

Performance Analysis of Efficient Framework of Image Segmentation using Energy Minimization Function

Pranoti P. Mahakalkar, Aarti J. Vyavahare

Abstract: Image segmentation plays very vital role in many image processing applications and domains. Efficient image segmentation leads to accurate results to end users. There are number of image segmentation techniques presented so far with different objectives. The existing segmentation techniques are based on various features of image. Target objects segmentation from the input image which may from different application areas such as medical, security systems etc. The segmentation of images those are having many complex areas, mixed pixel intensities or noise corrupted data. The existing level set based image segmentation methods needs the prior information about the total number of image segments which is practically impossible for each image. Therefore to overcome such limitations and research challenges of image segmentation, in this paper we proposed the new image segmentation energy function with two distribution descriptors in order to distinguish automatically background and target region from input image. This paper presents the extensive analysis of this proposal method against the existing method in terms of execution time and JD error rates. In this propose scheme, first single background descriptor models the heterogeneous background with multiple regions. Then, the target descriptor takes into account the intensity distribution and incorporates local spatial constraint. The proposed descriptors, which have more complete distribution information, construct the unique energy function to differentiate the target from the background and are more tolerant of image noise. The simulation and evaluation of this proposed method is done by using well known image processing tool MATLAB.

Index Terms: Image Segmentation, Image processing, Energy Minimization, Level Set Methods, Region based, Edge based, Minimizer

I. INTRODUCTION

Image segmentation, is widely used in content based image retrieval, Machine vision, Medical Imaging ,Object detection, Pedestrian detection, Face detection, Brake light detection, Locate objects in satellite images, Recognition Tasks, Iris recognition, Traffic control systems. The main applications of medical imaging are Locate tumors and other pathologies, Measure tissue volumes, Diagnosis & study of anatomical structure.

Medical imaging which consists mainly combination of sensors recording the anatomical body structure like magnetic resonance image (MRI), ultrasound or CT with sensors monitoring functional and metabolic body activities like positron emission tomography (PET), single photon emission computed tomography (SPECT) or magnetic resonance spectroscopy (MRS). Results can be applied, for instance, in radiotherapy and nuclear medicine. This paper mainly deals with the application on magnetic resonance image [21].

If an image has been pre-processed appropriately to remove noise and artifacts, segmentation is often the key step in interpreting the image. Image segmentation is a process in which regions or features sharing similar characteristics are identified and grouped together. Image segmentation may use statistical classification, thresholding, edge detection, region detection, or any combination of these techniques. Most segmentation techniques are either region-based or edge based- Region-based techniques rely on common patterns in intensity values within a cluster of neighboring pixels. The cluster is referred to as the region, and the goal of the segmentation algorithm is to group regions according to their anatomical or functional roles. Edge-based techniques rely on discontinuities in image values between distinct regions, and the goal of the segmentation algorithm is to accurately demarcate the boundary separating these regions.

Segmentation is a process of extracting and representing information from an image is to group pixels together into regions of similarity. The categories of image segmentation mainly include Clustering Methods - Level Set Methods, Histogram-Based Methods- Graph Partitioning Methods, Edge Detection Methods- Watershed Transformation, and Region Growing Methods- Neural Networks Segmentation.

The method was implemented by Osher and Sethian; mainly rely on the theory of curve and surface evolution and on the link between front propagation and hyperbolic conservation laws. The level Set Method is a numerical technique for tracking interfaces and shapes has been increasingly applied to image segmentation. This method makes it easy to follow shapes that change topology, means the shape can be split into two for developing holes and the reverse of these operations is also possible. In this method contours or surfaces are represented as a zero level set of a higher dimensional function, usually called a level set function.

Manuscript published on 30 June 2016.

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The image segmentation problem can be formulated and solved in a principled way based on well-established mathematical theories including calculus of variations and partial differential equations can be achieved with the help of level set representation. Level set approach is numerically most stable implicit representation. The method is acting as a great tool for modeling time varying objects. There are two types of level set formulations which include Time dependent level set formulation and Stationary level set formulation. Two important numerical issues regarding level set method is Level Set Method does not implicitly preserve the level set function as a distance function and the algorithm is slow because the time step is limited by the CFL condition.

The main advantage of Level Set Method is that one can perform numerical computations involving curves and surfaces on a fixed Cartesian grid without having to parameterize these objects. The level set method amounts to representing a closed curve using auxiliary function called level set function which is represented as the zero level set. The level set method defines problem in one higher dimension. The zero level set at one point in time as a slice of the level set surface. The formulation of level set implies that the level set value of a point on the contour with motion must always be zero. The level set method is boundary driven and region driven model free segmentation.

There are many methods presented fewer than two categories of level set methods, however they suffered from limitations. The existing level set methods are having disadvantage while producing the efficient segmentation without prior information over image segments and in noisy images. To overcome these limitations recently energy model proposed. However, such techniques required prior knowledge of number of level sets required. Also when images are corrupted with noise, then performance of existing segmentation methods becoming poor [1].

In this research work, the novel technique of image segmentation energy function is proposed with goal of overcoming the limitations of existing methods. In this proposed energy minimization based segmentation method, two distribution descriptors are used in order to distinguish the background and the target region. The single background descriptor models the heterogeneous background with multiple regions. Then, the target descriptor takes into account the intensity distribution and incorporates local spatial constraint. These descriptors are having more complete distribution information, construct the unique energy function to differentiate the target from the background and are more tolerant of image noise [2]. Remainder of paper is composed of sections listed as: in section II, literature review of different level set methods for energy minimization in image segmentation domain are discussed, section III will present the architecture of proposed algorithm and its mathematical formulation. IV the current results for proposed system elaborated and discussed. Section V presenting the conclusion and future work.

II. LITERATURE REVIEW

In this section we are discussing the different techniques based on level set approach for image segmentation.

A. Novel Shape Prior Based Level Set Method for Liver Segmentation From MR Images [3].

This paper propose a novel level set based variation approach that incorporates shape prior knowledge into the Chan Vese model which can overcome the leakage and over segmentation problems. Statistical methods are used to get the prior shape and the training process allows the prior shape not exactly at the location of desired object. But all these models above fail to segment object from images where the objects are occluded by other objects or some parts of them are in low gray contrast or missing because they are all gray intensity based. These methods do not allow for translation, rotation, and scaling of the prior shape. The proposed method allows translation, rotation and scaling of the prior shapes and also performs object supervision before segmentation to achieve better result and higher performance. This is mainly achieved with the help of affine transformation. The proposed method novel level set base variation model for segmentation using prior shapes which helps us to detect the liver perfusion position and measure the intensity. This helps to achieve faster speed and we propose a new measure which help to perform the training of given shapes. If some part of the image is occluded or missing, we can still get a reliable segmentation result

B. Variational Image Segmentation Models: Application to medical images MRI [4]

An important branch of computer vision is image segmentation. The image segmentation problem can be solved with the help of mathematical frame work based on variational model and partial differential equations. This framework is defined in a continuous setting which makes the proposed model independent with respect to the grid of digital images This paper mainly introduces a segmentation method based on Variational approach. These models are defined in a continuous setting and are mathematically well established. Two well-known image segmentation based on variational approach is specified. They are Mumford Shah model and level set approach. The Mumford Shah model is mainly used to minimize the energy function. The level set method was introduced for tracking moving fronts. The theory of curve/surface convolution methods are used here which efficiently solve the problem of moving fronts especially in the problem of change of topology. In traditional level set methods it is necessary to initialize the level set method function as a signed distance function. But the proposed method suggests the following function as the initial function. Hence this method is more suitable than other existing segmentation methods.

C. Variational Multi Phase Level Set Approach to Simultaneous Segmentation and Bias Correction [5]

This paper represents a novel level set approach to simultaneous tissue segmentation and bias correction of MRI. The intensity of each tissue is modeled as a Gaussian distribution of spatially varying mean and variance.

The sliding window is used to transform intensity from one dimension to another. The objective function is distributed over each point and integrated over the entire domain to form a variational level set evolution. With the help of level set evolution process tissue segmentation and bias correction are achieved. The proposed method uses a variational level set approach to simultaneous segmentation and bias correction. The method uses a k-Means clustering which is weighted k-means variational level set. The proposed method use a special case SVMLS-Statistical & variational multiphase level set. Advantage of this method is that the smoothness of the computed bias field is ensured by the normalized convolution without extra cost.

D. Model-Based Medical Image Segmentation: A Level Set Approach [6]

This method mainly requires the definition of a speed function that governs model deformation. This mainly considers the region intensity information while the existing methods usually do image gradient information. The region intensity information into the level set framework forms an accurate and robust segmentation. The level set approach the convergence to the final result may be relatively independent of the initial shape, and branches and splits and merges can develop without problems as the front moves. The challenges in level set approach are to construct an adequate model for the speed function. The main idea is to define a range of intensity values that classify the tissue type of interest and then base the propagation term on the level set equation for that intensity range. Using this approach the smoothness of the evolving surface can be used to prevent the leakage which is common in connected component schemes.

E. Variational Level Set SAR Image Segmentation Approach Based on Statistical Model [7].

Due to the presence of speckle noise SAR image automatic segmentation is difficult task. A variational level set approach for SAR image is presented in this paper which states a new energy functional is defined by taking account of a statistical model of speckle noise. The energy functional produced in this paper is different from the energy functional with respect to the parameterized curve in general level set approach. By minimizing the energy functional in level set approach segmentation is achieved. The energy functional well describes the property of SAR image accurately and automatically extracts the region of interest in SAR image without any speckle pre-processing step. The functional consist of a region based term and a boundary based term. The region based term is derived from SAR image statistical property and measure the conformity of image data to a gamma model. The boundary based term is related to the edge gradient and alliance the boundary to the image pixels with maximum gradient while keeping its smoothness which describes region property and boundary property of SAR image simultaneously. Target extraction is implemented by minimizing the energy criterion via variational level set approach. The segmentation method based on this paper is more accurate than segmentation method with active contours and level sets.

F. Variational Level Set Approach to Segmentation and Bias Correction of Images with Intensity Inhomogeneity [8]

This paper presents a variational level set approach to join segmentation and bias corrections of images with intensity inhomogeneity. Intensities in small local regions are separable despite of the inseparability of the intensities in the whole image caused by intensity inhomogeneity. A weighted k mean clustering objective function is defined for image intensities in a neighbourhood around each point, with the clusters centers having a multiplicative factor that estimate the bias with in the neighborhood. The objective function is then integrated over the entire domain and in cooperated in to a variational level set formulation. The energy minimization is performed by a level set evolution process. This method is able to estimate bias of general profiles. This method is robust to initialization and allows automatic applications. The level set formulation mainly consists of $N=2$ (two phase) and $N=4$ (multi-phase process)By this we can reduce energy formulation and hence the segmentation is done. The advantage of this method is not sensitive to initialization and thereby allowing automatic applications. Reinitialization is also possible in this method.

G. Variational and Shape Prior-based Level Set Model for Image Segmentation [9]

Radiographic medical Images are consider were boundaries are not silent and object having the same gray level as other structure in the image, so we need a prior information about the shape that forces the level set to be closed to a signed distance function which avoids the re initialization procedure. The proposed method uses a prior information about the boundary so that the active contour or level set will evolve according to a known shape. The proposed segmentation method needs a training shape. The shape variants are computed by principle component analysis (PCA). The new shape prior is the matrix of Eigen vectors and the vector of shape parameters to be determined. A new energy functional is defined for image segmentation which is used to avoid the re initialization process and its related draw backs

H. Multiphase Level Set Framework for Image Segmentation Using the Mumford and Shah Model [10].

This paper proposes a new multiphase level set frame work for image segmentation using the Mumford and Shah Model for piece wise constant and piece wise smooth optimal approximations. This method is a generalization of active contour model without edges based two phase segmentation in the case of piece wise smooth only two level set functions formerly suffix to represent any partition based on the four colour theorem in the piece wise constant case only $\log n$ level set functions for any phases in the piece wise constant case. This avoids the problem of vaccum and overlaps and represents boundaries with complex topology including triple junction.

The multiphase formulation is different than the classical approaches and has the advantages that the phases cannot produce vacuum or overlap by construction and it minimizes the computational cost by reducing the number of level set functions.

III. PROPOSED WORK

3.1. Energy Minimization

We first give an outline of the energy minimization for image segmentation process. We define a medical image I_i and its corresponding segmentation (i.e., pixel labels) S_i , each having N pixels. Then $\mathbf{I} = \{I_1, I_2, \dots, I_N\}$ and $\mathbf{S} = \{S_1, S_2, \dots, S_N\}$ are sets of images and their corresponding ground-truth segmentations. In a slight abuse of the notation, we occasionally omit the subscript i from I_i and S_i for clarity and instead use I and S . The first step in any energy minimization problem is the identification of the form of the energy function. In the next section, we will briefly group some popular energy terms into three main categories: boundary, region, and shape. Boundary terms are concerned primarily with the object boundary, region terms with the region inside or outside the object, and shape terms with the shape of the object. Other energy terms include spatial constraints on multipart objects, for example, containment (or layering), exclusion, or the number of labels.

Here we use these groupings to build a general energy functional. It may be convex or non-convex, as can the shape space over which it will be minimized. A general form is $E(S, I, \mathbf{w}) = w_1 \times \text{boundary}(S, I) + w_2 \times \text{region}(S, I) + w_3 \times \text{shape Prior}(S)$, with free parameter $\mathbf{w} = [w_1, w_2, w_3]$. Depending on the value of \mathbf{w} , minima of E tend toward best satisfying the boundary, region, or shape terms. We note that the boundary and region terms are often referred to as external terms, since they involve cues external to the shape model, while the shape prior is deemed an internal term. More generally, we can write

$$E(S, I, \mathbf{w}) = w_1 J_1(S, I) + w_2 J_2(S, I) + \dots + w_n J_n(S, I), \quad (2.1)$$

Where J_i is the i th energy term
 $\mathbf{w} = [w_1, \dots, w_n]$ are the weights

We note here that depending on the nature of S , E may be called an energy function or an energy functional, with the latter indicating S itself is a function. The segmentation problem is to solve

$$S^* = \arg \min_s E(S | I, \mathbf{w}), \quad (2.2)$$

which involves choosing a \mathbf{w} and, depending on the nature of the energy functional, may also require training appearance and/or shape priors using \mathbf{I} and/or \mathbf{S} and setting an initialization. A gradient-descent-based solver is typically used but combinatorial approaches have also been explored for discretized versions of the problem (Boykov and Kolmogorov, 2003). In either case, non-convexity, or super modularity for combinatorial problems, can be quite problematic. There is no guarantee that another solution does not exist that better minimizes the energy and thus is

potentially a better segmentation. Ideally both functional and shape spaces are convex, guaranteeing globally optimal solutions. However, whether convex or not the ground-truth segmentation, S_i , for image, I_i is not guaranteed to be an optima (local or global) of $E(S|I_i, \mathbf{w})$. The goal, in general, is to build an $E(S|I_i, \mathbf{w})$ such that S^* is as close as possible to S_i under some definition of closeness.

One of the earliest developed, and perhaps most recognized, examples of energy minimization methods being applied to image segmentation is that of deformable models. Deformable models for MIS gained popularity since the 1987 introduction of *snakes* by Terzopoulos et al. (Kass et al., 1987; Terzopoulos, 1987).

At a high level, energy-minimizing deformable models work by deforming a user provided initial shape to fit to a target structure in a medical image. Shape-changing deformations result from the minimization, with respect to the shape, of a cost function measuring how plausible is the shape model and how well it aligns with the boundaries of the target anatomy in the image. Since the shape model itself is most commonly represented by a function, the cost function is often termed an energy functional and its gradient is derived using methods from vibrational calculus.

The shape deformations are therefore typically simulated by solving an initial value problem using gradient descent optimization algorithms. One further development, "Deformable organisms," uses artificial life modelling techniques to augment the energy-minimizing deformable models with models of cognition, behaviours, and sensory input.

3.2. Proposed Solution

In this section we are presenting the architecture and algorithm design for proposed energy minimization based image segmentation method. Below figure 1 is showing the flowchart of proposed energy minimization method.

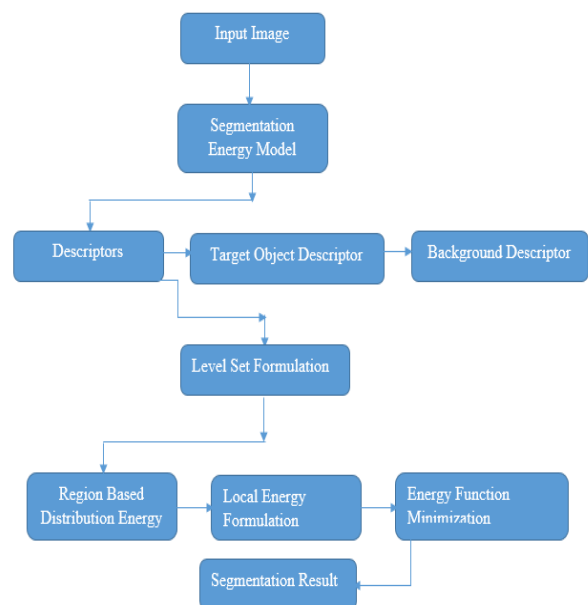


Figure 1: Proposed Algorithm Design



3.3. Algorithm Design

Step 1: Segmentation Energy Framework

1) Problem Description and Hypothesis on Segmentation

Model: A given image u_0 on an image domain Ω can be represented as N disjoint regions: $\cup_{i=1}^N \Omega_i = \Omega$ and $\cap_{i=1}^N \Omega_i = \emptyset$, and may be corrupted by different levels/types of image noises. These regions are to be partitioned into two major groups $\{ \Omega_W, \Omega_B \} = \Omega$ including the target object(s) W represented as $\Omega_W = \{ \Omega_{K_t} \}_{t=1}^W = 1$; and the background regions B denoted as Ω_B , and $\Omega_B = \{ \Omega_i \}_{i=K_W+1}^N$ with an unknown number of the background regions. Any specific target object t with K_t regions is represented as $\Omega_{K_t} = \{ \Omega_i \}_{i=K_{t-1}+1}^{K_t} + 1$. Deduced from the problem description, if we can define a unique descriptor for a target object and a single separate background descriptor, we should be able to distinguish the target object(s) from the background.

2) Formulation of Segmentation Energy Model: We define two unique descriptors to determine the label of every pixel belonging to the target (foreground) or the background, which are formulated as the target energy and background energy terms. By incorporating the energy terms from the target and the background, our energy functional is defined as below:

$$E = E_W + E_B$$

$$= \int_{\Omega_W} O_W(u_0(x)) dx + \int_{\Omega_B} O_B(u_0(x)) dx$$

(6)

Where E_W is the target energy and E_B is the background energy that are derived from the target descriptor $O_W(u_0(x))$ and background descriptor $O_B(u_0(x))$.

The minimization of energy function M is performed to optimize the descriptors to fit the target object(s) and background to classify each pixel of u_0 belonging to the target object(s) or to the background. The first component E_W of the energy formulation (6) can be further expanded to describe multiple target object(s) as:

$$[1] E_W = \int_{\Omega_W} O_W(u_0(x)) dx = \sum_{t=1}^W \sum_{i=K_{t-1}+1}^{K_t} \int_{\Omega_i} O_t(u_0(x)) dx$$

(7)

[2]

Where $O_t(u_0(x))$ is the descriptor of the target object t . Without loss of generality, in the following sections we present the energy formulation for segmenting a single target from a given image.

Step 2: The Descriptors Design

Two descriptors are designed such as target object descriptor and background descriptor.

Step 3: Minimization of the Energy Function

We introduced the distance regularized term [8] into our level set formulation to regulate the iterations of evolving surface without re-initialization. After embedding the distance regularized term with E_p and E_t^S . This function finally used to minimize the energy consumption performance of segmentation method.

IV. COMPARATIVE ANALYSIS

4.1. Performance Metrics

This section deals with presentation and analysis of simulation work with different types of images for segmentation. We evaluated the proposed energy minimization based segmentation method against two well-

known existing methods such as Level set method and Chan-Vese method in order to claim the efficiency of proposed technique. We are comparing three methods based on three important performance metrics:

Average CPU Time: This can be computed by using below performance metrics

$$CPU\ Time = I * CPI * T$$

Where,

I = number of instructions in program

CPI = average cycles per instruction

T = clock cycle time

Processing Time: This is performance metrics which is used to check total time required for image segmentation. This can be measured as

$$Processing\ time = end_time - start_time$$

Where,

$start_time$ is variable which holds current system time when input image is taken for processing of segmentation

end_time is variable which holds the current system time when segmentation process completed.

Average Jeccard Distance (JD) Error: error metric for precise evaluation of the segmentation results. This can be computed using below equation

$$JD(Is, Im) = 100\% \times (1 - |Is \cap Im| / |Is \cup Im|),$$

Where, Is is the segmentation result and Im is the groundtruth reference. The segmentation error rate is 0 for a perfect segmentation and 1 if segmentation and reference do not overlap at all.

4.2. Simulation Results

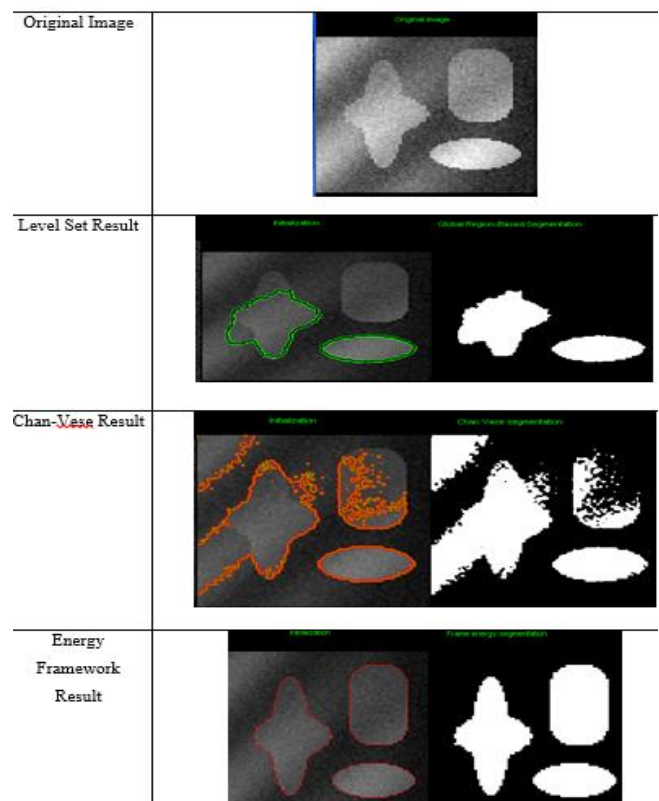


Figure 2: Image Segmentation Result with different methods



Table 1 is showing the comparative study for different kinds of images dataset for execution time.

Table 1: Comparative Analysis of Execution Time

Images Type	Multiple-Level Set	CV-Model	Proposed Method
30 Synthetic Images	3.8193	26.3358	0.10117
15 Medical Images	3.6406	25.8629	0.09
30 Biometric Images	3.8003	33.3758	0.0497
70 All types of images	4.8746	27.9554	0.067

Table 2: Comparative Analysis of JD Error Rate %

Images Type	Multiple-Level Set	CV-Model	Proposed Method
30 Synthetic Images	1.12	0.7	0.35
15 Medical Images	1.03	0.69	0.31
30 Biometric Images	1.1	0.7	0.32
70 All types of images	1.3	0.9	0.45

Above both tables are showing that proposed energy minimization based image segmentation method having less execution time and JD error rates percentage as compared to existing method.

V. CONCLUSION AND FUTURE WORK

In this paper we deals with energy minimized based image segmentation technique. Throughout the paper first we summarized background of image segmentation, fundamentals, and definitions, and methods, advantages of methods and disadvantages of methods. Our aim was to present review of different energy minimization based image segmentation techniques. Image segmentation is widely used process in many of real time applications for tracking, identification and analysis. We focused methods under the category of level set functions. Level set methods basically used in order to solve the problems of propagation of surfaces or curve implicitly. We listed the limitations and drawbacks of existing methods. To overcome the limitations of existing method in this project we proposed the innovative energy framework for the segmentation of complicated images. Our background descriptor provides a unified energy for the background that has arbitrary number of segments and regions. By integrating distribution information and local spatial boundary properties, our object descriptor is able to segment objects with multiple- and single regions. Our evaluation across synthetic and medical images showed that our segmentation model produced more accurate segmented target objects from images with complicated intensity backgrounds when compared to the geodesic edge-based level set model and the piecewise smooth/constant region based level set models including the multiphase level set and the Chan-Vese models. Real time

deployment, designing and testing will be the possible future work for this work.

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