

# Data Modeling, Estimation and Recovery of Dynamic and Static Sparse Signals-A Review

Sulthana Shafi, George M Joseph

**Abstract:** For sparse signal, compressed sensing is the present dogma, using only fewer measurements for sampling, compression and reconstruction of signals satisfying the Nyquist theorem. Here the outgrowth of compressive sensing using different algorithms for time invariant till time varying sparse signals and its recovery are surveyed. Thus these algorithms are effective in recovering dynamic and static sparse signal vectors. Algorithms exhibiting correlation and optimization approaches are reviewed. Also different mathematical models are reviewed which improves the quality of estimated solutions to best optimal solution.

**Keywords:** Compressed sensing, Multiple measurement vector, OFDM, Lasso, Homotopy, kalman filter, Expectation Maximization.

## I. INTRODUCTION

Compressed sensing (CS) is a convex optimization technique [1], has attracted considerable attention in various areas of engineering and statistics by suggesting that it may be possible to surpass the traditional dogma of Nyquist sampling theory [2]. However, in practice, we often compress the data soon after sensing in order to be represented without error. Clearly, this is wasteful of valuable sensing resources [3]. It is of crucial importance to start thinking about the signals exhibiting sparseness [4], which allow for an economic use of sensing resources, also lessens demands for communication, saving both bandwidth and power. So Compressive sensing has emerged, in which a signal is sampled and simultaneously compressed to its “information” rate using non-adaptive, linear measurements [5]. For “sparse” signals that can be represented using just a few terms from a basis expansion, this corresponds to sub-Nyquist sampling. The compressive sensing concept has led to the development of new signal acquisition hardware and has inspired a variety of new techniques for processing sparse data[5]. The recovery algorithms for the most part are static: they focus on finding the solution for a fixed set of measurements, assuming a fixed (sparse) structure of the signal[6],[7]. Nowadays it is necessary to deal with different applications such as Orthogonal Frequency Division Multiplexing and wireless sensors, anticipating the need for dynamical settings. So sparse recovery algorithms that cater to various dynamical settings[6] are also reviewed.

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In the beginning, the most frequently used techniques in sparse recovery was L1-Homotopy[8],[9] and now it is substituted with an efficient optimization algorithms using Expectation Maximization techniques[10]. The survey contains a discussion on the major algorithmic approaches to sparse recovery. In this review, we briefly describe these methods along with the mathematical modelings.

## II. MATHEMATICAL MODELLING

This paper mainly aiming to extend the mathematical models explained by the authors in [8],[9],[10],[11],[12],[13],[14].

### A. Single Measurement Vector (SMV) Model

The most basic model in sparse signal recovery is the single measurement vector (SMV) model [19],[20] given by:

$$Y = \phi X + V \dots \dots \dots (1)$$

where  $Y \in R^{M \times 1}$  is an available measurement vector,  $\phi \in R^{M \times N}$  ( $M \ll N$ ) is a known matrix[21],  $X \in R^{N \times 1}$  is an unknown vector which we want to estimate, and  $V \in R^{M \times 1}$  is an unknown noise vector[22]. Generally, the  $\phi$  matrix is assumed to satisfy the unique representation property (URP) [11],[23],[24]. In compressed sensing, it is called a sensing matrix or a measurement matrix [24] and in signal representation, it is called a basis matrix or a dictionary matrix [11],[25],[26]. The problem  $Y = \phi X + V$  is an underdetermined inverse problem. Generally, there are infinite solutions. Thus, it is impossible to find the true solution. However, when the true solution is sufficiently sparse, it is possible to find it with small errors [11],[27],[28] or even exactly in some cases [11],[29]. However, this approach is NP-hard [11],[30]. So alternative algorithms were proposed to seek the sparsest. In Section IV, we will discuss it in detail.

### B. Block Sparse Model

Another class of widely used model is block/group structure [19]. With this structure, X can be viewed as a concatenation of blocks, i.e.,

$$X = (x_1, \dots, x_{d_1}, \dots, x_{d_g - 1}, \dots, x_{d_g})^T \dots \dots (2)$$

Where  $d_i$  for all  $i$  are not necessarily identical [11],[31],[32],[33],[34],[35]. Among these blocks, only a few blocks are nonzero but their locations are unknown [11],[28],[36]. The SMV model with the block partition is called the canonical block sparse model [11],[28],[36]. In section IV, algorithms utilizing the intra-block correlation to improve recovery performance are discussed.



**C. Multiple Measurement Vector (MMV) Model**

The basic SMV model can be extended to the Following multiple measurement vector (MMV) model [11],[37],[38],[39].

$$Y = \phi X + V \dots\dots\dots(3)$$

Where  $Y = [Y1, \dots, YL] \in R^{M \times N}$  consists of measurement vectors,  $X = [X1, \dots, XL] \in R^{M \times N}$  is the desired solution matrix, and  $V$  is an unknown noise matrix[6],[37]. A key assumption in the MMV model is that the support (i.e. indexes of nonzero entries) of every column in  $X$  is identical (referred as the common sparsity assumption in literature [37],[39]). In addition, similar to the constraint in the SMV model, the number of nonzero rows in  $X$  has to be below a threshold to ensure a unique and global solution [11],[37],[39]. This leads to the fact that  $X$  has a small number of nonzero rows. It has been shown that compared to the SMV case, the successful recovery rate of the support can be greatly improved using multiple measurement vectors [37], [39], [40], [41],[42]. It is worth pointing out [11] that in practical applications (e.g. source localization), there is correlation among the entries in each nonzero row of  $X$ [43]. If ignoring the correlation, it can deteriorate algorithm's recovery performance. Unfortunately, most existing MMV algorithms ignored the correlation.

**D. Time-Varying Sparse Model**

MMV model cannot obtain the common sparsity assumption always. In many practical signals, the common sparsity assumption is valid for only a small  $L$  in the MMV model [37]. Thus, the time-varying sparsity model is a natural extension of the MMV model [43]. When MMV from different source signal with common support are available, accuracy of sparse signal recovery can be improved by performing joint processing of their vectors [11],[44]. It considers the case when the support of each column of  $X$  is time-varying[11],[43]. The measurement model in this case is given [43] by

$$Y^T = \phi X^T + V^T \dots\dots\dots(4)$$

Where,  $Y^T \in R^{M \times 1}$  is a measurement vector,  $X^T \in R^{N \times 1}$  is the sparse signal with time-varying sparsity [43], and  $V^T$  is a noise vector. To deal with time-varying sparsity[43], many algorithms have been proposed, such as Kalman lters [11],[45] or Bayesian approach estimation/prediction [11],[46],[47], algorithms based on the sparsity of the unknown part of the support [11],[48], algorithms based on message passing [11],[49], and algorithms based on homotopy continuation principles[11],[50]. Most of the algorithms employ SMV algorithms to find the turn on coefficients at each snapshot. However, since the support of  $x_t$  is changing slowly[43], we can view such a time varying sparsity model as a concatenation of several MMV models, wherein the temporal correlation[20] in each MMV model can be exploited to further improve the support recovery rate[11].

**E. Spatiotemporal Sparse Model**

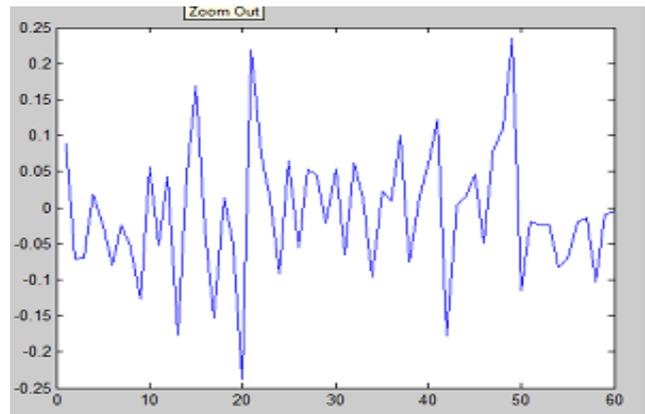
A MMV model with assumptions that matrix  $X$  has spatiotemporal correlation. It can be described as:

$$Y = \phi X + V \dots\dots\dots(5)$$

where  $Y \in R^{M \times 1}$  is an available measurement vector,  $\phi \in R^{M \times N}$  ( $M \ll N$ ) is a known matrix[20],  $X \in R^{N \times 1}$  is an unknown vector which we want to estimate, and  $V \in R^{M \times 1}$  is an unknown noise vector.

$$X = [X1, \dots, XJ] \dots\dots\dots(6)$$

Where  $X_i \in R^{d_i \times 1}$  is the  $i$ -th block of  $X$ , and  $\sum_{i=1}^g d_i = N$  and  $d_1, \dots, d_J$  is called the block partition. Among the  $g$  blocks, only a few are nonzero blocks [11],[51]. The entries in each column of  $X_i$  are correlated (intra-block correlation), and entries in each row of  $X_i$  are also correlated (temporal correlation)[11],[52]. Thus, the model can be seen as the combination of the canonical MMV model and the canonical block sparse model[11],[53].



**Figure 1. Time varying MMV Model**

An Experimental result of time-varying MMV model designed is shown in Figure 1.

**III. SIGNAL RECOVERY**

Signal recovery using Compressive Sensing are broadly categorized into 2 classes [54]

**(1) Deterministic signal model**

- i. Underlying signal is seen on deterministic vector [44].
- ii. Sparsity promoting cost function (e.g.:- L1 norm) is employed to solve the problem[44].
- iii. Approach include basis pursuit (BP), orthogonal matching pursuit (OMP) and its variants[44].
- iv. MMV based recovery algorithm targeted for deterministic signal recovery include mixed norm solution and convex relaxation.

**(2) Probabilistic signal model**

- i. Signal sparsity is described by apriori distribution of signal and Bayesian framework is employed in finding sparse solution[44].
- ii. MMV based recovery algorithm include MMV sparse Bayesian learning (SBL), Block SBL, Auto – regressive SBL and kalman filtering based SBL[44].



The apriori knowledge of probability is not easily predictable. So deterministic signal model is most appropriate in most applications. However, In OFDM signals, all signals are equally probable, it is possible to prefer either deterministic or probabilistic signal model.

**IV. OUTGROWTH OF ALGORITHMS FOR SIGNAL RECOVERY**

Algorithms for Signal recovery as described in [11],[12],[13],[14],[15],[16],[17],[18],[55] by the authors are discussed here.

To recover sparse time varying signals, impose sparse constraints on either particle level, estimate level or both [56]. Here, due to slow variation of signal; sub sparse, estimation of channel parameter is translated into unconstrained minimization problem. Optimization approach [11] can be of:

**a) Convex approach**

With some conditions on  $\phi$  and  $x$ , it can be shown [11],[27],[57],[58] that the true solution of (1) can be found within the noise level by solving the following minimization problem[59]:

$$\min: ||X||_1 \text{ st: } ||Y - \phi X||_2^2 \dots \dots \dots (7)$$

Where  $\phi$  is a regularizer. Note that there are other equivalent forms, such as the one using the Lagrange multiplier:

$$X = \operatorname{argmin}_x ||Y - \phi X||_2^2 + \lambda ||X||_1 \dots \dots \dots (8)$$

Where  $\lambda$  is another regularizer. There are many solvers [60],[61] to the convex relaxation problems. Group fused lasso assumes time invariant support but allows non zero components of signal vary over time. This is a batch algorithm which relies on quadratic programming to recover unknown signals [55],[56].

The Recursive-Lasso algorithm estimates the sparse signal at each point in time recursively [56]. It is computationally more efficient [55] than group fused lasso. SPARLS algorithm [55] uses Expectation Maximization technique, estimating output vector stream from its noisy observation.

The major drawbacks of these algorithms is that one needs to tune the regularizers although there are methods to guide the tuning, such as the L-curve method [62],[63], cross-validation, and model selection [64],[65] in some applications the tuning is very difficult or even impossible. Thus, the selection of optimal  $\lambda$  or  $\phi$  remains an important topic. Recent progress on this can be found in [66],[67]. Also the global minimum generally does not correspond to the sparsest solution unless strict conditions on  $\phi$  and  $x$  are satisfied[19].

**b) Non-convex minimization**

Algorithms [6],[68] in this family seek a solution with minimal  $l_p$  norm, where  $0 < p < 1$ . Mathematically, they solve the following non-convex minimization problem:

$$X = \operatorname{argmin}_x ||Y - \phi X||_2^2 + \lambda ||X||_p \dots \dots (9)$$

$$||X||_p = (\sum_i |x_i|^p)^{\frac{1}{p}} \text{ and } (0 < p < 1) \dots (10)$$

Solving this problem generally leads to an iterative reweighted algorithm. The most famous one may be the FOCUSS algorithm [23],[69] which has been widely used in Neuromagnetic source localization [70]. These non-convex minimization algorithms also need to solve the issue of optimal choice of  $\lambda$ .

**c) Smooth approximation of l0 norm of X**

$$||X||_0 = \sum_i I(x_i \neq 0) \dots \dots \dots (11)$$

Where  $I(\cdot)$  is the indicator function[37]. They have the quality of high speed, and have excellent recovery performance in noiseless case and they have the disadvantage of robustness to noise [11],[71],[72].

**d) Greedy algorithms**

It works with high speed. Their recovery performance is strongly affected by the coherence among columns of  $\phi$ , and doesn't work well in noisy scenarios[22]. These drawbacks limit their applications to some problems such as source localization and tracking [11],[19],[22],[73],[74],[75],[76].

e) Message passing algorithms [11],[77],[78],[79] is a young group, but recently developed algorithms have shown excellent performance in some applications in terms of both speed and recovery performance. However, most algorithms cannot be used in the case when columns of  $\phi$  are coherent. From the review, it is clear that Sparse recovery with partially known support known as modified CS gives good performance, for dynamic fitting.

f) LSCS Residual, thus imposed sparsity on least square residual computed using support from previous time instant. Optimization attempts to minimize the change in support which is expected to be much more sparse than original signal[55],[56].

g) Kalman filter/Unscerted Kalman filtering [55],[56] based method whose sparsity constraints is enforced via so called pseudo measurements. Two stages of Kalman filtering are employed

- i For tracking temporal changes
  - ii Other for enforcing sparsity constraints at each stage.
- h) Homotopy Algorithm:[8],[80] for quickly solving weighed l1 norm minimization problem for streaming signal recovery.

Lassohomotopy[8],[80]:Sequentially estimating overlapping components of streaming signal while adding and removing measurements, instead of solving new optimization problem at every iteration, existing signal estimate is used as starting point (warm-start).

LI-Homotopy[9]-For solving LI norm minimization problems using homotopy techniques. These problems are usually encountered in the recovery of sparse signals from linear incoherent measurements[7]. LI Homotopy Dynamic updating moves the solution along piecewise continuous homotopy path, using previous signal estimate as a starting point[81].



This homotopy package can solve following optimization problems:-

- (i) Basis pursuit denoising (BPDN)/LASSO
- (ii) Dantzig selector
- (iii) LI decoding, robust LI decoding
- (iv) Re-weighted L1-norm (iterative and adaptive reweighing)

In addition to solving these problems for any given set of parameters, we have some dynamic algorithm to update their solution in the case of streaming signal recovery, when new measurements are sequentially added to the system[82], when the unknown signals varies over time and we get a new measurement vector.

The linear model:  $Y=AX+e$  is used. It is to solve weighted l1 norm minimization program. Solution of homotopy program changes from warm start vector to desired solution, as  $e$  changes from 0 to 1.

i) Bayesian algorithms:

It is a powerful algorithm. Sparse Bayesian Learning Algorithms [83],[84],[85],[86],[87],[88],[89],[90],[91],[92] are again classified into its subgroups:

a) T-MSBL/T-SBL model as explained in [12],[13],[14],[15],[16],[17],[18] by the authors, is used for MMV model  $Y=AX+V$ ; It exploits the temporal correlation exist in each non zero row of  $X$ [37]. It is used when matrix  $A$  is highly coherent [37]. In MMV model, Temporal correlation is utilized in single MV(SMV), no temporal information can be exploited. In SMV, it still has better performance than many algorithms in noisy environment. It is used in time varying sparsity model/dynamic compressed sensing model as support of each source vector  $X(i)$  is slowly time varying we can use the concatenate of MMV model to approximate this scenario.

2 Algorithms in it are iterative reweighted L2 version of T-SBL and iterative reweighted LI version of T-SBL. Key idea used here is the replacement of  $L_q$  norm (eg:L2 norm) with Mahalanobis distance measure Eg: Original MFOCUSS uses iterative reweighted L2 algorithm. Here Mahalanobis distance measure replaces it.

b) B - SBL for recovery of block/Group Sparse signals[93]. The B-SBL as explained in [12],[13],[14],[15],[16],[17],[18] by the authors exploits intra-block correlation in the block sparse model[28]. It can successfully solve: (i) Recovery of Block/Group sparse with block partition[20] (partition can be known or unknown) (ii) Recovery of Non-sparse signals with or without any structure[21] (not necessary is the block structure).

Algorithms derived from it are (i)BS BL-EM, gives the best performance but slowest(ii)BS BL-BO is the one with the balanced performance and speed (iii)BS BL-LI is the fastest but performance slightly inferior to other two. It provide strategies to improve the existing algorithm of group lasso so that they can also effectively explain inter-block correlation.

j) EM-GM-GAMP:

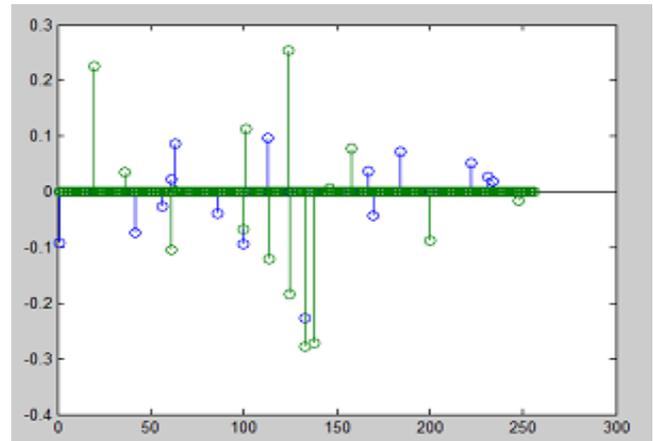
An algorithm[10],[78],[94] for sparse Representation Expectation maximization Gaussian mixture Approximate Message passing is an algorithm designed to recover a signal  $x$  from noisy measurements. The signal model is

$Y=Ax+w$ . Particularly, when measurements are under sampled ie,  $M < N$  with sufficient signal sparsity and well-conditioned mixing matrix, signal recovery is possible[94].

EM-GM-GAMP-assumes sparsity promoting iid Gaussian mixture prior [94]. After running GM-AMP, it then find Expectation Maximization (EM) update and repeats GM-AMP until convergence [94]. It support MMV model as well as both real and complex signals. EM algorithm [44] is efficient to find out ML or MAP estimate of signal model. It is more flexible version of EM Bernoulli Gaussian AMP[94].

An Experimental result of estimated and original signal designed with the basic time-varying MMV model (fig.1) using Expectation Maximization Algorithm in figure 2.

The accuracy of the result developed can be highly tuned by designing compressed sensing matrix/sensing matrix [95] more precisely and using the fast convergence algorithm for the recovery, which is an investigative area to be emphasized while designing the sparse signal.



**Figure 2. Estimated signal in blue and Original Signal in green**

## V. APPLICATIONS

It is used in applications such as Audio and image compression, DOA estimation, Wireless telemonitoring of physiological signal, Under sampled MRI Signal and Pattern recognition [96],[97],[98]. With the advances in sensor technology[55],[3], the amount of data produced by pervasive sensors is large. A huge volume of such data is generated by numerous systems. The spatially separated agents sample the data, which is then transported the data to a central server for further processing[3]. It is a crucial task to gather only the informative data for the requirement. Thus a reduction in the sensing cost, data storage, processing overhead, and communications bandwidth can be observed [3].

In OFDM[99] for channel estimation, channel response at pilot sub carriers are calculated with Least Square (LS) Methods[100], parameters of each path are estimated and tracked to improve accuracy of estimation at pilot sub carriers and finally channel response at all  $N$  sub carriers is obtained [101].



The technique to be applied for OFDM in sequence [102] are:

1. Translating estimated channel Parameters into unconstrained minimization Problem for low dimensional signal space.
2. Solving unconstrained minimization problem via tracking method. eg:-Kalman Filter..

## VI. CONCLUSIONS

The purpose of this paper is to discuss various algorithms to find solution to an under-determined inverse problem. A careful examination on the signal to be recovered is required such that it should be represented in its sparse representation to improve the quality to its best optimal solution. A detailed survey on theoretic, algorithmic, mathematical modeling on sparse signal recovery and its recovery problems in static and dynamic case are reviewed. The Algorithms exhibiting correlation and using optimization approach is giving the highest degree of performance. Utilizing these algorithms, effectiveness can be improved by combining a good design for the sensing matrix.

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