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Probability Distribution of Rainfall and Discharge of Kulsi River Basin

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Abstract: ***The frequency analysis of daily rainfall data of 24 years was carried out to determine the annual one day maximum rainfall and discharge of Ukiam. For evaluation of observed and expected values Weibull's plotting position Gumbel, Log Pearson and Log normal probability distribution functions were fitted. For determination of goodness of fit chi square test was carried out by comparing the expected values with the observed values. The results found showed that the Log Pearson and Log Normal were the best fit probability distribution for determination of annual one day maximum rainfall and discharge for different return periods respectively.***

Keywords: Probability distribution, Chi-Square Value

I. INTRODUCTION

Hydrologic design is the process of assessing the impact of hydrologic events on a water resource system and choosing values for the key variables of the system so that it performs adequately (Chow 1988). Surface water hydrology deals with movement of water along the watershed as a result of precipitation. Detailed analysis of surface water flow rate is highly important to fields such as water supply, flood control, reservoir design, stream flow forecasting, irrigation, and water quality control. For economic and efficient design of hydraulic structures, such as spillways, bridges, drainage works and aqueducts, a reliable estimate of the flood discharge is essential. One of the most important problems in hydrology deals with interpreting past records of hydrological events in terms of probabilities of occurrence. Probability and frequency analysis of rainfall data enables us to determine the expected rainfall at various chances. Hydraulic engineers require maximum daily rainfall of different return periods for planning and designing of small and medium hydraulic structures such as dams, culverts, bridges, drainage works etc. It also helps in forecasting floods at sections in a river basin. Probability distribution function is used for prediction of maximum daily rainfall for higher return periods which fit the observed rainfall data approximately.

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II. LITERATURE REVIEW

A brief review is covered in this section about frequency analysis in various studies of watersheds. A detailed statistical analysis of weekly and monthly rainfall for Kota was carried out using 35 years daily rainfall data.

Probability analysis of weekly and monthly rainfall series was carried out by employing four probability distributions namely Normal, Log Normal, Weibull and Gumbel distributions. Goodness of fit of these distributions was tested by Chi- square test. For weekly maximum rainfall Weibull's distribution is found best fit distribution for rainy season and Gumbel distribution is non rainy season. The Gumbel distribution was found best fit for monthly maximum rainfall at Kota (S. R. Bhakar et.al. 2008).

Estimation of probable one day maximum rainfall for different return periods was carried out in Ludhiana Punjab by using probability distribution function. The probability distributions namely Log Normal, Gumbel and Log Pearson Type-III distribution had been used to determine the best fit probability distribution for the annual one day maximum rainfall. The result showed that the average of annual one day maximum rainfall was 105.9 mm with standard deviation, coefficient of variation and coefficient of skewness of 64, 0.604 and 2.2 respectively. The study concluded that Log Pearson type-III distribution was found to be the best method through chi square test (Rajneesh Kumar 2015).

Weibull's plotting position was used to determine the maximum daily rainfall; estimate of't' year rainfall in Silchar, Assam. The expected values were estimated by using probability function such as Normal, Log normal, Log Pearson type III distribution and Gumbel distribution. The chi-square test was done to compare the expected values with the observed values for determination of goodness of fit. The result showed that the Log Pearson type III distribution was the best fit distribution for estimation of annual maximum daily rainfall for different return periods (kaushik bora and P. Choudhury, 2015).

III. STUDY AREA

The Kulsi River is a prominent south bank tributary of mighty river Brahmaputra. The river originates from the northern slope of the Khasi hills of Meghalaya and flows to the north and enters Kamrup district of Assam. The proposed dam site is located across the Kulsi River at Ukiam village. The latitude and longitude is between 25°48´58.88´´ N and 91°19´26.75´´ E.

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The soil in the Kulsi catchment is mostly reddish in colour and texturally loamy or sand. The soils are medium acidic and surface horizon are rich in organic matter.

Fig 1: Catchment of Kulsi River

IV. MATHEMATICAL MODEL AND METHODOLOGY

Annual maximum daily rainfall and discharge data of proposed Ukiam dam collected from Water Resource Department, Guwahati for 24 years. Three probability distribution functions, Log normal, Log Pearson type-III and Gumbel distribution were used to estimate daily maximum rainfall and daily maximum discharge.

Weibull's Formula:

Probability

$$(P) = \frac{m}{N+1} \quad (1)$$

$$\text{Return period } (T) = \frac{N+1}{m} \quad (2)$$

Where N = the number of years record

m= the rank of observed rainfall values arranged in descending order

A. Frequency Analysis

Based on theoretical probability distributions, it could be possible to forecast the incoming rainfall of various magnitudes with different return periods. The probability distributions, most commonly used to estimate the rainfall frequency are Log-normal distribution, Log-Pearson Type-III distribution, Gumbel distribution. Rainfall analysis by theoretical probability distributions can be done by using frequency factor 'K' which is based on some statistical parameters Chow (1964).

B. Log Normal Distribution

In this distribution, the logarithm of the annual one day maximum rainfall/discharge was taken to base 10. If X is the variate of a random hydrologic series, then the series of Z variates where

$$Z = \log X \quad (3)$$

For this Z series, for any recurrence interval T

$$Z = \bar{Z} + K_z \sigma_z \quad (4)$$

$$\sigma_z = \sqrt{\frac{1}{N-1} \sum_{i=1}^{n} (Z - \bar{Z})^2} \quad (5)$$

Where,

σ_z is standard deviation of the Z variate sample

$\bar{Z}$ is mean of Z

N is the sample size= number of years of record

The frequency factor for normal distribution is expressed as

$$K_T = w - \frac{2.515517 + 0.802853w + 0.01032w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3} \quad (6)$$

Where,

$$W = \left[ln\left(\frac{1}{P^2}\right) \right]^{\frac{1}{2}} \qquad 0 < P \leq 0.5 \quad (7)$$

Where

p is probability of exceedence and w is intermediate variable When P > 0.5, 1-p is substituted and K_T value computed is given as negative sign. The expected value of rainfall/discharge 'X_T', at return period T, can be obtained from the relation

$$X_T = \text{antilog}\ (Z_T) \quad (8)$$

C. Log Pearson Type III Distribution

In log-Pearson type-III distribution, the value of variate 'X' (rainfall/discharge) is transformed to logarithm (base 10). The expected value of rainfall X_T can be obtained by the following formulae

$$X_T = \text{Antilog}\ (Z_T) \quad (9)$$

$$\text{Log } Z = \bar{Z} + K_T \sigma_z \quad (10)$$

Where,

$\bar{Z}$ is the mean of logarithmic values of observed data

σ_z is the standard deviation as previously defined

Frequency factor K_T is taken from Benson (1968) corresponding to coefficient of skewness (C_s) of transformed variate as

$$K_T = \frac{2}{C_s} \left[\left\{ (Z - C_s)\frac{C_s}{6} + 1 \right\}^3 - 1 \right] \quad (11)$$

D. Gumbel Distribution

In Gumbel distribution, the expected rainfall /discharge X_T is computed by the following formula

$$X_T = \bar{X}(1 + C_V K_T) \quad (12)$$

Where,

$\bar{X}$ Is mean of the observed data

C_V *is* the coefficient of variation;

K_T is frequency factor which is calculated by the formula given by Gumbel (1958) as

$$K_T = -\frac{\sqrt{6}}{\pi} \left\{ 0.5772 + ln\left[ln\left(\frac{T}{T-1}\right) \right] \right\} \quad (13)$$

E. Testing The Goodness of Fit of Probability Distribution

The best fit distributions decided by chi-square test for goodness of fit to observed values. The chi-square test statistic is given by the equation as:

$$x^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \quad (14)$$

Where, O_i is the observed rainfall and E_i is the expected rainfall.

The best probability distribution function was determined by comparing Chi square values obtained from each distribution and selecting the function that gives smallest chi-square value.

Regression model: Regression model were developed for estimating the annual maximum daily rainfall and annual maximum discharge to return periods and found the coefficient of determination (R^2).

V. OBSERVATIONS AND RESULTS

In this section, the observation tables and related graphs of frequency analysis of rainfall and discharge of the study area have been presented. The expected values of maximum rainfall and discharge were calculated by probability distributions, *viz.* Gumbel, Log-Pearson Type-III and Log normal distribution at different selected probabilities. All probability distribution functions were compared by chi square test of goodness of fit and then selecting the function that gave the smallest chi-square value determined the best probability function. Regression models were developed from the observed data against different return period by using Weibull's method.

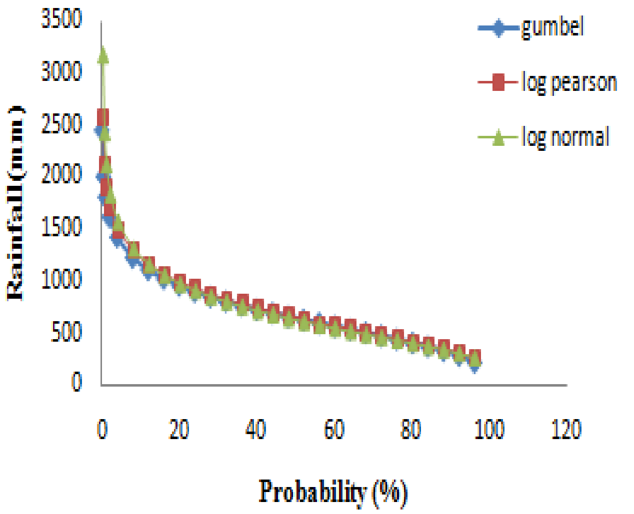

Fig.2: Estimated Annual One Day Maximum Rainfall at Different Probability Levels

Table 1: Comparison of Rainfall using Gumbel, Log Pearson Type III, Log Normal Method

Sl. No	Probability (%)	Return Period (years)	Observed Rainfall (mm)	Expected Rainfall (mm)		
				Gumbel	Log Pearson Type III	Log normal
1	96	1.042	60	19.047	56.234	48.035
2	92	1.09	61.2	33.549	61.659	55.498
3	88	1.136	63.4	44.05	64.565	61.525
4	84	1.19	68.58	52.917	67.608	66.927
5	80	1.25	68.8	60.851	70.795	71.939
6	76	1.316	70	98.163	74.131	76.789
7	72	1.389	72.39	75.085	77.625	81.599
8	68	1.471	81	81.852	81.283	86.28
9	64	1.563	82.2	88.463	85.114	91.048
10	60	1.667	84.2	94.997	89.125	95.048
11	56	1.786	98	101.686	93.325	101.086
12	52	1.923	98.04	108.453	97.724	105.773
13	48	2.083	100.6	115.454	102.329	111.34
14	44	2.273	108.8	122.765	107.152	117.084
15	40	2.5	112.4	130.465	112.2	123.001
16	36	2.778	112.6	138.632	120.23	129.992
17	32	3.125	123.19	147.499	125.89	137.244
18	28	3.571	137.16	157.222	134.89	144.756
19	24	4.167	138.5	168.112	144.54	154.592
20	20	5	172.72	180.634	154.88	164.849
21	16	6.25	173.25	195.646	173.78	177.902
22	12	8.333	225	214.469	194.98	195.656
23	8	12.5	290	240.371	234.42	219.401
24	4	25	391	283.695	301.99	259.883
25	2	50	-	326.319	389.045	301.762
26	1	100	-	368.709	489.78	345.1937
27	0.5	200	-	410.867	630.957	392.915

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Table 2: Chi-Square Values at Different Probability Levels for Different Distributions

Sl.no	Probability (%)	Return Period (years)	Gumbel	Log Pearson Type III	Log normal
1	96	1.042	88.05314	0.25221	2.980352
2	92	1.09	22.78988	0.003417	0.585837
3	88	1.136	8.499943	0.021021	0.057141
4	84	1.190	4.63612	0.013974	0.040827
5	80	1.25	1.038382	0.056219	0.136968
6	76	1.316	8.079975	0.230203	0.600223
7	72	1.389	0.096731	0.353046	1.039298
8	68	1.471	0.008868	0.000985	0.323115
9	64	1.563	0.443408	0.099765	0.859844
10	60	1.667	1.227146	0.272153	1.238102
11	56	1.786	1.33613	0.234188	0.094211
12	52	1.923	0.999793	0.001022	0.565355
13	48	2.083	1.911076	0.029214	1.035994
14	44	2.273	1.588573	0.025346	0.586115
15	40	2.5	2.501393	0.000357	0.913661
16	36	2.778	4.888229	0.4842130	2.326925
17	32	3.125	4.006315	0.057908	1.439152
18	28	3.571	2.559972	0.038201	0.398596
19	24	4.167	5.21599	0.252398	1.67507
20	20	5	0.346731	2.054917	0.375814
21	16	6.25	2.563716	0.001616	0.121646
22	12	8.333	0.5171	4.622015	4.40094
23	8	12.5	10.24682	13.17779	22.71739
24	4	25	40.58712	26.23524	66.15157
CHI-SQUARE VALUE = x^2			212.9452	48.5174	110.6641

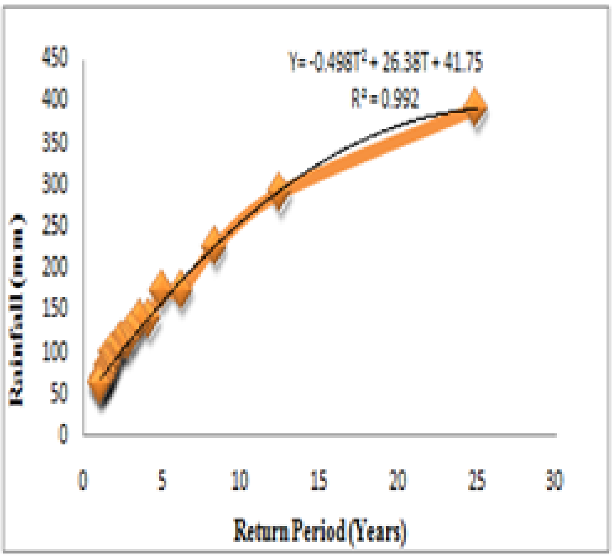

Fig.3: Annual One Day Maximum Rainfall Vs Return Period by Weibull's Method

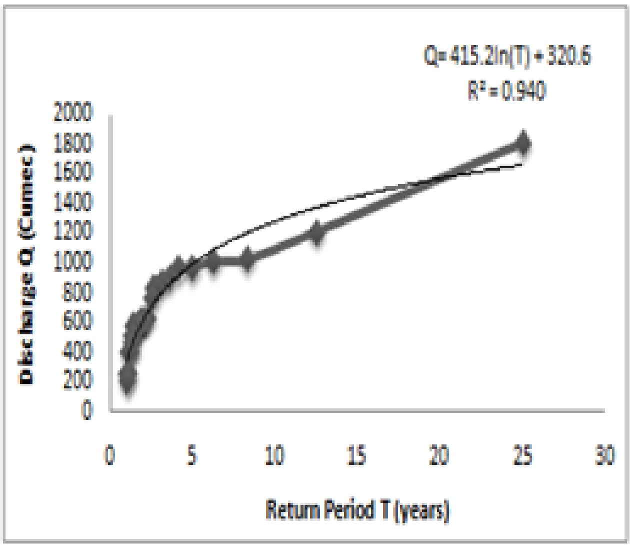

Fig.4: Annual One Day Maximum Discharge Vs Return Period by Weibull's Method

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Table 3: Comparison of Discharge Value using Gumbel, Log Normal, and Log Pearson Method

Sl. No	Probability (%)	Return Period (years)	Observed Discharge (Cumec)	Expected Discharge (CUMEC)		
				Gumbel	Log Pearson Type III	Log normal
1	96	1.042	200.000	224.761	255.548	266.385
2	92	1.09	246.550	290.949	303.707	309.771
3	88	1.136	267.179	338.73	344.339	345.633
4	84	1.190	401.800	379.081	378.739	377.602
5	80	1.250	406.770	415.183	411.379	407.348
6	76	1.316	449.615	448.454	442.642	436.309
7	72	1.389	503.090	479.955	473.548	464.837
8	68	1.471	590.800	510.748	503.441	492.889
9	64	1.563	563.330	540.833	533.823	521.721
10	60	1.667	576.060	570.565	563.969	550.486
11	56	1.786	600.000	601.049	579.066	582.386
12	52	1.923	617.060	631.797	626.279	610.534
13	48	2.083	623.370	663.652	660.850	644.364
14	44	2.273	632.450	696.923	695.982	679.313
15	40	2.500	761.160	731.963	732.214	715.442
16	36	2.778	845.880	769.127	774.374	758.250
17	32	3.125	863.430	809.477	818.105	802.767
18	28	3.571	895.371	853.719	862.498	849.083
19	24	4.167	961.880	903.272	919.827	909.656
20	20	5.000	969. 530	960.257	978.916	973.383
21	16	6.250	1012.260	1028.568	1052.759	1054.302
22	12	8.333	1022.470	1114.223	1153.095	1162.797
23	8	12.500	1209.780	1232.086	1280.292	1313.053
24	4	25.000	1807.07	1429.233	1490.057	1573.619
25	2	50.000	-	1623.195	1689.413	1833.011
26	1	100.000	-	1816.094	1895.502	2117.026
27	0.5	200.000	-	2007.932	2104.593	2425.926
28	0.1	1000.000	-	2452.840	2567.508	3171.686

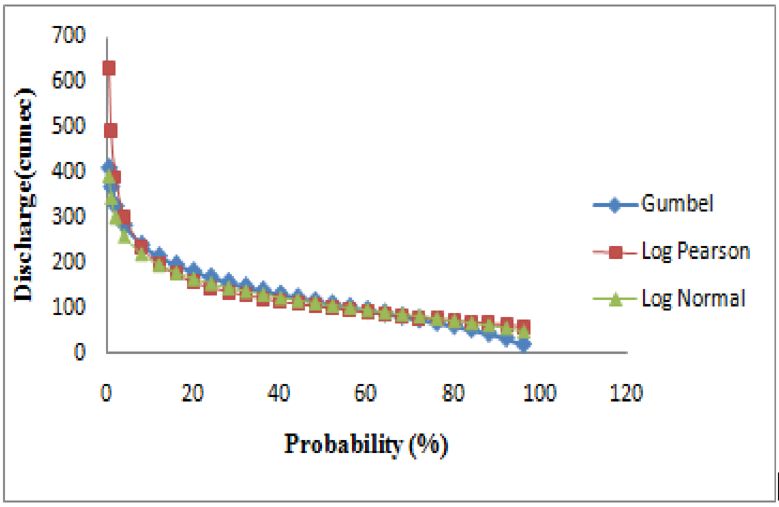

Fig: 5: Estimated Annual One Day Maximum Rainfall at Different Probability Levels

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Table.4: Chi-Square Values at Different Probability Levels for Different Distributions

Sl. No	Probability (%)	Return Period (years)	Gumbel	Log Pearson Type III	Log normal
1	96	1.042	2.728	12.074	16.544
2	92	1.09	6.775	10.757	12.903
3	88	1.136	15.114	17.290	17.808
4	84	1.190	1.3616	1.4042	1.5507
5	80	1.25	0.17048	0.0516	0.00082
6	76	1.316	0.00301	0.1098	0.4058
7	72	1.389	1.11516	1.8429	3.1479
8	68	1.471	12.547	15.159	19.449
9	64	1.563	0.9358	1.631	3.3185
10	60	1.667	0.05292	0.259	1.1881
11	56	1.786	0.00183	0.7568	0.53273
12	52	1.923	0.34375	0.1357	0.0698
13	48	2.083	2.44502	2.126	0.6840
14	44	2.273	5.9645	5.799	3.2328
15	40	2.5	1.1646	1.144	2.9215
16	36	2.778	7.6596	6.603	10.127
17	32	3.125	3.5961	2.511	4.584
18	28	3.571	2.0322	1.253	2.523
19	24	4.167	3.8027	1.923	2.998
20	20	5	0.08955	0.0899	0.0153
21	16	6.25	0.2585	1.558	1.676
22	12	8.333	7.5555	14.797	16.935
23	8	12.5	0.40383	3.883	8.123
24	4	25	99.886	67.445	34.633
CHI-SQUARE VALUE $=x^2$			176.007	170.605	165.372

The frequency analysis of ADMR (annual daily maximum rainfall) and ADMD (annual daily maximum discharge) for identifying the best fit probability distribution can be studied for probability distributions such as Gumbel, Log-Pearson Type III, and Log-normal. Waybill's method was used to estimate the probability in percentage and return period in years. The expected ADMR and ADMD were calculated by using probability distribution functions such as Gumbel, Log-Pearson Type-III and Log normal distribution. The expected ADMR and ADMD were graphically presented in fig (2) and (5). The distribution functions were compared by chi square test of goodness of fit and found that the smallest chi square value which would be considered as the best probability distribution function. The chi square values for Gumbel, Log-Pearson Type III and Log normal in case of ADMR and ADMD were 212.9452, 48.5174, 110.6641 and 176.007, 170.605, 165.372 respectively. Regression models for ADMR and ADMD were developed by using Weibull's method to different return periods in fig (3) and (4).

VI. CONCLUSION

The expected ADMR and ADMD for different probability distributions such as Gumbel, Log-Pearson Type-III and Log-normal were calculated for different return periods. Log-Pearson Type-III distribution gave the lowest calculated chi-square value for ADMR and Log Normal distribution gave the lowest chi-square value for ADMD among the probability distribution. Regression models for ADMR and ADMD were developed by using Weibull's method to predict the rainfall and discharge for different return period .The trend analysis for prediction of one day maximum rainfall and discharge for different return period was carried out and it is found that the polynomial trend line gave better coefficient of determination (R^2) = 0.992 for ADMR and (R^2)=0.942 for ADMD. This study helps for prediction of ADMR and ADMD to design hydraulic structure.

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