OPEN ACCESS
International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-6 Issue-1, October 2016

Turbulent Functions and Solving the Navier-Stokes Equation by Fourier series

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***Abstract:** I give a resolution of the Navier-Stokes [2] equation by using the series of Fourier. Résumé: Je donne une résolution de l'équation de Navier-Stokes [2] par les séries de Fourier.*

***Keywords:** Navier-Stokes, Fourier, Séries de Fourier.*

I. INTRODUCTION

The Navier–Stokes equations is considered to be the first step to understanding the elusive phenomenon of turbulence, the Clay Mathematics Institute in May 2000 made this problem [2] one of its seven Millennium Prize problems in mathematics. In this article I will prove that the Navier-Stokes equation has a solutions and I will give techniques to resolve this beautiful equation. The Navier-Stokes equation, established in the nineteenth century by the French Navier and the British Stokes. It is an equation that describes the velocity field of a fluid. More specifically, it is a differential equation whose velocity field is unknown.

The Navier-Stokes equation is also used to predict the weather, the oceans simulate, optimize aircraft wings ... Knowing that a link between the Boltzmann equation and the Navier-Stokes equation was established, by studying the latter problem, I found that for to solve it we can reduce the problem of the heat-equation which is known can be solved by several methods : one of the first methods of solving the heat-equation was proposed by Joseph Fourier in his treatise analytical Theory of heat [1] in 1822. After giving a specific solution to the Navier-Stokes equation, I will demonstrate how to find all solutions of this equation if they exist, and I give the necessary and sufficient conditions for their existence. It will be seen in a remark that if the turbulence function is negligible, then the fluid will tend to behave like an ideal gas.

II. RECALL, NOTATIONS AND DEFINITIONS

Here are the Navier-Stokes equation:

$$\rho\left(\frac{\partial u}{\partial t}+(u.\ \nabla u)\right)=-\nabla p+\mu\ \nabla^2 u$$

$$\operatorname{div} u=0$$

Where u is the velocity field, p is the pressure, the density of the fluid, and μ its viscosity.

Manuscript published on 30 October 2016.
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And:

$$\nabla=\left(\frac{\partial}{\partial x_1},\ldots,\frac{\partial}{\partial x_n}\right)$$

$$(u.\ \nabla u)=\sum_1^n u_i\frac{\partial u}{\partial x_i}$$

$$\nabla^2 u=\sum_1^n\frac{\partial^2 u}{\partial x_i^2}$$

$$\operatorname{div} u=\sum_1^n\frac{\partial u}{\partial x_i}$$

In the following, by dividing by ρ , the Navier-Stokes equation is of the form:

$$\frac{\partial u}{\partial t}+(u.\ \nabla u)=\alpha\ \nabla p+\beta\ \nabla^2 u$$

$$\operatorname{div} u=0$$

III. EXISTENCE OF THE SOLUTIONS FOR THE NAVIER-STOKES EQUATION:

On each axis i, try to find the solutions of the form:

$$\frac{\partial u_i}{\partial t}+\left(u_i\frac{\partial u_i}{\partial x_i}\right)=\alpha\frac{\partial p_i}{\partial x_i}+\beta\frac{\partial^2 u_i}{\partial x_i^2}$$

This is equivalent to:

$$\frac{\partial u_i}{\partial t}+\frac{\partial\left(\frac{1}{2}u_i^2-\alpha p_i\right)}{\partial x_i}=\beta\frac{\partial^2 u_i}{\partial x_i^2}$$

If u_i is a solution of the equation:

$$\frac{\partial u_i}{\partial t}=\beta\frac{\partial^2 u_i}{\partial x_i^2}$$

Such solutions u_i exist because the equation is analogous to the heat-equation which is resolvable by the Fourier series [1].

If p_i is such $\frac{1}{2}u_i^2-\alpha p_i=f_i(t)$, then

$$\frac{\partial\left(\frac{1}{2}u_i^2-\alpha p_i\right)}{\partial x_i}=0$$, and the equation is solved.

We do the same for all axes i until $i=n-1$.

For the axe $i=n$:

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Let be $u_n = -\sum_{1}^{n-1} u_i$.

We have: $\frac{\partial u_n}{\partial t} = \beta \frac{\partial^2 u_n}{\partial x_i^2}$, and if p_n is such

$\frac{1}{2} u_n^2 - \alpha\, p_n = f_n(t)$, then $\frac{\partial \left(\frac{1}{2} u_n^2 - \alpha\, p_n \right)}{\partial x_n} = 0$, and the equation is solved for the axe n.

It is clear that if e_i is the vector for the axes i, then $u = \sum_{1}^{n} u_i e_i$ is one solution of the Naviers-Stokes equation if $\operatorname{div} u = 0$.

Else, to have $\operatorname{div} u = 0$, we take u_i of the form:

$$u_i = e^{\beta t + \left(\sum_{1}^{n} x_i \right)}, \ \forall i \square \{1, \dots, n-1\}$$

So we have solutions of the Navier-Stokes equation.

IV. NECESSARY CONDITIONS

Any solution (u, p) of the Navier-Stokes equation verifies that:

$$u_i^2 - \alpha\, p_i = f_i(t), \ \forall i \square \{1, \dots, n\}$$

Indeed:

If (u, p) is a solution of the Navier-Stokes equation , we must have :

$$\frac{\partial u_i}{\partial t} + \frac{\partial \left(\frac{1}{2} u_i^2 - \alpha\, p_i \right)}{\partial x_i} = \beta \frac{\partial^2 u_i}{\partial x_i^2}$$

Therefore:

$$-\frac{\partial u_i}{\partial t} = \frac{\partial \left(\frac{1}{2} u_i^2 - \alpha\, p_i - \beta \frac{\partial u_i}{\partial x_i} \right)}{\partial x_i}$$

And:

$$-\partial x_i \frac{\partial u_i}{\partial t} = \partial \left(\frac{1}{2} u_i^2 - \alpha\, p_i - \beta \frac{\partial u_i}{\partial x_i} \right)$$

When the fluid flows in one direction, then the space-time flows in the opposite direction with the same speed value:

We deduce that:

$$-u_i \partial u_i = \partial \left(\frac{1}{2} u_i^2 - \alpha\, p_i - \beta \frac{\partial u_i}{\partial x_i} \right)$$

Therefore:

$$0 = \partial \left(u_i^2 - \alpha\, p_i - \beta \frac{\partial u_i}{\partial x_i} \right)$$

And:

$$u_i^2 - \alpha\, p_i - \beta \frac{\partial u_i}{\partial x_i} = f_i(t)$$

So:

$$\sum_{1}^{n} u_i^2 - \alpha\, p_i - \beta \frac{\partial u_i}{\partial x_i} = f(t)$$

And consequently $\sum_{1}^{n} u_i^2 - \alpha\, p_i = g(t)$ because $\operatorname{div} u = 0$.

We deduce therefore that:

$u_i^2 - \alpha\, p_i = h_i(t), \ \forall i \square \{1, \dots, n\}$ because: $\forall i \square \{1, \dots, n\}, u_i^2 - \alpha\, p_i = l_i(x_i, t)$, and we must have $\frac{\partial (u_i^2 - \alpha\, p_i)}{\partial x_j} = 0 \ \forall j \square \{1, \dots, n\}$.

V. CONCLUSION

Theorem:

The Navier-stokes equation have a solution, Moreover, any solution (u, p) must check: $u_i^2 - \alpha\, p_i = f_i(t)$ and $\frac{\partial u_i}{\partial t} = \beta \frac{\partial^2 u_i}{\partial x_i^2}$, $\forall i \square \{1, \dots, n\}$. where $u = \sum_{1}^{n} u_i e_i$ and $p = \sum_{1}^{n} p_i e_i$.

Conversely any pair (n, p) satisfying these conditions with $\operatorname{div} u = 0$ is solution of the Navier-Stokes equation.

REMARKS

1- We note in the above equations the dependence between pressure, density, speed vector fields and viscosity

2- By dividing by α in the equation $u_i^2 - \alpha\, p_i = f_i(t)$ we deduce that: $\rho u_i^2 - p_i = \rho f_i(t)$ where $\frac{1}{\alpha} = \rho$ is the density of the fluid.

Let's $\rho = \frac{m}{V}$, we will have : $m u_i^2 - V p_i = m f_i(t)$.

And the equation $m u_i^2 - V p_i = m f_i(t)$ is linking energy, mass, pressure, temperature, volume and time ... This may not be surprising since a link between the Boltzmann equation and the Navier-Stokes has been established.

When $f_i(t)$ tends to 0, we will have $m u_i^2 \square V p_i$, there is therefore a tendency towards the law of an ideal gas, and the function $f_i(t)$ can be regarded as a turbulent function.

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