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Digital Topology and Edge Detection as Application

Mohanad Abdulkareem Hasan Hasab

Abstract— Digital topology refers to the use of topological properties and features that could be extracted from images defined as digital grid, In this paper we defined the basic and well known concepts of digital topology and how it represents an image as digital array of different dimension with some operations could be used for enhancing and processing the image for different practical purpose , Then producing an algorithm to detect the edges of images that are considered a type of crucial information needed for segmentation and recognition . and also presented a brief study of the fundamental concepts of the edge detection methods.

Index Terms— Digital topology ,digital image processing , edge detection

I. INTRODUCTION

Topology grew out of geometry and set theory to study both the fine structure and global structure of space. Digital topology deals with the topological properties of digital image represented as discrete array of n-dimension .It gives a perfect model with theoretical fundamentals and operations on images that make use of information extracted from them and invests it in many application like image segmentation ,manipulation, interpolation and analysis based on objects. One of the important operation on digital image is edge detection that is the process of detection of discontinuity or sudden change in certain visual properties (such as the intensity of lighting, composition, color) and the processes and treatments is very important to the understanding of images and analysis may be used in the process of distinction objects in digital images The detection operation begins with the examination of the local discontinuity at each pixel element in an image. Amplitude, orientation, and location of a particular subarea in the image that is of interest are essentially important characteristics of possible edges .Based on these characteristics, the detector has to decide whether each of the examined pixels is an edge or not. Frei and Chen [17] suggested that edge detection is best carried out by simple edge detector, followed by a morphological thinning and linking process to optimize the boundaries. This paper gives an overview of first and second order derivative edge detections, edge fitting detection model as well as the detector performance evaluation. Here we introduce a simple but effective algorithm to detect edges in digital picture using the concepts of digital topology [1][3][4][17].

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II. DIGITAL TOPOLOGY: BASIC CONCEPTS

The image in this paper represented as an array of 2-dimensions and 3-dimensions, all of them its elements either logical (0 or 1) that is Black and White images or numerical (0-255) which is colored or grayscale ones .The elements of a two-dimensional image array are called *Pixel* while the elements of a three-dimensional image array are called *vixels*[1][6] .

Let f be a digital picture on a digital set X then we called it a binary picture , The ordered pair (p,f(p)) p$\in X$ is called a pixel ,where p is the coordinates and f(p) is the brightness level of the pixel .The foreground (object points) of the binary picture is the set $F = \{p \in X | f(p) = 1\}$,and Its background (background points) is the set $B = \{p \in X | f(p) = 0\}$.

Each pixel or voxel are associated with lattice point (i.e with integer coordinates) and these points have a relation in its plane or in 3-space called adjacency or neighborhood [8].

2.1 Neighborhood structure

a. *Connectivity in 2D* : let p(x, y) and g(x`, y`) $\in X$ (digital set) and belonging to the same neighborhood structure we can define the following relations :

- ***N_4 or 4-adjacent*** if they are differ in at most one of their coordinates , mathematically as :

$$(p,q) \in N_4 \Leftrightarrow |x-x'|+|y-y'|=1 \qquad (1)$$

- ***N_I or i-adjacent*** this called indirect neighborhood when each coordinate of one differs from the corresponding of the other by 1 , mathematically as:

$$(p,q) \in N_I \Leftrightarrow |x-x'|=|y-y'|=1 \qquad (2)$$

fig (1) illustrates the defined concepts .

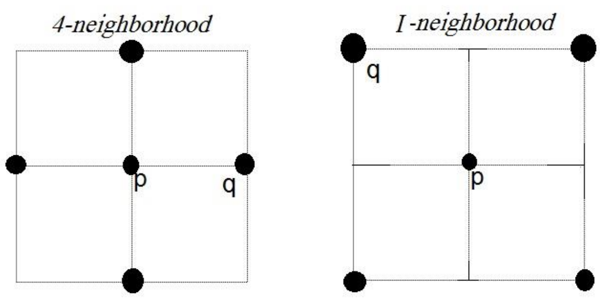

Fig (1) the neighborhood relations between two adjacent pixel in a 2D digital image

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- ***N_8 or 8-adjacent*** is $N_4 \cup N_I$ as in fig (2).[1][2][7]

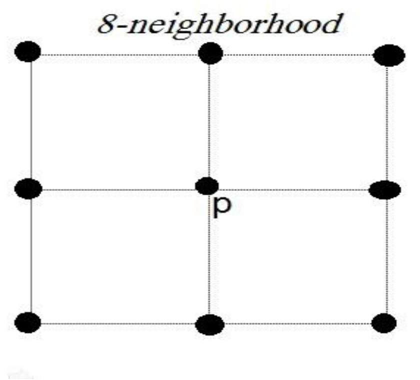

Fig (2) shows the N_8 in a 2D digital image

b. ***Connectivity in 3D*** :if p lattice point in 3d space then $N_m(P)$,m=6,18 ,26 denotes the set consisting of p and its *m-neighbors* that are distinct and each coordinate of them differs from the corresponding coordinate of p by at most one[8][9][10][5],fig(3).

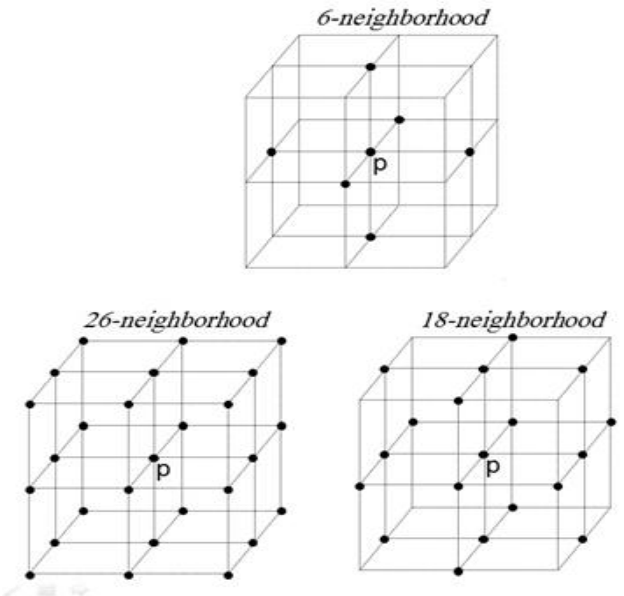

Fig (3) The 26, 18 and 6 neighborhood in a 3d images

c. ***Neighborhood or adjacency between a point and set*** :A point p is said to be *neighbor* to a set of points S if p is neighbor to some point in S. Two sets A,B are *n-neighborhood* if there are points : $a \in A , b \in B$ which are *n-neighborhood*, fig(4).

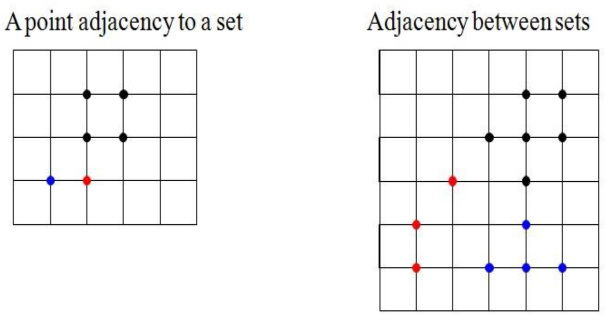

Fig (4) adjacency between point and set, set and set of points in a 2D images

2.2 Operations on Digital Picture

There are many operations to manipulate the digital picture which are defined on the same digital set,Let f and g are digital pictures :

- addition:

  $f + g = \{(p, h(p)) | h(p) = f(p) + g(p), P \in X\}$
- multiplication :

  $f * g = \{(p, h(p)) | h(p) = f(p). g(p), P \in X\}$
- Logical OR :

  $f \vee g = \{(p, h(p)) | h(p) = f(p) \vee g(p), P \in X\}$
- Logical AND :

  $f \wedge g = \{(p, h(p)) | h(p) = f(p) \wedge g(p), P \in X\}$

2.3 Connected Set And Component Set In A Digital Picture

Having a digital picture with a set S of white and black pixels. We say a set S of black and/or white points in a digital picture is *connected* if cannot be partitioned into two subsets that are not adjacent to each other . A component of a set of black and/or white points S is a non-empty connected subset of S which is not adjacent to any other points in S. In an (m,n) digital picture a component of a set of black points is an m-component, whereas a component of a set of white points is an n-component[10], fig(5).

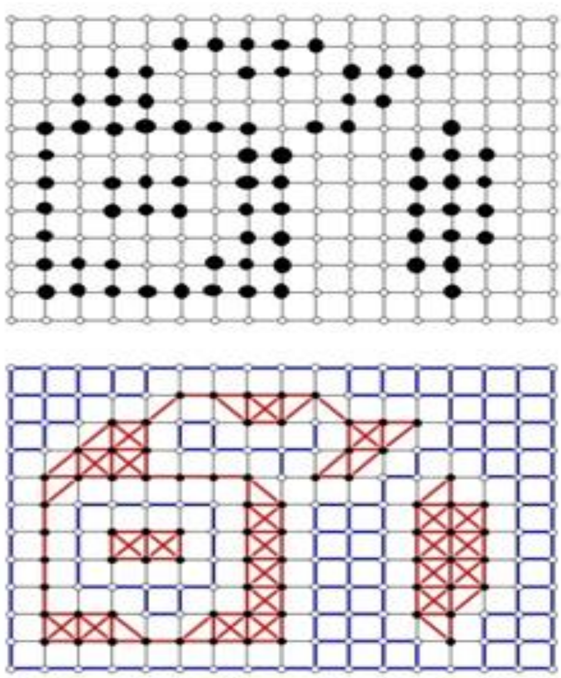

Fig (5) This picture is an (8,4) digital picture that has 3 8-componants and 3 4-components

2.4 Hole, cavity

In a digital picture P ,a white component that is adjacent to and surrounded by a black component C is called a Hole in C if P is a two-dimensional digital picture and a cavity in C if P is a three-dimensional digital picture .By a hole of P (cavity of P) we mean a hole[2][7],fig(6).

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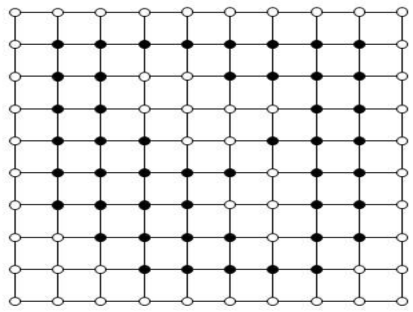

Fig (6) The black component here has two holes if we take it as (8,4) digital picture or has one hole if it regarded as (4,8) digital picture

2.5 Isolated, Border and Interior Point

An object point is said to be *isolated* if it is not adjacent to any other object point, while *a border* point (which are squared black ones ◙)is the point that is adjacent to one or more background points ; otherwise the object point is called an *Interior* point(rounded black points), fig (7).

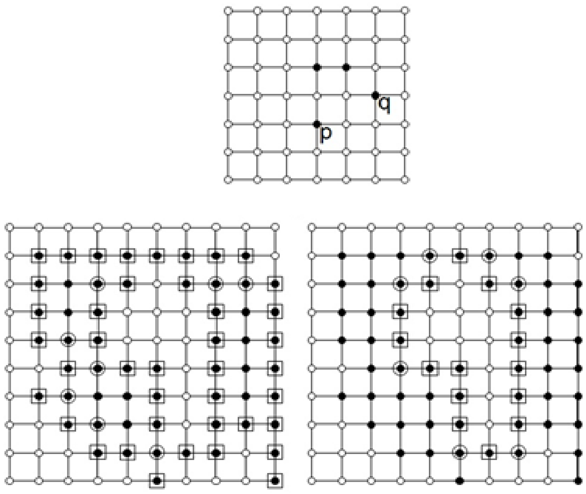

Fig (7) shows the concepts of isolated , border and interior points in a digital picture

III. EDGE DETECTION

Edge detection is a type of image segmentation techniques which determines the presence of an edge or line in an image and outlines them in an appropriate way [17][13].

The main purpose of edge detection is to simplify the image data in order to minimize the amount of data to be processed [13]. Edges are significant local changes of intensity in an image, and typically occur on the boundary between two different regions in an image, the aim of it is to produce a line drawing of a scene from an image of that scene and to provide important features can be extracted from the edges of an image (e.g., corners, lines, curves) and these features are used by higher-level computer vision algorithms (e.g., recognition), fig (8).

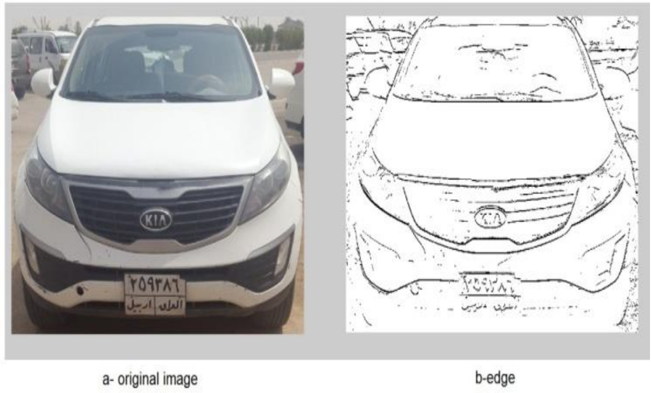

Fig (8) a-original true color image b- Logical edged image

The changing in intensity causes by either Various physical or Geometric events like object boundary (discontinuity in depth and/or surface color and texture)and surface boundary (discontinuity in surface orientation and/or surface color and texture),while sometimes by Non-geometric events as secularity (direct reflection of light, such as a mirror) or shadows (from other objects or from the same object) [13][14][16][11] .

3.1 The edge detection general algorithm

all edge detection algorithm are following the same basic steps to determine edge of an image that are starting with suppressing the noise as much as possible ,then apply a filter to enhance the quality of the edges in the image (i.e sharpening). after that it determines which edge pixels should be discarded as noise and which should be retained (thresholding).the final step is localization or determination of the exact location of an edge (sub-pixel) resolution might be required for some applications, Edge thinning and linking are usually required in this step.

3.2 Edge-detection methods

The edge detection algorithms can be generally classified based on the behavioral study of edges with respect to the operators. Different edge-detection approaches can be broadly classified under Classical or Gradient based edge detectors (first derivative), Zero crossing (second derivative) and Optimal edge-detector Calculus describes changes of continuous functions using *derivatives*[15][13].

a) First Order Derivative Based Edge Detection (Gradient method):

It detects the edges by seeking the maximum and minimum in the first derivative of the image. The gradient vector points in the direction of maximum rate of change. For a function f(a,b), the magnitude of the gradient of f at coordinates (a,b) is defined as:

$$\nabla f(a,b) = \left(\partial_x f(a,b)\right)^2 + \left(\partial_y f(a,b)\right)^2$$

3)

while the gradient orientation is given by:

$$\angle\nabla f(a,b) = \arctan\left(\partial_y f(x,a,b) \quad \partial_x f(a,b)\right) \qquad (4)$$

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b) Second Order Derivative Based Edge Detection (Laplacian based Edge Detection)

To find the edges, Laplacian method search for zerocrossings in the second derivative of the image. An edge has the 1-D shape of a ramp and its location can be highlighted by calculating the derivative of the image. This method of locating an edge is characteristic of the "gradient filter" family of edge detection filters and includes the Sobel method.

c) Canny Operator

It is a method to find edges by isolating noise from the image without affecting the features of the edges in the image and then applying the tendency to find the edges and the critical value for threshold. The canny edge detector first smoothens the image to eliminate noise. Then it finds the image gradient to highlight regions with high spatial derivatives. After that it perform tracking along these regions and suppresses any pixel that is not at the maximum. The gradient array at this moment can further be reduced by hysteresis which is used to track along the remaining pixels that have not been suppressed. Hysteresis uses two thresholds and if the magnitude is below the first threshold, it is set to zero . If the magnitude is above the high threshold, it is made an edge. Major application of canny edge detector is for remote sensing images which are inherently noisy, fig(9).

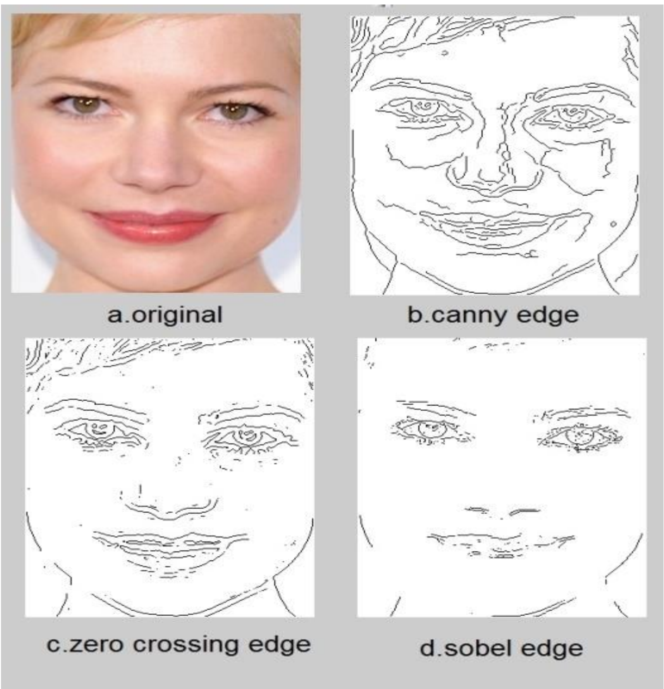

Fig (9) different type of edge detection methods applied to an image b. canny algorithm c. zerocrossing algorithm and d. sobol edge detector.

3.3 Criteria for optimal edge detection

- The optimal detector must minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges).
- The edges detected must be as close as possible to the true edges.
- The detector must return one point only for each true edge point; that is, minimize the number of local maxima around the true edge (created by noise).
- There are also missing edges so edge linking may try to recover missing edges and edge following will form contours from existing and recovered edges.

IV. PROPOSED MODEL

We introduce a simple algorithm to detect edges in a digital picture depending on operations had given by digital topology as an example to its uses. The algorithm works on showing the contrast between the grayscale values of the picture and determines the regions that represent a switching among picture components to consider it as an edges then applies a step of thinning to that regions to give as thin as possible edge lines. several steps are followed in this algorithm and they are showed in fig(10) to manipulate the digital images were taken by a digital camera of different resolution .

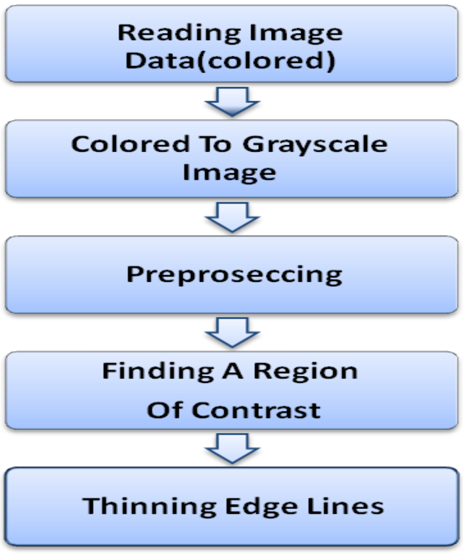

Fig (10) The main steps of proposed edge detector

4.1 Reading image data

The colored image is represented by 3-dimensional matrix ,each point of it (voxel) take a three value to reflect the intensity of the three main color (Red, Green and Blue) and each value is in range [0,255],fig(11).

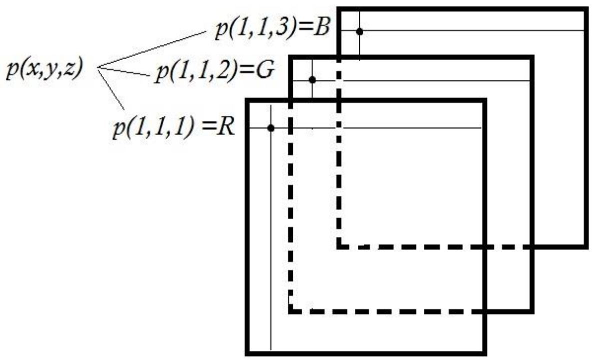

Fig (11)RGB image consists of three 2D arrays each one represents a color

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4.2 Colored To Grayscale Image

In this step the 3-dimensional truecolor image (rgb) is converted to a 2-dimensional grayscale intensity image(*I*). It converts RGB images to grayscale by eliminating the hue and saturation information while retaining the luminance. The RGB values are converted to a grayscale values by forming a weighted sum of the R, G, and B components:

$$I = 0.2989 * R + 0.5870 * G + 0.1140 * B \quad (5)$$

Mathematically:

$$I(x,y) = 0.2989 * rgb(x,y,1) + 0.5870 * rgb(x,y,2) + 0.1140 * rgb(x,y,3) \quad x=1,2\ldots,n \ , \ y=1,2,\ldots,m . \quad (6)$$

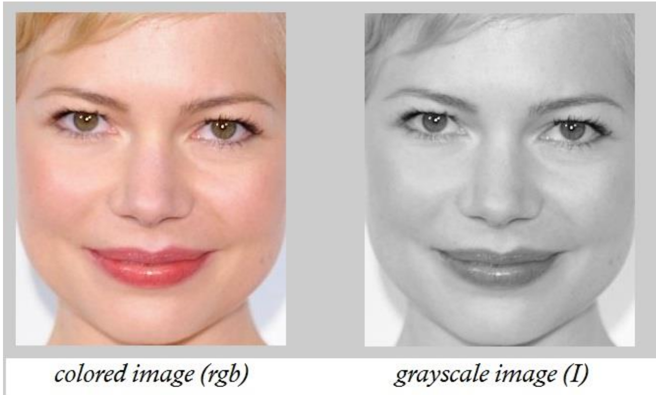

Fig (12) A 3D truecolor image was converted to 2D grayscale image

4.3 Preprocessing

Noise contained in image is smoothed by convolving the input image *I* with Gaussian filter G. Mathematically, the smooth resultant image is given by:

$$I_{enhanced}(x,y) = G * I(x,y) \quad (7)$$

A rotationally symmetric Gaussian lowpass filter of size h (squared or different number of rows and colomns)with standard deviation sigma (positive) ,the default value for sigma is 0.5 while default value of h is 3×3 ,fig(13).

Fig (13) shows the difference in noised and noise reduced image with Gaussian filter

4.4 Finding A Region of Contrast

several step take place to achieve this mission as is detailed in below:

- Calculating the difference in intensity between each adjacent point of $I_{enhanced}$ image:

$$I(x,y) = I(x+1,y) - I(x:y) .. \ x=1,2,\ldots,n \ , \ y=1,2,\ldots,m \quad (8)$$

This step showed the place of intensity variance which usually represents edge (black or most darker regions), Fig (14).

Fig (14) The right image is the result of subtract each adjacent point in a picture.

- Strengthen the darkest regions or object point by squared the values of all points that make the largest values reach to 255 which represent the highest limit [0 255] that grayscale point belongs to; while the smallest value of subtraction is stay rationally low, fig(15).

$$I_{squred}(x,y) = I(x,y) * I(x,y) \quad x=1,2,\ldots,n \ , \ y=1,2,\ldots,m \quad (9)$$

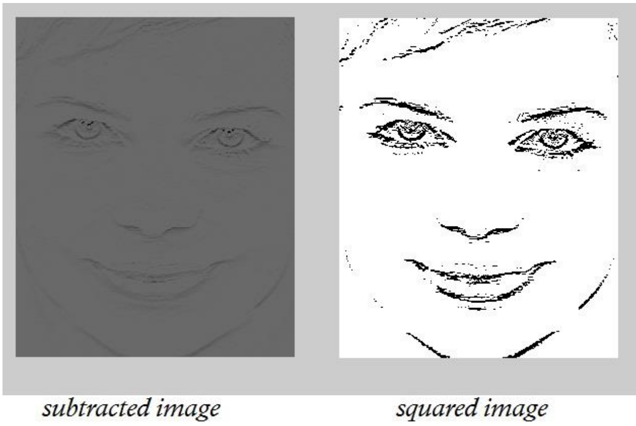

Fig (15) maximize the Subtracted Values

Notice: the squared image is also a grayscale image.

- constructing a digital Black and White image that represents the line of edges with black color and background with white color by applying the following function :

$$f(I(x,y)) = \begin{cases} 1 & I(x,y) > 250 \\ 0 & otherwise \end{cases} \quad (10)$$

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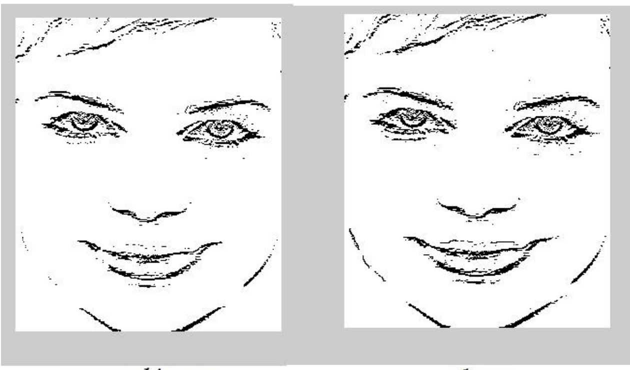


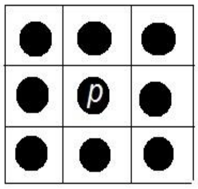
squared image *edges*

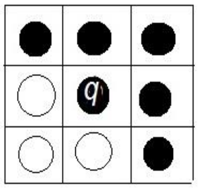
Fig (16) The squared image is a grayscale image transformed to edge binary image

4.5 Thinning Edge Lines

The edges resultant by previous step represent a thick line , the edge should be as thin as possible at most should be one point (pixel) thickness[1] ; thus ,in this step of algorithm we use some operations to delete some of the point that called *interior* points as in above and keep only the *border* points of the edge object .Every interior point that N_8 is deleted while keeping the border point that is connected with one or more background points.

The point is interior or 8-neighborhood if and only if all the points around it is black.





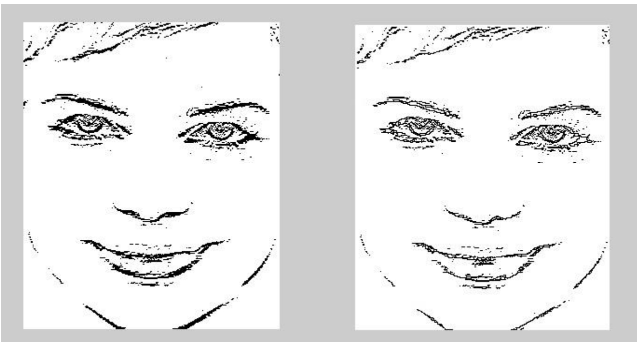
$\forall p(N_8) \in I(logical\ set)\ \ p(x,y) is\ an\ interior\ point$

$$\Leftrightarrow \sum_{x-1}^{x+1}\sum_{y-1}^{y+1} I = 9 \qquad (11)$$

$\forall q(N_8) \in I(logical\ set), q(x,y) is\ a\ border\ point$

$$\Leftrightarrow \sum_{x-1}^{x+1}\sum_{y-1}^{y+1} I < 9 \qquad (12)$$

The resulted picture as in fig (17).



edge *thinning edge*

Fig (17) the thinning edge image is tha last result of the edge detection algorithm suggested in this research .

V. CONCLUSION

Digital topology is fairly understood ,and its basic concepts were based and developed since 1960s .2 and 3 dimensional spaces are identified and invested in image representation and its operations used to manipulate pictures either in transformations , segmentation or recognition .

Various edge detection algorithms and detector have been showed based on mathematical approaches. In this paper the presented algorithm are proved to be partially effective in edge detection and reduction of noise-induced edges with keeping simplicity in implementation .The main aim of designing this detector is to show the uses of digital topology concepts in practical and it does rather succeed.

There is too many ideas should be taken in account for the future edge detection techniques such as reducing the noise without blurring the edges as well as precise edge detections with a minimum error detection possibility that can detect edges within a single pixel.

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