

Mathematical Modeling to Establish the Balance of Heat in a Capacitor

Diejo Jara, Estefania Salinas, Julio Romero, Michael Valarezo

Abstract: *The teaching-learning process in the field of exact sciences strengthened by the practical activity of a technological nature, in which to facilitate the safe reasoning and concise leads to the application of principles of physics and chemistry as well as updating processes industrial in the field of Mining, Pulp, Forest, Food, Chemical and Process. Which have potentiated a high degree of modelling and automation? This automation involves some advantages that have just moved to the quality and improvement of the final product. In this case, establishing the heat balance in a condenser. Includes ensure both a more competitive cost and simultaneously strengthening formation activity and the mathematical model to determine the hot balance in a capacitor means using parameters dependent pressure define variables as the volume of water and the amount of steam saturation entry and quantified by developed and simplified quantification and analysis of material balance equations. Thus, in this article the calculations used are presented to establish the mathematical modeling for the heat balance in a capacitor, for it was selected and implemented, with teams making and data records, pointing to possible strategies to conceive established the study of the processes of heat transfer and control systems as an integral part of an automation project*

Index Terms: *Automation, analytical calculation, mathematical modelling, analytical, design and construction.*

I. INTRODUCTION

Traditionally, in the technological field, it has resorted to the use of equipment, which gives an approximation of reality based on experimentation and modelling. Relating the use of control systems it is intended to integrate within a particular working group with the task of monitoring and open a major way for teaching using computer systems such as MATLAB® application specially designed for mathematical calculations that although has certain functions to more specific issues, such as control ToolBox, which facilitates the study of dynamical systems and their regulation. Also, a supplement called MATLAB®/SIMULINK, which gives a more graphic approach control systems. (Adolfo Anta Martínez 2001).

Applications that allow us simulation and development of measurement applications and providing us with monitoring tools that allow us to understand the behaviour of the input

variables and output, with lower cost, besides having a greater number of operations human supervision on automated systems forming unit operations within the process.

The project presented has come from the need to implement the principles governing the thermal analysis in a boiler and thus its application in line with technological advances in a highly industrialised environment in which energy saving is necessary and thus cheapen costs of the final product.

Finally, it should note that the technique used for this work based on the identification system, this method established in two stages namely: input-output and internal process that occurs in the block (Figure 1). The primary objective will be to find a transfer function that will be similar to the actual plant in its main features.

The obtained data aims to have the necessary information input, and output is one of the priorities to take to the workspace of Matlab, this objective another powerful tool Matlab-Simulink, where will be built the mathematical model used.

II. MATERIALS AND METHODS

A. Description of the system

When James Watt noted that steam could use as an economic force that would replace animal and manual power, it began to develop the manufacture of boilers, to reach that currently have greater use in different industries.

The first boilers have the drawback that the hot gases were in contact only with their base and consequently preyed fuel heat poorly. Because of this and was subsequently introduced tubes, to increase the heating surface if the inside of the tubes gases or fire are classified firetube (smoke machines) and water tube boiler (water tubes).

The basic theory of a boiler sustained as a machine or device engineering designed to generate saturated steam. This steam is produced by heat transfer at constant pressure, in which the fluid, originally in a liquid state, is heated and changes state.

Terminology used:

○ m_h : Mass flow hot liquid, kg / h or lib / h.

○ m_c : cold fluid mass flow rate kg / h or lib / h

H_{h1} : Enthalpy hot fluid inlet $\left(\frac{Kcal}{kg}\right), \left(\frac{BTU}{lb}\right)$

H_{h2} : Enthalpy hot fluid outlet $\left(\frac{Kcal}{kg}\right), \left(\frac{BTU}{lb}\right)$

Manuscript published on 30 August 2016.

* Correspondence Author (s)

Diego Jara Delgado, Geology and Mines, University National of Loja, Loja, Ecuador.

Estefanía Salinas Nalvay, University National of Loja, Loja, Ecuador.

Michael Valarezo, Geology and Mines, University National of Loja, Loja, Ecuador.

Julio Romero, Geology and Mines, University National of Loja, Loja, Ecuador.

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an [open access](https://creativecommons.org/licenses/by-nc-nd/4.0/) article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>.

Mathematical Modeling to Establish the Balance of Heat in a Capacitor

H_{C1} : Enthalpy cold fluid inlet $\left(\frac{Kcal}{kg}\right), \left(\frac{BTU}{lb}\right)$

H_{C2} : Enthalpy hot fluid outlet $\left(\frac{Kcal}{kg}\right), \left(\frac{BTU}{lb}\right)$

q_h : lost by the hot liquid $\left(\frac{Kcal}{kg}\right), \left(\frac{BTU}{lb}\right)$

T_e : Boiling Point ($^{\circ}C$)

UT : feed water inlet (lib/h)

OUT : output saturated steam (lib/h)

V_{vap} : vapour volume within the boiler (m^3)

V_w : Volume of water within the boiler (m^3)

ρ_w : specific density liquid water in $\left(\frac{kg}{m^3}\right)$

ρ_{vap} : Vapor density specified in $\left(\frac{kg}{m^3}\right)$

M_c : Total mass stored in the cylinder.

h_c : Condensation enthalpy

Submit your manuscript electronically for review.

B. Theoretical considerations on the design and calculations for Mathematical Modeling

In a heat transfer when heating an object increases with its temperature perception that the heat and temperature are the same. However, this is not the case. Heat and temperature are related, but different concepts.

While the temperature is a measure of the average molecular energy. The temperature depends on the speed of the particles, their number, size and type. The temperature does not depend on the magnitude, the number or the type and temperature of a small cup of water can be the same as the temperature of a bucket of water, but the bucket has more heat because it has more water and therefore a total thermal energy.

The container Calderín is a horizontal configuration of varying thickness depending on the steam produced Figure 1. The size of it will be necessary to contain the elements of separation and support possible changes according to the load level. The mission is to accumulate reboiler at the bottom the water is distributed to the various manifolds of the radiant chamber, and in turn separated into the upper vapour particles carrying water, prepare it in two-column format, including figures and tables.

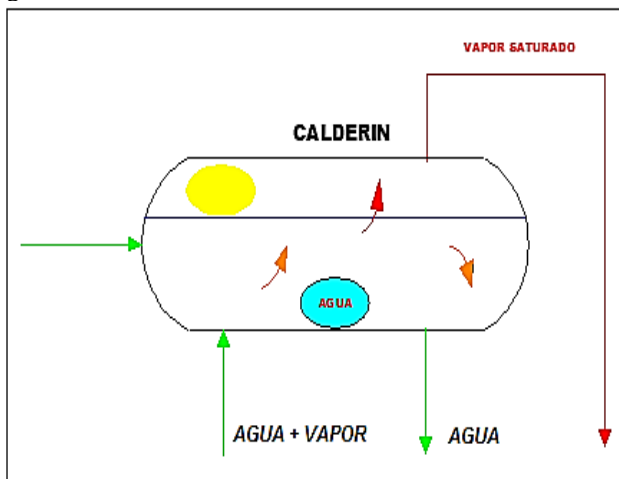


Fig. 1 Calderin

In this area by applying an energy balance for the fluid such as water, considering the input and output has the equation (1):

$$h_1 + \frac{u_1^2}{2g} + JH_1 + Jq = h_2 + \frac{u_2^2}{2g} + JH_2 + W_o \quad (1)$$

If we consider this expression in the calderín no moving parts and therefore with work is done then the value of $w = 0$. As in the condenser is in a horizontal position and to be at the same height, $tieneh_1 = H_2$. In the Calderín there is no direct heat input so it considered that potential energy and kinetic energy are zero so the above expression (2), (3), (4) is synthesised:

$$q = H_2 - H_1 \quad (2)$$

To the hot fluid F_h in the equation (3).

$$q = (H_{h2} - H_{h1}) * m^{\circ} h \quad (3)$$

For cold fluid f_c in the equation (4)

$$q_c = (H_{c2} - H_{c1}) * m^{\circ} c \quad (4)$$

The heat balance in all types of heaters must be satisfied that the waste heat is equal to the heat gained with which we have:

waste heat = heat gained.

$$F_h = F_c$$

$$-q_h = q_c$$

Replacing have, the equation (6) and (7):

$$-m_h * (H_{h2} - H_{h1}) = m_c * (H_{c2} - H_{c1}) \quad (6)$$

$$-m_h * (H_{h1} - H_{h2}) = m_c * (H_{c2} - H_{c1}) \quad (7)$$

According to the above expressions have the inputs and outputs and disturbances within represented in the the block diagram for the reboiler according to what shown in Figure 2

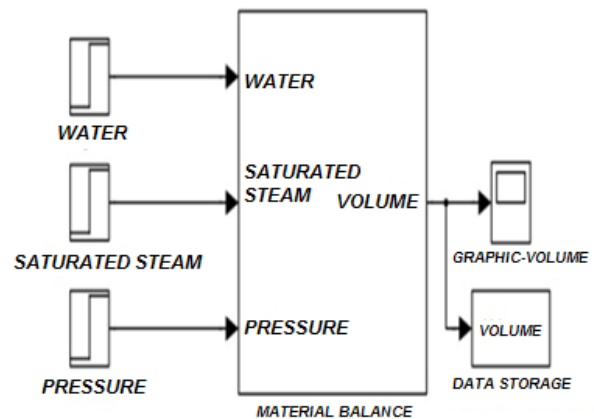


Fig. 2 Variables involved in the system

In the analysis of the variables the equipment used to determine the heat, balance is the cauldron, the same one that provides the dry saturated steam; and a series of thermometers and manometers; well as two tanks to place the condensate to receive it.

In the condenser water flow is determined. Also, you have a meter fluid temperature for both the output to the input consists of a meter for vapour pressure and one for the steam. So that way determine the time and collect water in a container that volume then be measured, and this experience conducted ten times, results detailed in the following Table 1:

Table 1. Obtained data Testing Laboratory Unit Operations

Temperature (°C)		Pressure		Temperatura (°C)
T _{c1}	T _{c2}	Manometric	Barometric	T _h
13.1	71.0	0	0.7972	97
13.5	62.5	0.1	0.7972	99
13.5	71.0	0	0.7972	99
13.2	64.0	0	0.7972	97
13.2	65.0	0.05	0.7972	99
13.2	64.0	0	0.7972	98
13.2	63.0	0	0.7972	99
13.2	61.0	0	0.7972	99
13.2	65.0	0.1	0.7972	99
13.2	64.0	0.05	0.7972	99
13.2	62.0	0	0.7972	99
13.245	64.772	0.0273	0.7972	98.70

Time used: 0.161516 hours
 V_h: Volume of hot water 0.01135 m³
 V_c: Volume of cold water 0.1080 m³

C. Ranking of the parameters

In the equation (9)

$$-m_c \cdot C_{pc} \cdot (T_{c2} - T_{c1}) = m_h \cdot C_{ph} \cdot (T_{h1} - T_{h2}) \quad (9)$$

Formula for calculating q_c, in the equation (10)

$$q_c = m_c \cdot C_{ph} \cdot (T_{c2} - T_{c1}) \quad (10)$$

$$-q_h = q_c$$

Calculation of [(ṁ)]_c

$$[(\dot{m})_c = \text{mass}/\text{time} \rightarrow \text{density}) = \text{mass}/\text{volume}$$

$$\text{mass} = \rho \cdot V_c \equiv (0.108 \text{ [m]}^3) \cdot \rho$$

Calculation of the density ρ

$$\rho \rightarrow (T_c)^{-}$$

$$(T_c)^{-} = (T_{c1} + T_{c2})/2 \rightarrow (13,2455 + 64,7727)/2 = 39,0091$$

$$\delta_{(39,0091^\circ\text{C})} = 992,9 \text{ Kg/m}^3$$

$$\text{mass} = 0.108 \text{ [m]}^3 \cdot \rho \equiv 0.108 \text{ [m]}^3 \cdot 992,9 \text{ Kg/m}^3 = 107,2332 \text{ kg}$$

$$[(\dot{m})_c = 107,2332 \text{ Kg} / 0,161516 \text{ h} = 663,917 \text{ Kg/h}$$

Calculation of C_{pc}

$$C_{pc} \rightarrow (T_c)^{-}$$

$$C_{pc}(39,0091^\circ\text{C}) = 0,9987 \text{ kcal/(kg}^\circ\text{C)}$$

Replacing in the equation for q_c it has:

$$q_c = 663,917 \text{ (kg/h)} \cdot 0,9987 \text{ (kcal/(kg}^\circ\text{C))} \cdot (64,7727 - 13,2455)^\circ\text{C}$$

$$q_c = 34165,3113 \text{ kcal/h}$$

Where:

Calculation with equation (11).

$$m_c^\circ = \frac{\text{masa}}{\text{tiempo}} > \rho(\text{densidad}) = \frac{\text{masa}}{\text{volumen}} \quad (11)$$

Calderín assumes that always exist in a biphasic mixture of saturated liquid and saturated vapour, As it is shown in Figure 3

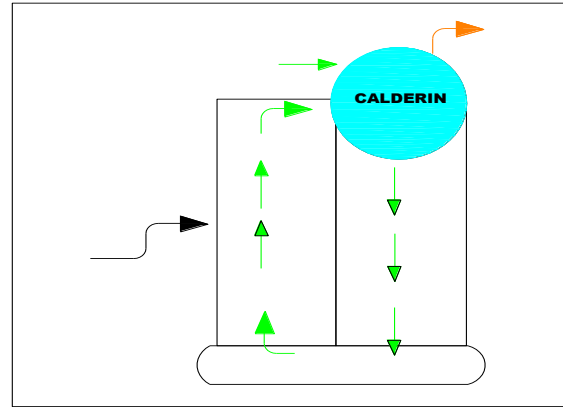


Fig. 3 Fluid mixture in the reboiler

Formula for calculating q_h

$$q_h = [(\dot{m})_h] \cdot \lambda = m_h^\circ \cdot h_{fg}$$

Calculation the [(ṁ)]_h.

$$[(\dot{m})_h = \text{mass}/\text{tiempo}$$

Calculation of the mass:

$$\text{mass} = [\text{Kg}]_{\text{liquido}} / ((1-x))$$

Calculation of the [(kg)]_{liquido}

$$[\text{kg}]_{\text{liquido}} = V_h \cdot \rho \rightarrow 0,01135 \text{ m}^3 \cdot \rho$$

$$\rho \rightarrow (T_h)^{-}$$

Boiling temperature = 92,8°C at a pressure of 0,7972 (Kg)/(cm)²

$$(T_h)^{-} = (T_h + T_e)/2 \rightarrow (98,70 + 92,8)/2 = 95,75$$

$$\delta_{(95,75^\circ\text{C})} = 968,75 \text{ Kg/m}^3$$

$$\text{mass} = 0,01135 \text{ [m]}^3 \cdot \rho$$

$$\text{mass} = 0,01135 \text{ [m]}^3 \cdot 968,75 \text{ Kg/m}^3$$

$$\text{mass} = 10,9951 \text{ kg}$$

Calculation of x (Title steam).

$$x = (H_{(liq.1)} - H_{(liq.2)}) / \lambda_2$$

Calculation of λ₂.

$$P_a: \text{absolute pressure} = 0,0273 + 0,7972$$

$$P_a = 0,08245 \text{ (} \rightarrow \text{Kg) / (cm)}^2$$

Interpolating determine the saturation pressure. However, in modelling these values they quantified in the expression as a function of pressure, as it is shown in Table :

Table 2. Interpolating determine the saturation pressure

Presión de saturación	Pérdidas		Presión de saturación externa
	hf	hg	
0,7149	89,88	635,1	0,7149
0,8619	95,01	637,0	

Mathematical Modeling to Establish the Balance of Heat in a Capacitor

$\Delta P = 0,147$	5,13	1,9	
$\Delta P = 0,1096$	$x = 3,1408$	$y = 1,4166$	

$$h_f = 89,88 + 3,1408 = 93,021$$

$$h_{(g \text{ (foreign)})} = 635,10 + 1,41166 = 636,5166$$

$$h_{(f \text{ (foreign)})} = 89,88 + 2,9721 = 92,8521$$

Calculation of x (Title steam).

$$y = (636,5166 - 93,021) / (636,5166 - 92,8521) = 0,99839$$

Total mass:

$$M_C = (0,01135 * 992,9) / 0,99839 = 11,2876$$

$$\lambda_2 = \lambda_g - \lambda_f = 93,021$$

Interpolating it is the Table 3.

Table 3. Interpolating results

Presión de saturación	Perdidas	Presión de saturación externa
	hg	
0,7149	635,1	0,7149
0,8619	637,0	

$\Delta P = 0,147$	1,9
$\Delta P = 0,1096$	$y = 1,4166$

$$h_{(g \text{ (foreign)})} = 635,10 + 1,41166 = 636,5166$$

$$\lambda_{(f \text{ (foreign)})} = 636,5166 - 93,021 = 543,4956$$

$$q_h = [\dot{m}]_h * \lambda$$

$$q_h = 543,4956 * 11,2876 / 0,161516 = 37982,373$$

$$q_h = 37982,373 \text{ kcal/h}$$

D. Mathematical Model Calderin

It raises the mathematical model of Calderin, it is necessary to build some working assumptions by the calculations made to facilitate the process, as shown in Figure 4.



Fig. 4 Issues to consider in the serpentin

Based on the view that the boiler is always in motion, i.e., inside the Calderin there is always approximately the same conditions of temperature, pressure and volume. If we assume that the boiler has a maximum capacity of 4m³ that correspond to the 100% level. In our work testability is V_h: Average volume of water clients 0.01135m³; and the average amount of cold water 0.1080m³ and time spent for testing is equivalent to 0.161516 hours,

A. Material Balance of Calderin

The fact that the boiler has two variables the same as to integrate their difference to determine the total mass that

stored in the reboiler is synthesised, as shown in the equation (12):

$$M_C = \int ut - out \quad (12)$$

Once I have calculated the mass of Calderin, you should find the volume using the following formula: Accumulation = Input - Output. This is shown in equation (13)

$$\frac{d}{dt} (\rho_{vap} * V_{vap} + \rho_w * V_w) = ut - out \quad (13)$$

Being: $V_t = V_{vap} + V_w$

The overall volume Allocated for modelling is 10m³ in the boiler if we know the maximum volume of 4.59 that is m³, we will know at all times the size and level the boiler required expressed, this is shown in equation (14).

$$\frac{d}{dt} (\rho_{vap} * V_t - \rho_w * V_w) = ut - out \quad (14)$$

It must also take into account variations in specific gravity of water and steam in pressure function. These variables obtained through Las Tablas account corresponding parameter or diagram pressure and volume.

To close the material balance, discussed above, we know at any time the pressure we have in the tank. Although we are aware that the steady pressure that works the boiler is 120 PSI, at any time you can suffer a disturbance either temperature (heat), pressure or flow, which make conditions change.

The overall energy balance is, this is shown in equation (15):

$$\frac{d}{dt} [\rho_{vap} * u_{vap} * V_{vap} + \rho_w * u_w * V_w + \rho_w * V_w] = Q + ut * h_f - out * h_{vap} \quad (15)$$

If the internal energy is $u = h - p/\rho$ so, This is shown in equation (16):

$$\frac{d}{dt} [\rho_{vap} * h_{vap} * V_{vap} + \rho_w * h_w * V_w + m_t * C_p * t * m] = Q + ut * h_f - out * h_{vap} \quad (16)$$

To make further simplifications then it is required to know the pressure in the boiler. To do this, we multiply the mass balance equation by how (enthalpy of water) and then we subtract the energy balance equation (17) and (18).

$$h_w * \frac{d}{dt} (\rho_{vap} * V_{vap} + \rho_w * V_w) = h_w * (ut - out_{pur}) \quad (17)$$

$$\frac{h_c * \frac{d}{dt} (\rho_{vap} * V_{vap}) * \rho_{vap} * V_{vap} * \frac{dh_s}{dt} + \rho_w * V_w * \frac{dh_w}{dt} - V_t * dp}{dt + m_t * C_p * (dt - s)} = Q - ut(h_w - f) - out * h_c \quad (18)$$

If $h_c = h_s - h_w$

From the above equation, we can find the relationship of the pressure with the other terms, if we consider that the level of the boiler is well controlled, volume changes will be small. If we neglect these variations (volume almost constant), we can reach the following expression (19) and (20):

$$K_{-1} * \frac{dp}{dt} = Q - ut (h_{-W} - h_{-f}) - out * h_{-c} \quad (19)$$

$$\frac{h_{-1} = h_{-c} V_{-v} a p_{-} (\partial p)_{-} \frac{s}{\partial p + p_{-s} V_{-} (va p) * (\partial h)_{-} \frac{s}{\partial p + p_{-W} V_{-W} * (\partial h)} - \frac{\partial p + m_{-t} (VC)_{-} (p) * (dt)}{\partial p - V_{-t}} \quad (20)$$

The physical phenomenon, which dominates the dynamic pressure of a Calderín, are water and metal mass reboiler. Therefore, a good approximation of K1 would be, as shown equation (21):

$$K_{-1} = p_{-W} * V_{-W} * \frac{(\partial h_{-W})}{\partial p + m_{-t} * C_{-p} * \frac{(\partial t_{-s})}{\partial p}} \quad (21)$$

From the above equations, all data are known less the enthalpies hs and how that will depend on the pressure. For modelling, the pressure ranges defined value ranging from 0 to 2 atmospheres. The operations performed through tables and a spreadsheet that has given us an equation for each enthalpy as a function of pressure, as shown equation (22):

$$h_{-W} * \frac{KJ}{T} = 0.2769 - 48.949 * (10)^{-3} * p - 7.054 * (10)^{-4} * p^2 \quad (22)$$

The model obtained for the reboiler shown in Figure 6 so that in this case has not been considered control for the hot water flow hence the behaviour of the water volume in the boiler has no restrictions in the simulation and the equal balance.

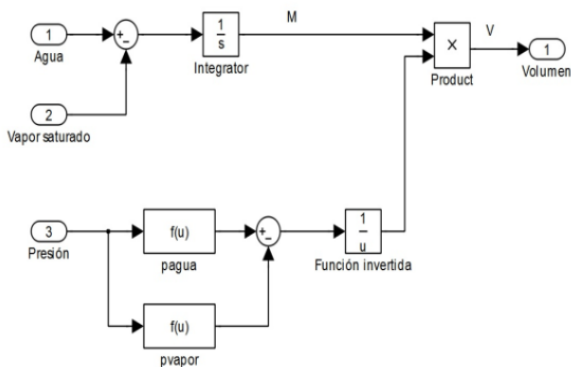


Fig. 2 Binary distillation column with pressure variable.

III. CONCLUSIONS

This paper has presented the corresponding calculation the heat balance in a capacitor, work that has been done by the Laboratory of Automation and Control at the National University of Loja. Electro mechanics race to provide students with a study of heat transfer work closer to actual industrial processes. In the construction of mathematical modelling, it has been used materials, tools, and equipment to ensure thermodynamic processes in research and teaching. Therefore it is necessary to perform this thermal and practical analysis to understand the heat balance.

In addition to the already mentioned, indicate that this type of structures which aim is to open a path for the development and enhancement of the teaching-learning process by creating an enabling environment for creativity, analysis, opening the possibility of incorporating practices in units of Thermodynamics,

Process Automation and Instrumentation in the field of training of future professionals and can also be applied together disciplines allowing introduce teaching on the concept of ergonomics and technical information concerning the interaction between people and automated subsystems and operations within a process (man-machine-environment).

REFERENCES

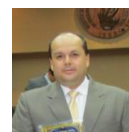
1. Aplein Ingenieros: Diseño y realización de la sala de control y operaciones de la planta Bilbao Bizkaia Gas. En URL: <http://www.apleiningenieros.com/bbg.pdf>, (2009)
2. Beyer, H., Hotzblatt, K.: Contextual design. Defining customer-centered systems. Morgan Kaufmann, San Francisco, (1998)
3. Constantine, L.L., Looockwood, L.A.D.: Software for use: a practical guide to the models and methods of usage-centered design. Addison-Wesley, (1999)
4. Good, M., Spine, T.M., Whiteside, J., George, P.: User-derived impact analysis as a tool for ACM, (1986)
5. Granollers, T.: User centred design process model. Integration of usability engineering and software engineering. Proceedings of INTERACT 2003, Zurich, Suiza, (2003)
6. Smith., Van Ness. Introducción a la Termodinámica en Ingeniería Química. Editorial McGrawHill. México 1981



Diego Jara Delgado received the title of Chemical Engineer, State University of Cuenca, in the province of Azuay. Ecuador in 1979 has a Master's Degree in Educational Research University Teaching 2002. His interest in the field of research in thermochemical and practice with projection to the empowerment of new forms of energy



Estefania Salinas received Engineering degree in Systems from Universidad Nacional de Loja, Loja, Ecuador, in 2015. She is a research interest in network security and communications



Michael Valarezo received the title of a mining engineer, at the Universidad Nacional de Loja, in the province of Loja Ecuador in 1998, through interinstitutional agreement with the Instituto Superior metallurgical and mining of Cuba has a Master's Degree in Environmental Management. Its interest in the field of research concerns the behavior of the practice and its involvement in Geotechnics and natural hazards.



Julio Eduardo Romero Sigcho received the title of a mining engineer, at the Universidad Nacional de Loja, in the province of Loja Ecuador in 2001, Master's degree in Environmental Management. I am interested in research in the mining field, environmental and new environmental technology for mining development

Mathematical Modeling to Establish the Balance of Heat in a Capacitor