefficients of the signals, we find that the defect signals have a very low value of variance when compared to the noise signals. The values of variance for defect signals fall in and around 0.1. While the variance of noise are much higher ranging from 2 to 45. This provides a much more conspicuous and better distinction than when taking the variance of the signal only. Thus the method of analysis using variance of FFT coefficients of the signal is very efficient method to distinguish the defect signal from noise.

VII. CONCLUSION

The proposed method attempts to separate the defect signal from the noise by making use of the variance of the signals. First the variance of the signals are taken directly. It is observed that the values of variance of signals lie around 0.04. Whereas for the noise signals the values lie around 0.1. Subsequently, the Fast Fourier transform of the signals are taken and the values of its variance are calculated. The values of variance for defect signals fall in and around 0.1. While the variance of noise are much higher ranging from 2 to 45. This provides a much more conspicuous and better distinction than when taking the variance of the signal only. Thus both methods can be used to separate the defect signals from noise. The second method proves to be a better choice as the separation of values are more evident.

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