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# Smart Grid State Estimation by Weighted Least Square Estimation

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***Abstract— The smart grid is an advanced power grid with many new added functions and more reliability than the traditional grid. More controlled power flow is enabled in the smart grid by use of features from fields of communication, control system, signal processing etc. Knowing the present condition of the system is critical for signal processing applications and hence more accurate state estimation is important. State of the system along with information about the network topology will give complete information about the power grid network. In this paper the network topology is modeled using the MATPOWER package, a powerful software package of MATLAB. Weighted Least Square (WLS) state estimation is used to develop equations and algorithms for state estimation. The linear state estimation problem is formulated with linear methods using phasor measurement unit (PMU) data. The measurements which are included in the observation vector and also the size of the system (given by number of busses in the system) are important and these features affect the accuracy of the system state estimate. In this paper, state estimates of IEEE standard bus system of different size are stimulated using MATPOWER package. Also state estimates are stimulated, with different measurement parameters in the observation vector and the stimulation result obtained are compared.***

***Index Terms—Smart Grid, State Estimation, Weighted Least Square Estimation, Modeling of Smart Grid.***

## I. INTRODUCTION

The use of electricity and electric devices has been an integral part of human society for decades and we humans in the present days depend on the electricity heavily. Hence we require a very reliable grid to produce and deliver electricity. One of the key aspects of maintaining reliability of a large system such as the electric power grid is finding a way to provide feedback to those that control it. The control systems that control the power grid require appropriate and reliable information in order to make decisions that will improve not only the day-to-day reliability of the system but allow for engineers to plan more effectively for the future.

The growing dependency of humans in electricity have pushed the need for more reliability and this demands for a new and improved electric distribution system. The smart grid in an advanced and intelligent power grid that have many added features from various fields like communication, control system, energy management and signal processing.

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The smart grid add many new features to the grid like distributed generation and two-way flow current, more advanced data flow layer with two-way flow of data, more intelligent data processing centers that provide more efficient grid management etc.

The power grid includes different grid elements like switches and circuit breakers that determines how the current flows through the network. In other words the present status of these circuit elements determines the topology of the grid. The information about the present status of these system elements is called the status data. If we know the status data completely and accurately at the data processing center, then by also using the information about physical power grid network parameters, we can accurately determine the topology of the grid. However for large power grids, it is not possible to collect the status data with reliable accuracy.

The second type of data used by the energy management system is the analog data which include the bus voltage and angle, power injection, power flow, and reactance. This analog data can be used to find the loading profile of line and transformers. Both status data and analog data are related to each other as they both are part of the same system. Any error in status data will lead to error in determining network topology and this will also get reflected in state estimation.

The state of the system is a vector including the voltage and angle at all bus. This information, along with the knowledge of the topology and impedance parameters of the grid, can be used to characterize the entire system. The EMS/supervisory control and data acquisition (SCADA) system is a set of computational tools used to monitor, control, and optimize the performance of a power system [1]. Power system State Estimation (SE) has become a critical part of the operation and management of transmission systems worldwide [2].

## II. IEEE STANDARD BUS SYSTEMS

The actual power grid is a very large system which is continuously on use by millions of people. Hence any experiment on the actual power grid is a tedious process. Hence the IEEE Distribution System Analysis Subcommittee developed and published data for many unbalanced radial distribution test feeders. These standard bus systems can be used for the stimulation and evaluation purpose using various software [3]. These test feeders may be either a hypothetical test feeder network prepared for analysis purpose or a network model of an actual power grid system.


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For instance the IEEE 34 node test feeder is modeled from an actual test feeder in Arizona. The IEEE standard data for an IEEE 5 bus system is given in table1, table2 and table3.

**Table1: Bus Data for IEEE 5-Bus System**

| Bus Code P | Assumed Bus Voltage | Generation | | Load | |
|---|---|---|---|---|---|
| | | Mega watts | Mega vars | Mega watts | Mega vars |
| 1 | 1.06 + j0.0 | 0 | 0 | 0 | 0 |
| 2 | 1.0 + j0.0 | 40 | 30 | 20 | 10 |
| 3 | 1.0 + j0.0 | 0 | 0 | 45 | 15 |
| 4 | 1.0 + j0.0 | 0 | 0 | 40 | 5 |
| 5 | 1.0 + j0.0 | 0 | 0 | 60 | 10 |

**Table2: MW Limits for Branches in IEEE 5 Bus System**

| Line | MW Limits (P.u) |
|---|---|
| 1 - 2 | 0.8 |
| 1 - 3 | 0.3 |
| 2 - 3 | 0.2 |
| 2 - 4 | 0.2 |
| 2 - 5 | 0.6 |
| 3 - 4 | 0.1 |
| 4 - 5 | 0.1 |

**Table3: Line Data for IEEE 5-Bus System**

| Bus Code p - q | Line Impedance $Z_{pq}$ | | Line charging |
|---|---|---|---|
| | R per unit | X per unit | |
| 1 - 2 | 0.02 | 0.06 | X per unit |
| 1 - 3 | 0.08 | 0.24 | 0.0 + j0.025 |
| 2 - 3 | 0.06 | 0.25 | 0.0 + j0.020 |
| 2 - 4 | 0.06 | 0.18 | 0.0 + j0.020 |
| 2 - 5 | 0.04 | 0.12 | 0.0 + j0.015 |
| 3 - 4 | 0.01 | 0.03 | 0.0 + j0.010 |
| 4 - 5 | 0.08 | 0.24 | 0.0 + j0.025 |

The main power grid elements include busses, branches, generators, loads etc. Apart from these, the switches and circuit breakers in the power grid determine the real tine topology of the power grid. All transmission lines, transformers and phase shifters are modeled with a common branch model, consisting of a standard transmission line model, with series impedance $z_s = r_s + jx_s$ and total charging susceptance '$bc$', in series with an ideal phase shifting transformer. Mathematically a power generator is modeled as a complex power injection at a specific bus. Similarly constant power loads can be modeled as real and reactive power consumed at a bus. Shunt elements connected in the network is modeled as fixed impedance to ground at bus. One line diagram of some IEEE standard bus systems are given here as example [4].

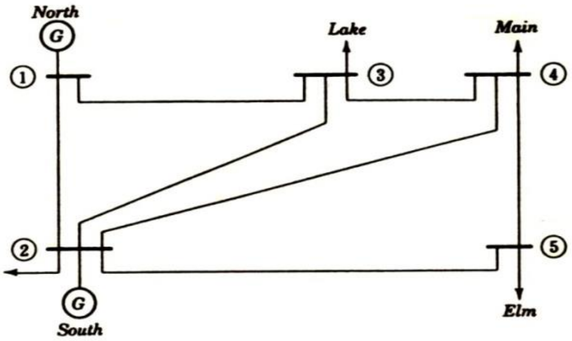


**Fig1: IEEE 5 Bus System**

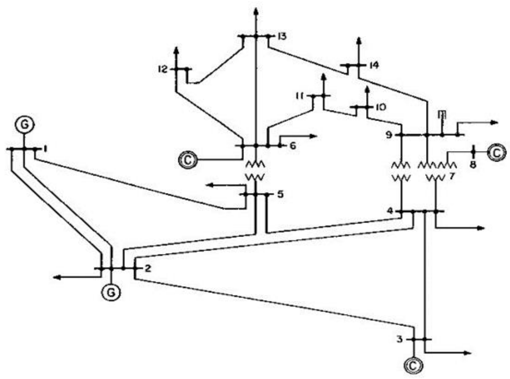

**Fig2: IEEE 14 Bus Systems**

## III. MATPOWER

The MATPOWER is a powerful MATLAB package consisting of M-files. This package helps in solving the power flow and optimal power flow problems. It is intended as a simulation tool for researchers and educators that are easy to use and modify. Matpower is designed to give the best performance possible while keeping the code simple to understand and modify [4].



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### A. System Modeling Grid Elements in Matpower

MATPOWER uses the MATLAB M-file or MAT file. These return a single MATLAB structure as output. The M-file looks just as if a plane text and hence can be easily edited. BaseMVA, bus, branch, gen are the permanent fields of the structure out of which BaseMVA is a scalar and the rest are matrices. Also the gencoast is an optional matrix field. Each row in the matrices corresponds to a single bus, branch, or generator.

We can use Matpower to model different standard steady state power grid network, which can be used for power flow analysis. For modeling, no offline elements are considered. Consecutive numbering is given to busses. Generators are numbered according to busses to which they are connected. As the Matlab is good in handling matrices, networks are usually modeled using matrices. Graph theory is used to prepare mathematical model of the grid. Topology is converted into a di-graph consisting of nodes (vertexes) and edges. Unlike ordinary graphs, edges in di-graphs allow bidirectional flow. As current and power in the modern smart grid can flow in both directions, a di-graph best models the smart grid.

The generators and loads are modeled mathematically simply as complex power injection or consumption respectively. For modeling the transmission lines, two port ∏ model equivalents is used as given in figure3. Different lumped elements are used to represent different losses in the transmission line. Series resistor is used to represent the copper losses. Series inductor is used to represent the energy stored in magnetic field surrounding the conductor. Shunt impedances are used to represent line charging of conductors.

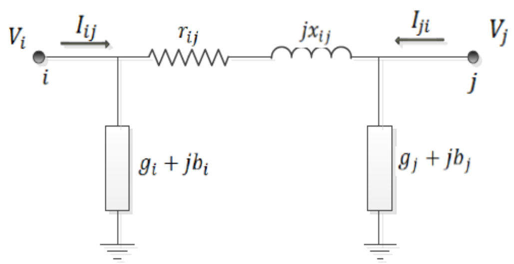


**Fig3: Two port ∏ model for transmission lines**

### B. Optimization Problem in Matpower

Mathematically the power grid can be modeled as a non-linear system. Different measurement parameters like power flow and power injection are non-linear function of the system state variables. In power flow problem, we solve for the state voltage and current flow according to the load and generation pattern. For solving DC and AC power flow problems, the Matpower tries to solve a subset of power balance equations represented as function of unknown voltage quantities as given in equation1 and equation2. In the traditional formulation of the AC power flow problem, the power balance equation in is split into its real and reactive components, expressed as functions of the voltage angles and magnitudes and generator injections $Pg$ and $Qg$, where the load injections are assumed constant.

$$g_p(\theta;\ V_m;\ P_g) = P_{bus}(\theta;\ V_m) + P_d - C_g P_g = 0 \quad (1)$$

$$g_p(\theta;\ V_m;\ Q_g) = Q_{bus}(\theta;\ V_m) + Q_d - C_g Q_g = 0 \quad (2)$$

## IV. STATE ESTIMATION PROBLEM

Estimation means guessing an unknown parameter accurately from the known parameters. For state estimation problem, we have to estimate the state vector from the measured parameters like power flow and power injection. With the collection and use of PMU data in present day advanced systems, we are able to get more accurate estimation of the system state in a timelier manner.

### A. WLS State Estimation

Traditionally the power grid state estimation means using the measurement data measured from different points spread through the power grid to estimate the state of the power grid. The statistical concept of the maximum likelihood estimation is used for the state estimation. Here the likelihood function is simply the product of each of the probability density functions of each measurement. Maximum likelihood estimation aims to estimate the unknown parameters of each of the measurements probability density functions through an optimization [5].

Maximum likelihood estimation aims to maximize this function to determine the unknown parameters of the probability density function of each of the measurements. This can be done by maximizing the logarithm of the likelihood function or minimizing the weighted sum of squares of the residuals [5]. The solution to this problem is referred to as the *weighted least squares* estimator for *x*.

Power system state estimators use a set of redundant measurements taken from the power system to determine the most likely system state from the given information and assumptions. The state estimator becomes a weighted least squares estimator with the inclusion of the measurement error covariance matrix which serves to weigh the accuracy of each of the measurements. The physical system model information and measurements are part of the equality constraints of the basic weighted least squares optimization and are what make this algorithm specific to power systems. This section presents the solution to the weighted least squares problem and the system matrix formulation including the measurement function matrix and measurement Jacobian matrix.

$$J(X) = [Z - h(X)]^T [C]^{-1} [Z - h(X)] \quad (3)$$

### B. State of the system

State of any system is a minimal set of parameter that can completely represent the present system condition. If we know the present topology of the system accurately, then all we require in order to accurately know the present condition of the system are the voltages at different busses in the power grid.


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Hence the state variable is actually a vector that holds the magnitude and phase details of the bus voltages. This can be fulfilled either, by representing the voltage by its real and imaginary part (in Cartesian form) or by representing the voltage by magnitude and phase angle of the voltage (in polar form).

$$X = [V_1, V_2 \dots V_n, \theta_1 \theta_2 \dots \theta_n]^T \quad (4)$$

The state of the power system is a function of several things. These include system parameters such as real and reactive power flows, current injections, & voltages which are unknown but measured, network topology, and parameters such as resistance, reactance, and shunt susceptance of transmission lines which is assumed to be known [5].

### C. System Equations

The state vector that holds complete details about the system includes the voltage at each bus. Different measurements in the power grid can be expressed as a non-linear function of the power grid state variable. Different measurements made at different points in the grid are sent to the central control center. With the use of PMU measurements in todays advanced power grid monitoring systems, we get these measurements in a geo-temporally synchronous manner. This data forms the observation vector. As discussed, the observation vector will be a non-linear function of the state vector.

$$Z_n = h(x) + e_n \quad (5)$$

Here $Z_n$ in the control center observation vector, h() is a non-linear function of the power grid state variable X and $e_n$ is an error vector representing Gaussian measurement noise with zero mean and covariance $C_e$.

In real life scenario the phase difference that exists between any two busses in the network is important. By assuming that this phase difference is small we can create a linear approximation of the system. The linear model of the equation can be given as follows.

$$Z_n = Hx + e_n \quad (6)$$

Here H is the measurement Jacobian matrix. The Jacobian matrix can be obtained by partially differentiating the non-linear function defined in the system with each of the state vector elements.

$$[\mathrm{H}] = \left[\frac{\delta h(X)}{\delta x}\right] \quad (7)$$

Since the function is a highly non-linear function and each elements of the system of equations are highly dependent to one another, we cannot obtain the H matrix simply by dividing the output vector of the system of equations by the state vector. Instead we have fall back to the use of equations to find each of the Jacobian Matrix. Now as the H matrix also depends on the present topology of the power grid and since it can be used to estimate the topology of the power grid, this matrix is also called the topology matrix.

### D. Solving For System State

The state of the system X can be estimated by solving for the X vector in the equation. The error included in the observation vector during its measurements should be removed to get good estimation of the power grid state. Hence we use the measurement error covariance matrix as a weight for the state estimation by solving the system of equations. Hence the equation for the solution will be as given.

$$x = [H^T C^{-1} H]^{-1} H^T C^{-1} Z_n \quad (8)$$

If we directly measure the state vector variables due to the measurement error and error during communication the state vector will be error prone. Hence we will not be able to get good approximation of state vector by direct measurement. By estimation we get more concurrent and more accurate estimation.

This is a linear approximation of an actual non-linear system. Solving the linear system of equations directly will give satisfactory results for smaller systems. However for larger systems if we solve the linear system of equations we will get an inaccurate approximation of the system state. This is because for larger systems the non-linearity of the system will be more dominant. Hence instead of directly solving these systems we will have to use some iterative methods by also updating the result in each of the iteration. Ignoring the higher order terms of the Taylor series expansion of the derivative of the objective functions yields an iterative solution known as the Gauss-Newton method.

$$x^{k+1} = x^k = [H(x^k)]^T [C]^{-1} [H(x^k)]^{-1} [H(x^k)]^T [C]^{-1} [Z - h(x^k)] \quad (9)$$

Here k is the iteration index, C is the covariance matrix, H is the Jacobian measurement matrix, Z is the observation vector, X is the state variable and h( ) is the non-linear function that relates the observation vector to the state variable X. For the first iteration of the optimization the measurement function and measurement Jacobian should be evaluated at flat voltage profile, or flat start. A flat start refers to a state vector where all of the voltage magnitudes are 1.0 per unit and all of the voltage angles are 0 degrees. In conjunction with the measurements, the next iteration of the state vector can be calculated again and again until a desired tolerance is reached.

## V. RESULTS

The state estimation of the power grid can be done in many ways. The WLS state estimation method is an important technique used for the state estimation. Direct measurement of the state vector parameters will be prone to error. The estimation technique helps us to obtain better measurement of the state vector. For power grid state estimation we use the grid parameter measurements obtained at different parts of the grid. These measurements are noted as observation vector Z. The relation between the state vector X and the observation vector Z is used to develop a system of equations, which can be used for the purpose of state estimation. The figure5 and figure 8 respectively gives state estimates when real and reactive power flow, voltage and angle are used in observation vector and their error from actual value.



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State estimates for when only real and reactive power flow is used and their error from actual value are given in figure 6 and figure 9 respectively. Similarly state estimates for when only voltage and angle measurements are used and their error from actual value are given in figure 6 and figure 9 respectively.

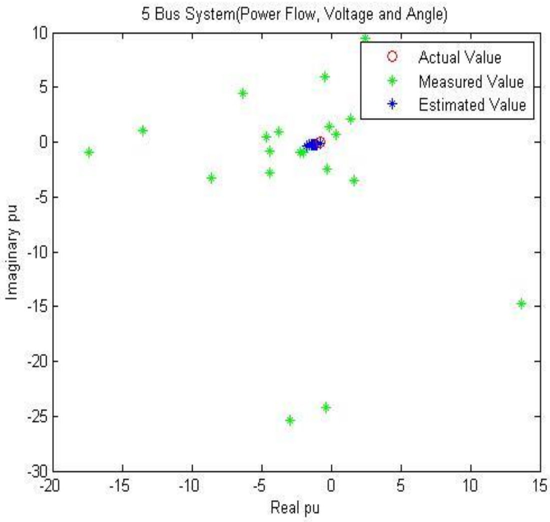


**Fig5: State Estimation Using Power flow, Voltage and Angle**

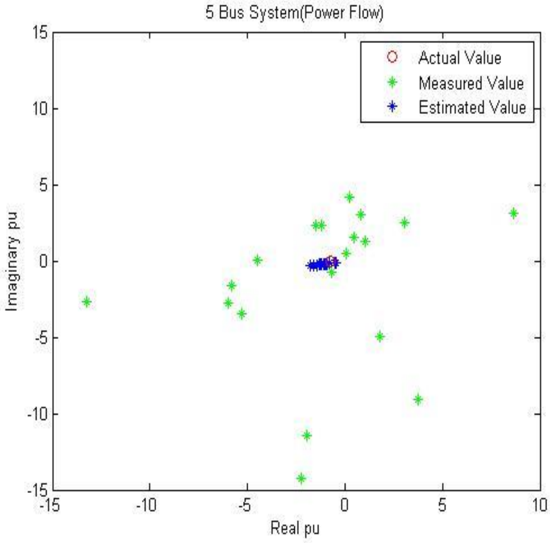


**Fig6: State Estimation Using Power flow only**

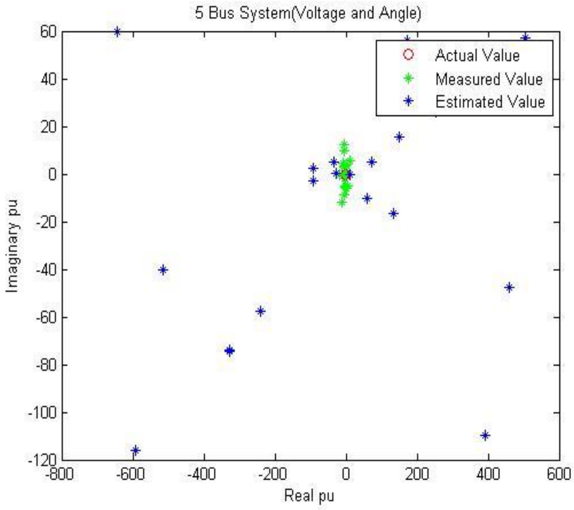


**Fig7: State Estimation Using Voltage and Angle only**

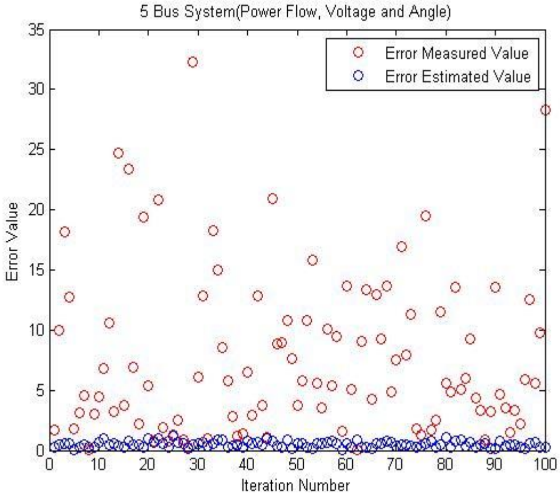


**Fig8: Error From Actual Value When Using Power flow, Voltage and Angle**

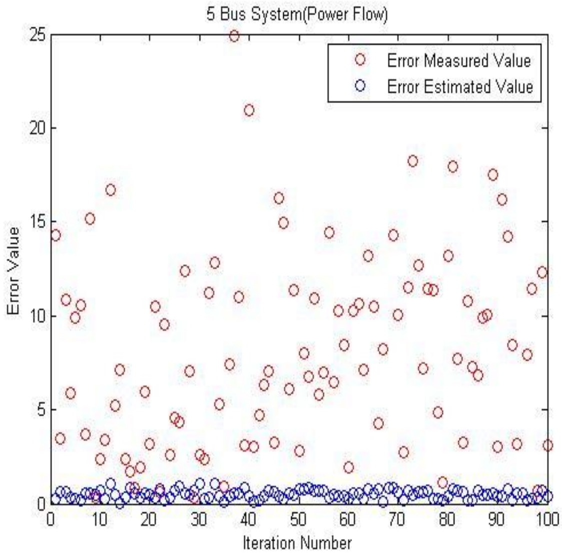


**Fig9: Error from actual value when using Power flow only**

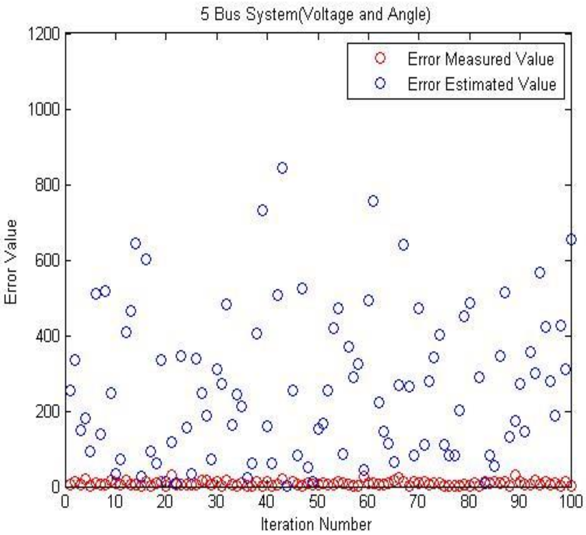


**Fig10: Error from actual value when using Voltage and Angle**



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The smaller systems give better results when we use the linear approximation model in place of the non-linear system. As we scale up the number of busses in the system, the effect of the non-linearity become more and more dominant and we tend to get more error prone estimates. As the system is made much large, the effect of non-linearity becomes so much dominant. Hence we can see that this estimation method is better for smaller systems.

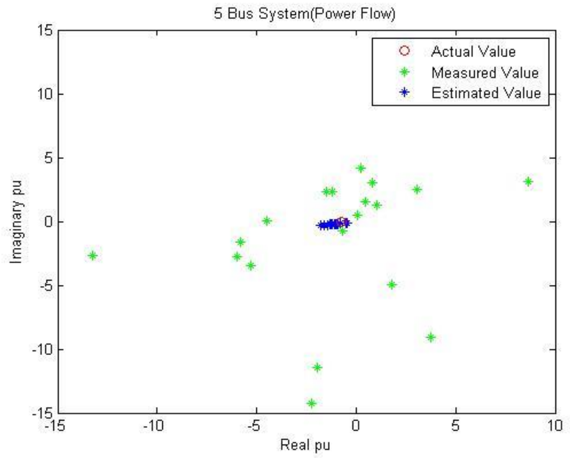


**Fig11: State Estimation Using Power flow only for 5 bus system**

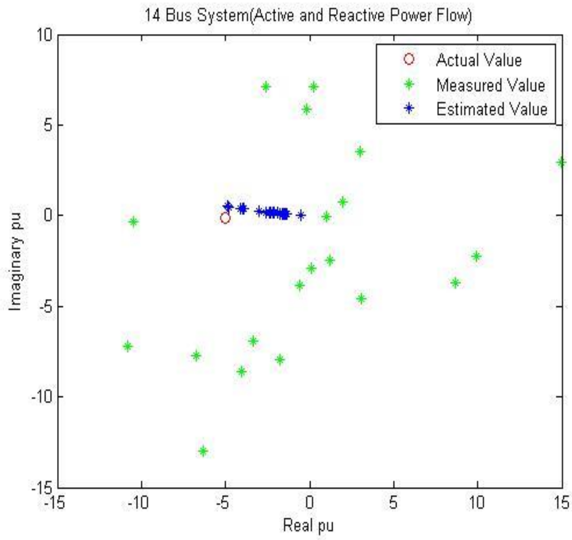


**Fig12: State Estimation Using Power flow, Voltage and Angle for 14 bus system**

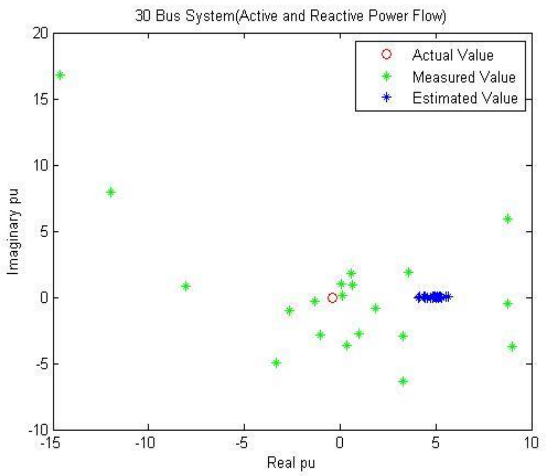


**Fig13: State Estimation Using Power flow, Voltage and Angle for 30 bus system**

## VI. COLLUSION

The accuracy, consistency and unbiased nature of the state vector estimate depends on which among the power flow measurements we use in the observation vector and hence also in the system of equations. For better estimation, we can use the redundancy in the measured parameters to our advantage. Hence power flow parameters that have non-linear relation with the state vector are found to give better estimation of those parameters that have linear relation with the state vector. From the below figures it can be seen that we get an acceptable estimate when the power flow parameters are used. However if only the power flow parameters are used, the estimate obtained is more prone to be biased. An unbiased and good estimate can be obtained if we use both power flow parameters and voltage and angle for estimation. Also as we scale us the size of the grid accuracy reduces.

## ACKNOWLEDGMENT

The preferred spelling of the word “acknowledgment” in American English is without an “e” after the “g.” Use the singular heading even if you have many acknowledgments. Avoid expressions such as “One of us (S.B.A.) would like to thank ... .” Instead, write “F. A. Author thanks ... .” Sponsor and financial support acknowledgments are placed in the unnumbered footnote on the first page.

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