International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-5 Issue-5, June 2016

A Review on Compressive Sensed Image Reconstruction using Group-based Sparse Representation

Divya Velayudhan, Salim Paul

Abstract – Compressive Sensing (CS) – a novel sensing paradigm asserts that signals can be reconstructed from fewer samples than that recommended by Nyquist sampling theorem, when it can be expressed in a sparse basis. Conventional approaches for compressive sensed image recovery utilized fixed basis (DCT, wavelets) that do not yield higher level of sparsity for the entire signal resulting in poor performance. This paper reviews the performance of Group-based sparse representation (GSR) model for CS recovery which yields high degree of sparsity for natural images in the domain of group. GSR stacks together non-local similar patches in an image to form a group and the sparse representation of each group is achieved using self-adaptive dictionary learning technique. Thus GSR takes advantage of the intrinsic local sparsity and non-local self-similarity of images simultaneously in a unified framework. The GSR driven optimization problem is solved using split-bregman iteration. Experimental results obtained on images for CS recovery reveals the performance achieved by GSR over many current state-of-the-art schemes.

Index Terms— Compressive sensing, Sparse representation, self-similarity, split-Bregman.

I.INTRODUCTION

Compressive sensing (CS) [1][2] has piqued the interest of researchers in many field including applied mathematics and signal processing to explore the chances of sparse representation of signals as an alternative to the conventional technique of data acquisition and processing. The principle of CS stating "Sample only what you need" compared with the traditional method of "Sample first, discard unwanted later" shifts the technical burden from the sensing end to the reconstruction end. CS theory states that a signal can be recovered with very high probability from very few samples than that recommended by the Nyquist sampling theorem, if expressed suitably in a sparse domain. This has revolutionized the data acquisition paradigm especially in astronomy, geology and medical image processing where data acquisition is either expensive or difficult or where oversampling might harm the object (as in medical imaging). Compressive Sensing relies on both inherent sparsity of the signal and incoherence of the sensing modality [3].

Manuscript published on 30 June 2016.
***** Correspondence Author (s)
Divya Velayudhan*, Student, Department of Electronics and Communication Engineering, Sree Chitra Thirunal College of Engineering, Pappanamcode, Thiruvananthapuram (Kerala). India.
Salim Paul, Associate Professor, Department of Electronics and Communication Engineering, Sree Chitra Thirunal College of Engineering, Pappanamcode, Thiruvananthapuram (Kerala). India.

Higher the sparsity, better will be its reconstruction from fewer samples. Sparse modeling of signals assumes that signals can be represented using very few elements of a dictionary or basis. Hence there has been extensive research in this area to find suitable basis to represent signals. DCT, wavelets, Contour lets or so, have been used for CS recovery [4][5] but, gives poor rate-distortion performance.

This problem was rectified by including prior knowledge about transform coefficients for CS recovery such as tree-structured DCT [6] and tree-structured wavelet [7]. Chen et. al. [8] exploited multi-hypothesis predictions to generate a residual in the domain of the compressed-sensing random projections, which leads to improved reconstruction quality since the residual is more compressible than the original signal. Moreover, latest works also utilized both local and non-local statistics for image restoration [9][10]. CS recovery via collaborative sparsity utilizing both local 2-D sparsity and non-local 3-D sparsity in an adaptive hybrid space-transform domain was proposed in [11].

Recently impressive performance in image restoration was obtained using local patch-based sparse representation using dictionary learned from natural images [12]. Dictionaries learned from natural images give better performance than mathematically designed dictionaries like bandlets or wavelets. Thus learned dictionaries have been used for various image processing applications including denoising [13] and deblurring [14] even though dictionary learning is non-convex and computationally complex. However in patch-based sparse representation approach, each image patch is considered independently for dictionary learning which results in ignoring the relation between patches. However the promising result obtained by nonlocal means (NLM) filter [15] has encouraged the grouping of similar patches. Simultaneous sparse coding in [16] enforced that similar patches have the same dictionary elements. Zhang et al. [17] proposed a structural group sparse representation model for CS recovery where similar patches are grouped together and each group is used for the dictionary learning process, thereby more adaptively sparsifying the entire image. These results were further improved by using the split-bregman iteration for solving the optimization problem in [18]. This paper gives a brief review on the group-based sparse representation (GSR) framework [18]. The paper is organized as follows: Section II gives a brief overview on the background about Compressive Sensing (CS) and patch-based sparse representation model.

*Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)*

Group-based sparse representation model (GSR) is discussed thoroughly in Section III. Section IV elaborates on the implementation details for CS recovery. The simulation results are given in Section V followed by conclusion in Section VI.

II.BACKGROUND

A. Compressive Sensing

If the transform coefficients of a signal $x \in R^N$ is mostly zeroes or close enough to zero when expressed in a basis ψ, then it is said to be sparse in that domain or basis. The signal is said to be K-sparse if there is K significant elements within the transform coefficient vector.

Compressive Sensing recovery of x from $b \in R^M\,(M \geq cK)$ can be represented using the following constrained optimization problem:

$$\min_u \| \psi^T x \|_p \quad s.t. \quad b = Ax \qquad (1)$$

where $\alpha = \psi^T x$ represents the transform coefficient vector and A represents the set of random projections. If p = 0, it represents l_0 norm counting the number of non-zero entries in the vector, whereas p = 1 represents l_1 norm.

B. Patch-based Sparse Representation

Sparse representation of images has been exploited with great success in various image processing tasks [12][13]. The approach in these works consists of dividing an image into overlapping patches. Assuming the image x is divided in to patches such that $x \in R^N$ and $x_k \in R^{B_s}$ are vector representations of the original image and an image patch of dimension $\sqrt{B_s}$ x $\sqrt{B_s}$ at location k where k= 1, 2,….n, then each patch x_k can be represented using a dictionary $D \in R^{B_s x M}$ given by

$$\alpha_k = D^T x_k \qquad (2)$$

Then the complete image can be sparsely represented by the set $\{\alpha_k\}$. Thus recovering a compressive sensed image using the dictionary D can be formulated as:

$$\min_x \sum_{k=1}^{n} \| D^T x_k \|_p \quad s.t. \quad b = Ax \qquad (3)$$

However the patch-based approach ignores the relation between similar patches. The GSR model on the other hand, takes into account the self-similarity of images thereby grouping similar patches in to groups and then sparse coding each such group.

III.GROUP-BASED SPARSE REPRESENTATION

A. Group Creation and GSR Modeling

Group-based sparse representation (GSR) approach stacks together similar patches to form a group. Such groups form the basic component of GSR model. This takes into account the self-similarity of images and hence reduces the complexity of the dictionary learning process. The groups are sparsified using a group-based self-adaptive dictionary which is designed using Singular Value Decomposition (SVD). An efficient split-Bregman method is used to find the solution for the optimization problem. An image x is divided into n overlapping patches, each of size $\sqrt{B_s}$ x $\sqrt{B_s}$. Each patch is represented as $x_k \in R^{B_s}$ (k = 1, 2, ...n). For each patch in a training window, c best matching patches are found as shown in Fig. 1.

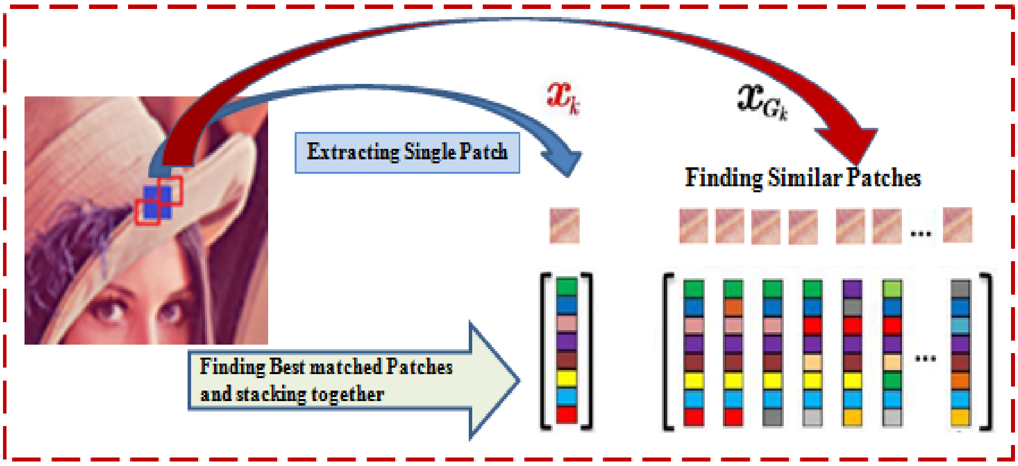

Fig. 1. A patch vector x_k is extracted from image and then c best matching patches are found and stacked to form a group x_{G_k}

The similar patches are then stacked together to form a matrix x_{G_k} of size B_s x c called the group matrix. The similar patches form the columns of x_{G_k}. Since similar patches grouped together share the same under-lying structure, a group-based dictionary learning method is designed.

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

Each group x_{G_k} is modeled using a group-based self-adaptive dictionary D_{G_k} obtained using SVD as follows:

$$r_{G_k} = U_{G_k}\Sigma_{G_k}V_{G_k}^T = \sum_{i=1}^{m} \gamma_{r_{G_k\otimes i}}(u_{G_k\otimes i}v_{G_k\otimes i}^T) \quad (4)$$

Where $\gamma_{r_{G_k}} = [\gamma_{r_{G_k\otimes 1}}; \gamma_{r_{G_k\otimes 2}}; \dots \gamma_{r_{G_k m}}]$,

$\Sigma_{G_k} = \mathrm{diag}(\gamma_{r_{G_k}})$ is a diagonal matrix , r_{G_k} is the group estimate obtained by the optimization technique and $u_{G_k\otimes i}$ and $v_{G_k\otimes i}$ are the columns of U_{G_k} and V_{G_k} respectively.

Therefore the self-adaptive dictionary for x_{G_k} is defined as

$$D_{G_k} = [d_{G_k\otimes 1}, d_{G_k\otimes 2} \dots, d_{G_k\otimes m}] \quad (5)$$

$$d_{G_k\otimes i} = u_{G_k\otimes i}v_{G_k\otimes i}^T \quad i = 1,2 \dots m \quad (6)$$

The sparse representation of x_{G_k} over D_{G_k}, results in the sparse vector α_{G_k}. If D_G represents the concatenation of all D_{G_k} and α_G represents the concatenation of all α_{G_k} and H represents the set of random projections, then the scheme for image compressive sensing recovery using adaptive learned sparsifying basis via l_0 minimization is formulated as

$$\widehat{\alpha_G} = \min_{\alpha_G} \frac{1}{2} \| HD_G \circ \alpha_G - y \|_2^2 + \lambda \| \alpha_G \|_0 \quad (7)$$

Eq. (7) reveals that GSR for Compressive sensed image recovery efficiently exploits both intrinsic local sparsity and non-local self-similarity of natural images within the domain of group.

B. Optimization Method

GSR driven l_0 minimization for CS recovery expressed by Eq. (7) is complex and NP-hard. So it's solved by optimal convex approximation such as l_1 minimization, which is equivalent to l_0 minimization under certain conditions. The l_1 minimization can be solved efficiently by split-Bregman iterative algorithm (SBI) [19].

Applying a variable u to split the above equation into two sub-problems yield:

$$\min_{\alpha_G, u} \frac{1}{2} \| Hu - y \|_2^2 + \lambda \| \alpha_G \|_0$$

$$\text{s.t.} \quad u = D_G \circ \alpha_G \quad (8)$$

Applying SBI gives:

$$u^{(t+1)} = \mathrm{argmin}_u \frac{1}{2} \| Hu - y \|_2^2 + \frac{\mu}{2} \| u - D_G \circ \alpha_G^{(t)} - b^{(t)} \|_2^2 \quad (9)$$

$$\alpha_G^{(t+1)} = \min_{\alpha_G} \lambda \| \alpha_G \|_0 + \frac{\mu}{2} \| u^{(t+1)} - D_G \circ \alpha_G^{(t)} - b^{(t)} \|_2^2 \quad (10)$$

$$b^{(t+1)} = b^{(t)} - (u^{(t+1)} - D_G \circ \alpha_G^{(t+1)}) \quad (11)$$

The algorithm for optimizing the solution for CS recovery using group-based sparse representation can be summarized as below:

ALGORITHM FOR COMPRESSIVE SENSED IMAGE RECOVERY

Input: The CS measurements y and the random projection operator H.

Initialization: $t = 0;\ b^{(0)} = 0;\ \alpha_G^{(0)} = 0;\ u^{(0)}, B_s, c,\ \lambda, \mu$

For Iteration i = 0, 1, 2,….Maximum iteration

Update $u^{(t+1)}$ by

$$u^{(t)} - \eta[H^T H u^{(t)} - H^T y + \mu(u^{(t)} - D_G^{(t)}\alpha_G^{(t)} - b)]$$

$$r^{(t+1)} = u^{(t+1)} - b^{(t)} ;\ \tau = \frac{\lambda K}{\mu N}$$

for each of the groups x_{G_k}

Build dictionary D_{G_k} by Eq. (5)

Rebuild $\widehat{\alpha_{G_k}}$ by $\widehat{\alpha_{G_k}} = \mathrm{hard}(\gamma_{G_k}, \sqrt{2\tau})$

end for

Form $D_G^{(t+1)}$ by concatenating all D_{G_k} .

Form $\widehat{\alpha}_G^{(t+1)}$ by concatenating all $\widehat{\alpha_{G_k}}$

Modify $b^{(t+1)}$ by Eq. (11)

$t = t + 1$

End for

Final restored image $\hat{x} = D_G \circ \widehat{\alpha_G}$

IV.SIMULATION RESULTS

The GSR algorithm for compressive sensing was tested on standard test images to evaluate its performance efficiency and compared with other methods. CS measurements were obtained using Gaussian random projection matrix on the original image at block level. The settings used are : 32 x 32 block size for CS. The size of each group for GSR is 64 x 60, B_s equals 64 and c equals 60. The overlapping between adjacent patches is 4 pixels. The training window is of size 40 x 40.

GSR is compared with three other methods- wavelet method (DWT) , total variation (TV) method [20] and multi-hypothesis method (MH) [21].The PSNR values obtained for all the four methods are compared in Table 1. The visual results of the four methods on image Barbara are shown in Fig. 2.and it reveals the efficiency of the GSR method.

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

TABLE I. Psnr Values for Various Cs Recovery Methods

Ratio	Algorithm	House	Barbara
20%	DWT	30.70	23.96
	TV [20]	31.44	23.70
	MH [21]	33.60	31.09
	GSR	36.78	34.59
40%	DWT	35.69	28.53
	TV [20]	35.56	26.56
	MH [21]	37.04	35.20
	GSR	40.60	38.99

(a) Original (b) DWT (23.96 dB) (c) TV (23.79 dB)

(d) MH (31.09 dB) (e) GSR (34.59 dB)

Fig. 2. Results obtained for CS recovery on Barbara with ratio = 20%

V.CONCLUSION

Group-based sparse representation method for CS recovery effectively exploits the internal local sparsity and non-local self-similarity of images and has been proved to provide better performance than many state-of-the-art methods.

REFERENCES

1. D. L. Donoho, "Compressed sensing," IEEE Trans. Inf. Theory, vol.52, no. 4, pp. 1289–1306, 2006.
2. E. J. Candès and T. Tao, "Near-optimal signal recovery from random projections: Universal encoding strategies," IEEE Trans. Inf. Theory, vol. 52, pp. 5406–5425, 2006.
3. E. Candes and M. Wakin, "An introduction to compressive sampling," IEEE Signal Process. Mag. vol. 25, no. 2, pp.21–30, Mar. 2008
4. M. N. Do and M. Vetterli, "The contourlet transform: An efficient directional multiresolution image representation," IEEE Trans. on Image Processing, vol. 14, no. 12, pp. 2091–2106, Dec. 2005
5. C. Li, W. Yin, and Y. Zhang, "TVAL3: TV Minimization by Augmented Lagrangian and Alternating Direction Algorithm," 2009
6. L. He, H. Chen and L. Carin, "Tree-structured compressive sensing with variational Bayesian analysis," IEEE Signal Processing Letter, vol. 17, no. 3, pp. 233–236, 2010
7. L. He and L. Carin, "Exploiting structure in wavelet-based Bayesian compressive sensing," IEEE Trans. Signal Process., vol. 57, no. 9, pp. 3488–3497, 2009
8. C. Chen, E. W. Tramel, and J. E. Fowler, "Compressed-Sensing Recovery of Images and Video Using Multihypo-thesis Predictions," Proc. of the 45th Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, pp. 1193–1198, Nov. 2011
9. J. Zhang, R. Xiong, S. Ma, and D. Zhao, "High-Quality Image Restoration from Partial Random Samples in Spatial Domain", Proc. of IEEE Visual Communications and Image Processing, pp. 1–4, Tainan, Taiwan, Nov. 2011.
10. J. Zhang, R. Xiong, C. Zhao, S. Ma, D. Zhao. "Exploiting Image Local and Nonlocal Consistency for Mixed Gaussian-Impulse Noise Removal", Prof. of IEEE Int. Conf. on Multimedia & Expo, pp. 592–597, Melbourne, Australia, Jul. 2012.
11. J. Zhang, D. Zhao, C. Zhao, R. Xiong, S. Ma, and W. Gao, "Compressed Sensing Recovery via Collaborative Sparsity", Proc. of IEEE Data Compression Conference, pp. 287–296, Snowbird, Utah, USA, Apr. 2012
12. M. Aharon, M. Elad, and A. Bruckstein, "K-SVD: An Algorithm for Designing Overcomplete Dictionaries for Sparse Representation," IEEE Trans. on Signal Process., vol. 54, no. 11, pp. 4311–4322, 2006
13. M. Elad and M. Aharon, "Image denoising via sparse and redundant representations over learned dictionaries," IEEE Trans. Image Process. vol. 15, no. 12, 2006, pp. 3736–3745.
14. W. Dong, L. Zhang, G. Shi, and X. Wu, "Image deblurring and super resolution by adaptive sparse domain selection and adaptive regularization," IEEE Trans. Image Process., vol. 20, no. 7, Jul. 2011, pp. 1838–1857.
15. Buades, B. Coll, and J. M. Morel, "A non-local algorithm for image denoising," in Proc. Int. Conf. CVPR, Jun. 2005, pp. 60–65
16. J. Mairal, F. Bach, J. Ponce, G. Sapiro, and A. Zisserman, "Non-local sparse models for image restoration," in *Proc. IEEE 12th Int. Conf. Comput. Vis.*, Tokyo, Japan, Sep. 2009, pp. 2272–2279
17. J. Zhang, D. Zhao, F. Jiang, and W. Gao, "Structural group sparse representation for image compressive sensing recovery," in Proc. IEEE DCC, Snowbird, UT, USA, Mar. 2013, pp. 331–340.
18. J. Zhang, C. Zhao, D. Zhao, and W. Gao, "Image compressive sensing recovery using adaptively learned sparsifying basis via L0 minimization," Signal Process., vol. 103, pp. 114–126, Oct. 2014
19. T. Goldstein and S. Osher, "The split Bregman algorithm for L1 regularized problems," SIAM J. Imaging Sci, vol. 2, Apr.2009, pp. 323-343
20. C. Li, W. Yin, H. Jiang, and Y. Zhang, "An efficient augmented Lagrangian method with applications to total variation minimization," Computational Optimization and Applications, Vol. 56, no. 3, pp. 507–530, Dec. 2013
21. C. Chen, E. W. Tramel, and J. E. Fowler, "Compressed-sensing recovery of images and video using multi-hypothesis predictions," in Proc. 45th Asilomar Conf. Signals, Syst., Comput., Pacific Grove, CA, USA, Nov. 2011, pp. 1193–1198

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)