International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-4 Issue-5, June 2015

Fuzzy Inventory Model for Weibull Deteriorating Items, with Time Depended Demand, Shortages, and Partially Backlogging

Wasim Akram Mandal, Sahidul Islam

***Abstract:** In this paper fuzzy inventory model for deteriorating item with time depended demand rate, shortages under partially backlogged is formulated and solved. The backlogging rate is taken to be inversely proportional to the waiting time for the next replenishment. Fuzziness is applying by allowing the cost components (holding cost, shortage cost, etc). In fuzzy environment it considered all required parameter to be triangular fuzzy numbers. The purpose of the model is to minimize total cost function.*

***Keywords:** Inventory, Deteriorating, Fuzzy number, Shortages, Partially backlogged, Triangular fuzzy number.*

I. INTRODUCTION

An inventory deal with decision that minimum the total average cost or maximize The total average profit. For this purpose the task is to construct a mathematical model of the real life Inventory system, such a mathematical model is based on various assumption and approximation. In a inventory model deterioration play an important role. Deterioration is defined as decay or damage in the quality of the inventory. Foods, Drugs, pharmaceuticals etc are deteriorating items. During inventory there have some losses of this deteriorating items, consequently this loss must be taken into account when analyzing the system. Shortages is also very important condition. There are several type of customer . At shortage period some customers are waiting for actual product and others do not it. For this it consider partially- backlogging. In ordinary inventory model it consider all parameter like shortage cost, holding cost, unit cost as fixed. But in real life situation it will have some little fluctuations. So consideration of fuzzy variables is more realistic. The study of inventory model where demand rates varies with time is the last decades. Datta and pal investigated an inventory system with power demand pattern and deterioration. Park and Wang studied shortages and partial backlogging of items. Friedman(1978) presented continuous time inventory model with time varying demand. M.Roychowdhury and K.S Chaudhuri(1983) studied an order level inventory for deteriorating items with finite rate of replenishment. Ritchie(1984) studied in inventory model with linear increasing demand.

Manuscript published on 30 June 2015.
* Correspondence Author (s)
Wasim Akram Mandal, Beldanga D.H.Sr. Madrasah, Beldanga-742189, Murshidabad, WB, India.
Sahidul Islam, Department of Mathematics, University of Kalyani, Kalyani, W.B, India.

Goswami, Chaudhuri(1991) discussed an inventory model with shortage.Gen et. Al. (1997) considered classical inventory model with Triangular fuzzy number. Yao and Lee(1998) considered an economic production quantity model in the fuzzy sense. Sujit Kumar De, P.K.Kundu and A.Goswami(2003) presented an economic production quantity inventory model involving fuzzy demand rate. J.K.Syde and L.A.Aziz(2007) applied sign distance method to fuzzy inventory model without shortage . D.Datta and Pravin Kumar published several paper of fuzzy inventory with or without shortage. In this paper we first consider crisp inventory model with time depended demand where shortage are allowed and partially backlogged. Thereafter we developed fuzzy inventory model with fuzzy time depended demand rate under partially backlogged. All inventory cost parameters are fuzzyfied as triangular fuzzy number.

II. PRELIMINARIES

For graded representation method to defuzzyfy, we need the following definitions,

Definition2.1: A fuzzy set $\tilde{A}$ on the given universal set X is a set of order pairs,

$\tilde{A}=\{(x,\mu_A(x)): x\epsilon X\}$ where $\mu_A(x)\to[0,1]$ is called a membership function.

Definition2.2:The α-cut of $\tilde{A}$, is defined by $A_\alpha=\{x: \mu_A(x)=\alpha, \alpha\geq 0\}$

Definition2.3:$\tilde{A}$ is normal if there exists $x\epsilon X$ such that $\mu_A(x)=1$

Definition2.4:A triangular fuzzy number $\tilde{A}$=(a,b,c) is represented with membership function $\tilde{A}$.

$\tilde{A}$ is defined as,

$$\mu_A(X)=\begin{cases} L(x)=\frac{x-a}{b-a}, & a\leq x\leq b \\ R(x)=\frac{c-x}{c-b}, & b\leq x\leq c \\ 0 & ,otherwise \end{cases}$$

when a=b=c , we have fuzzy point (a,a,a)=$\tilde{a}$. The family of all triangular fuzzy number on R,

denoted as $F_N=\{(a,b,c), a<b<c, \forall a,b,c\in R\}$. the α-cut of $\tilde{A}$=(a,b,c)$\in F_N$, $0\leq\alpha\leq 1$ is

$A(\alpha)=[A_L(\alpha),A_R(\alpha)]$ where $A_L(\alpha)=a+(b-a)\alpha$ and $A_R(\alpha)=c-(c-b)\alpha$ are the left and right end Point of A(α).

Definition2.5: If $\tilde{A}$=(a,b,c) is a triangular fuzzy number then the graded mean integration of $\tilde{A}$ is defined as,

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

$$P(\widetilde{A})=\frac{\int_0^{W_A}\left(\frac{L^{-1}(h)+R^{-1}(h)}{2}\right)dh}{\int_0^{W_A} h\,dh},\ (0\leq h\leq w_A \text{ and } 0\leq w_A\leq 1)$$

$$P(\widetilde{A})=\frac{\int_0^1\left[\frac{a+(b-a)h+c-(c-b)h]dh}{2}\right]}{\int_0^1 h\,dh}$$

$$=\frac{a+4b+c}{6}$$

Suppose $\tilde{a}=(a_1,a_2,a_3)$ and $\tilde{b}=(b_1,b_2,b_3)$ are two fuzzy triangular number then

(1) $\tilde{a}+\tilde{b}=(a_1,a_2,a_3)+(b_1,b_2,b_3)=(a_1+b_1,\ a_2+b_2,\ a_3+b_3)$.
(2) $\tilde{a}-\tilde{b}=(a_1,a_2,a_3)-(b_1,b_2,b_3)=(a_1-b_1,\ a_2-b_2,\ a_3-b_3)$.
(3) $\tilde{a}\times\tilde{b}=(a_1,a_2,a_3)\times(b_1,b_2,b_3)=(a_1b_1,a_2b_2,a_3b_3)$.
(4) $\frac{1}{\tilde{b}}=(\frac{1}{b_3},\frac{1}{b_2},\frac{1}{b_1})$ where b_1,b_1,b_1 are all non zero positive real number, then
$\frac{\tilde{a}}{\tilde{b}}=(\frac{a_1}{b_3},\frac{a_2}{b_2},\frac{a_3}{b_1})$.
(5) Let k∈R, then $k\tilde{a}=k(a_1,a_2,a_3)=(ka_1,ka_2,ka_3)$ for $k\geq 0$.

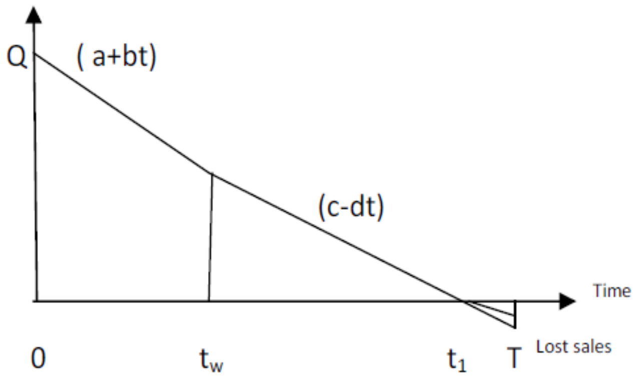
III. NOTATION

I(t)=Inventory level at any time, t≥0.
T:Cycle of length.
t_w: Time point, when demand rate start with (c-dt).
t_1: Time point when stock level reaches to zero.
c_1:Fixed cost.
c_2:Shortages cost per unit.
c_3:Deteriorating cost per unit.
c_4:Holding cost per unit.
c_5:Opportunity cost due to lost sales.
Q:Highest stock level at the beginning of the cycle.
TAC(t_w,t_1):Toal average cost per unit.
$\tilde{c}_1$=Fuzzy fixed cost.
$\tilde{c}_2$=Fuzzy shortage cost per unit.
$\tilde{c}_3$=Fuzzy deteriorating cost per unit.
$\tilde{c}_4$=Fuzzy holding cost per unit.
$\tilde{c}_5$=Fuzzy opportunity cost due to lost sale.
$\widetilde{TAC}(t_w,t_1)$=Fuzzy total cost per unit.

3.1. ASSUMTION:

a: The inventory system involves only one item.
b: The replenishment occur instantaneously at infinite rate.
c: The lead time is negligible.
d: Demand rate is time depended, we assume it (a+bt) in $0\leq t\leq t_w$ and (c-dt) in $t_w\leq t\leq t_1$.
e: $\alpha\beta t^{\beta-1}$, the two-parameter Weibull distribution deterioration rate. Where 0<α<<1 is called the scale parameter, β>0 is the shape parameter.
f: Backlogging rate is $\frac{1}{1+\delta(T-t)}$, $t_1\leq t\leq T$.
g: At time $t=t_w$ it should be $(a+bt_w)=(c-dt_w)$.

3.2. MODEL DEVOLOPMENT (CRISP MODEL)



During $0\leq t\leq t_w$ the inventory level decrease due to customer demand(rate of demand=a+bt) and deteriorating items. During $t_w\leq t\leq t_1$ the inventory level decrease due to customer demand (rate of demand=c-dt), deteriorating items and reaches to zero at $t=t_1$. In the time interval $t_1\leq t\leq T$ shortages with partially backlogged allowed.

The differential equation describing I(t) as follows

$$\frac{dI(t)}{dt}+\alpha\beta t^{\beta-1}I(t)=-(a+bt),\quad 0\leq t\leq t_w \tag{3.1}$$

With boundary condition I(0)=Q.

$$\frac{dI(t)}{dt}+\alpha\beta t^{\beta-1}I(t)=-(c-dt),\quad {}_w\leq t\leq t_1 \tag{3.2}$$

With boundary condition $I(t_1)=0$

$$\frac{dI(t)}{dt}=-\frac{(c-dt)}{\{1+\delta(T-T)\}},\quad t_1\leq t\leq T \tag{3.3}$$

With boundary condition $I(t_1)=0$.

α, δ is too small, so neglecting higher power of α, δ.

From (3.1) we get,

$$I(t)=Q(1-\alpha t^{\beta})-a(t+\frac{\alpha}{\beta+1}t^{\beta+1})+a\alpha t^{\beta+1}-b(\frac{t^2}{2}+\frac{\alpha}{\beta+2}t^{\beta+2})+\frac{\alpha b t^{\beta+2}}{2}. \tag{3.4}$$

From (3.2) we get,

$$I(t)=c(t_1-t)(1-\alpha t^{\beta})+\frac{c\alpha}{\beta+1}({t_1}^{\beta+1}-t^{\beta+1})-\frac{d(1-\alpha t^{\beta})({t_1}^2-t^2)}{2}-\frac{c\alpha}{\beta+2}({t_1}^{\beta+2}-t^{\beta+2}) \tag{3.5}$$

From (3.3) we get,

$$I(t)=c(1-\delta t)(t_1-t)-\frac{d(1-\delta t)({t_1}^2-t^2)}{2}+\frac{\delta c({t_1}^2-t^2)}{2}-\frac{\delta d({t_1}^3-t^3)}{3} \tag{3.6}$$

So the fixed cost per cycle is,

FC=c_1

Shortage cost per cycle is,

$$Sc=-c_2\int_{t_1}^{T} I(t)dt$$

$$=-c_2[c(1-\delta t)t_1(T-t_1)-\frac{c(1-\delta t)(T^2-{t_1}^2)}{2}-\frac{d(1-\delta t){t_1}^2(T-t_1)}{2}+\frac{d(1-\delta t)(T^3-{t_1}^3)}{6}+\frac{\delta c{t_1}^2(T-t_1)}{2}-\frac{\delta c(T^3-{t_1}^3)}{6}-\frac{\delta d{t_1}^3(T-t_1)}{3}+\frac{\delta d(T^4-{t_1}^4)}{12}.$$

Deteriorating cost per cycle is,

$$Dc=c_3\int_0^{t_w}\alpha\beta t^{\beta-1}I(t)dt+c_3\int_{t_w}^{t_1}\alpha\beta t^{\beta-1}I(t)dt$$

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

$$=c_3[Q\alpha t_w{}^{\beta}-a\alpha\beta\frac{t_w{}^{\beta+1}}{\beta+1}-b\alpha\beta\frac{t_w{}^{\beta+2}}{\beta+2}+c\alpha t_1(t_1{}^{\beta}-t_w{}^{\beta})-\frac{c\alpha\beta(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{\beta+1}-\frac{d\alpha t_1{}^2(t_1{}^{\beta}-t_w{}^{\beta})}{2}+\frac{d\alpha\beta(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{2(\beta+2)}$$

Holding cost per cycle is,

$$HC=c_4\int_0^{t_w} I(t)\,dt + c_4\int_{t_w}^{t_1} I(t)\,dt$$

$$=c_4[Qt_w-\frac{\alpha Q t_w{}^{\beta+1}}{\beta+1}-a\{\frac{t_w{}^2}{2}+\frac{\alpha t_w{}^{\beta+2}}{(\beta+1)(\beta+2)}\}+\frac{a\alpha t_w{}^{\beta+2}}{\beta+2}-b(\frac{t_w{}^3}{6}+\frac{\alpha t_w{}^{\beta+3}}{(\beta+2)(\beta+3)}+\frac{b\alpha t_w{}^{\beta+3}}{\beta+3}+ct_1(t_1-t_w)-\frac{c\alpha t_1(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{\beta+1}-\frac{c(t_1{}^2-t_w{}^2)}{2}+\frac{c\alpha(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{\beta+2}+\frac{c\alpha t_1{}^{\beta+1}(t_1-t_w)}{\beta+1}-\frac{c\alpha(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{(\beta+1)(\beta+2)}-\frac{dt_1{}^2(t_1-t_w)}{2}+\frac{\alpha d t_1{}^2(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{2(\beta+1)}+\frac{d(t_1{}^3-t_w{}^3)}{6}-\frac{\alpha d(t_1{}^{\beta+3}-t_w{}^{\beta+3})}{2(\beta+3)}-\frac{\alpha d t_1{}^{\beta+2}(t_1-t_w)}{\beta+2}+\frac{\alpha d(t_1{}^{\beta+3}-t_w{}^{\beta+3})}{(\beta+2)(\beta+3)}$$

Opportunity cost due to lost sales is,

$$OC=c_5\int_{t1}^{T} R(t)[1-\frac{1}{1+\delta(T-t)}]dt$$

$$=c_5\delta[cT(T-t_1)-\frac{c(T^2-t_1{}^2)}{2}-\frac{dT(T^2-t_1{}^2)}{2}+\frac{D(T^3-t_1{}^3)}{3}]$$

So total average cost per cycle is,

$$TAC(t_1,t_w)=\frac{1}{T}[FC+SC+DC+HC+OC]$$

$$=\frac{1}{T}[c_1-c_2[c(1-\delta t)t_1(T-t_1)-\frac{c(1-\delta t)(T^2-t_1{}^2)}{2}-\frac{d(1-\delta t)t_1{}^2(T-t_1)}{2}+\frac{d(1-\delta t)(T^3-t_1{}^3)}{6}+\frac{\delta c t_1{}^2(T-t_1)}{2}-\frac{\delta c(T^3-t_1{}^3)}{6}-\frac{\delta d t_1{}^3(T-t_1)}{3}+\frac{\delta d(T^4-t_1{}^4)}{12}+c_3[Q\alpha t_w{}^{\beta}-a\alpha\beta\frac{t_w{}^{\beta+1}}{\beta+1}-b\alpha\beta\frac{t_w{}^{\beta+2}}{\beta+2}+c\alpha t_1(t_1{}^{\beta}-t_w{}^{\beta})-\frac{c\alpha\beta(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{\beta+1}-\frac{d\alpha t_1{}^2(t_1{}^{\beta}-t_w{}^{\beta})}{2}+\frac{d\alpha\beta(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{2(\beta+2)}+c_4[Qt_w-\frac{\alpha Q t_w{}^{\beta+1}}{\beta+1}-a\{\frac{t_w{}^2}{2}+\frac{\alpha t_w{}^{\beta+2}}{(\beta+1)(\beta+2)}\}+\frac{a\alpha t_w{}^{\beta+2}}{\beta+2}-b\{\frac{t_w{}^3}{6}+\frac{\alpha t_w{}^{\beta+3}}{(\beta+2)(\beta+3)}\}+\frac{b\alpha t_w{}^{\beta+3}}{\beta+3}+ct_1(t_1-t_w)-\frac{c\alpha t_1(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{\beta+1}-\frac{c(t_1{}^2-t_w{}^2)}{2}+\frac{c\alpha(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{\beta+2}+\frac{c\alpha t_1{}^{\beta+1}(t_1-t_w)}{\beta+1}-\frac{c\alpha(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{(\beta+1)(\beta+2)}-\frac{dt_1{}^2(t_1-t_w)}{2}+\frac{\alpha d t_1{}^2(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{2(\beta+1)}+\frac{d(t_1{}^3-t_w{}^3)}{6}-\frac{\alpha d(t_1{}^{\beta+3}-t_w{}^{\beta+3})}{2(\beta+3)}-\frac{\alpha d t_1{}^{\beta+2}(t_1-t_w)}{\beta+2}+\frac{\alpha d(t_1{}^{\beta+3}-t_w{}^{\beta+3})}{(\beta+2)(\beta+3)}+c_5\delta[cT(T-t_1)-\frac{c(T^2-t^2)}{2}-\frac{dT(T^2-t_1{}^2)}{2}+\frac{D(T^3-t_1{}^3)}{3}]]$$

For minimum cost it should be,

$$\frac{\partial TAC(t_w,t_1)}{\partial t_w}=0,\quad \frac{\partial TAC(t_w,t_1)}{\partial t_1}=0$$

Provided it satisfies,

$$\frac{\partial^2 TAC(t_w,t_1)}{\partial t_w{}^2}>0,\quad \frac{\partial^2 TAC(t_w,t_1)}{\partial t_1{}^2}>0$$

And $\left[\frac{\partial^2 TAC(t_w,t_1)}{\partial t_w{}^2}\right]\left[\frac{\partial^2 TAC(t_w,t_1)}{\partial t_w{}^2}\right]-\left[\frac{\partial^2 TAC(t_w,t_1)}{\partial t_w\,\partial t_1}\right]>0.$

3.3 FUZZY MODEL:

Due to uncertainly lets us assume that,

$\tilde{c}_1=(c_1{}^1,c_1{}^2,c_1{}^3)$, $\tilde{c}_2=(c_2{}^1,c_2{}^2,c_2{}^3)$, $\tilde{c}_3=(c_3{}^1,c_3{}^2,c_3{}^3)$, $\tilde{c}_4=(c_4{}^1,c_4{}^2,c_4{}^3)$, $\tilde{c}_5=(c_5{}^1,c_5{}^2,c_5{}^3)$, be triangular fuzzy number then the total average cost is given by,

$$\widetilde{TAC(t_w},t_1)=\frac{1}{T}[\tilde{c}_1-\tilde{c}_2[c(1-\delta t)t_1(T-t_1)-\frac{c(1-\delta t)(T^2-t_1{}^2)}{2}-\frac{d(1-\delta t)t_1{}^2(T-t_1)}{2}+\frac{d(1-\delta t)(T^3-t_1{}^3)}{6}+\frac{\delta c t_1{}^2(T-t_1)}{2}-\frac{\delta c(T^3-t_1{}^3)}{6}-\frac{\delta d t_1{}^3(T-t_1)}{3}+\frac{\delta d(T^4-t_1{}^4)}{12}+\tilde{c}_3[Q\alpha t_w{}^{\beta}-a\alpha\beta\frac{t_w{}^{\beta+1}}{\beta+1}-b\alpha\beta\frac{t_w{}^{\beta+2}}{\beta+2}+c\alpha t_1(t_1{}^{\beta}-t_w{}^{\beta})-\frac{c\alpha\beta(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{\beta+1}-\frac{d\alpha t_1{}^2(t_1{}^{\beta}-t_w{}^{\beta})}{2}+\frac{d\alpha\beta(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{2(\beta+2)}+\tilde{c}_4[Qt_w-\frac{\alpha Q t_w{}^{\beta+1}}{\beta+1}-a\{\frac{t_w{}^2}{2}+\frac{\alpha t_w{}^{\beta+2}}{(\beta+1)(\beta+2)}\}+\frac{a\alpha t_w{}^{\beta+2}}{\beta+2}-b\{\frac{t_w{}^3}{6}+\frac{\alpha t_w{}^{\beta+3}}{(\beta+2)(\beta+3)}\}+\frac{b\alpha t_w{}^{\beta+3}}{\beta+3}+ct_1(t_1-t_w)-\frac{c\alpha t_1(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{\beta+1}-\frac{c(t_1{}^2-t_w{}^2)}{2}+\frac{c\alpha(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{\beta+2}+\frac{c\alpha t_1{}^{\beta+1}(t_1-t_w)}{\beta+1}-\frac{c\alpha(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{(\beta+1)(\beta+2)}-\frac{dt_1{}^2(t_1-t_w)}{2}+\frac{\alpha d t_1{}^2(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{2(\beta+1)}+\frac{d(t_1{}^3-t_w{}^3)}{6}-\frac{\alpha d(t_1{}^{\beta+3}-t_w{}^{\beta+3})}{2(\beta+3)}-\frac{\alpha d t_1{}^{\beta+2}(t_1-t_w)}{\beta+2}+\frac{\alpha d(t_1{}^{\beta+3}-t_w{}^{\beta+3})}{(\beta+2)(\beta+3)}+\tilde{c}_5\delta[cT(T-t_1)-\frac{c(T^2-T_1{}^2)}{2}-\frac{dT(T^2-t_1{}^2)}{2}+\frac{D(T^3-t_1{}^3)}{3}]]$$

We defuzzifi the fuzzy total cost $\widetilde{TAC}(t_1)$ by graded mean representation method as follows,

$$\widetilde{TAC}(t_1)=\frac{1}{6}[\widetilde{TAC}^1(t_w,t_1),\widetilde{TAC}^2(t_w,t_1),\widetilde{TAC}^3(t_w,t_1)]$$

Where

$$\widetilde{TAC}^r(t_w,t_1)=\frac{1}{T}[\widetilde{c^r}_1-\widetilde{c^r}_2[c(1-\delta t)t_1(T-t_1)-\frac{c(1-\delta t)(T^2-t_1{}^2)}{2}-\frac{d(1-\delta t)t_1{}^2(T-t_1)}{2}+\frac{d(1-\delta t)(T^3-t_1{}^3)}{6}+\frac{\delta c t_1{}^2(T-t_1)}{2}-\frac{\delta c(T^3-t_1{}^3)}{6}-\frac{\delta d t_1{}^3(T-t_1)}{3}+\frac{\delta d(T^4-t_1{}^4)}{12}+\widetilde{c^r}_3[Q\alpha t_w{}^{\beta}-a\alpha\beta\frac{t_w{}^{\beta+1}}{\beta+1}-b\alpha\beta\frac{t_w{}^{\beta+2}}{\beta+2}+c\alpha t_1(t_1{}^{\beta}-t_w{}^{\beta})-\frac{c\alpha\beta(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{\beta+1}-\frac{d\alpha t_1{}^2(t_1{}^{\beta}-t_w{}^{\beta})}{2}+\frac{d\alpha\beta(t_1{}^{\beta+2}-t_w{}^{\beta+2})}{2(\beta+2)}+\widetilde{c^r}_4[Qt_w-\frac{\alpha Q t_w{}^{\beta+1}}{\beta+1}-a\{\frac{t_w{}^2}{2}+\frac{\alpha t_w{}^{\beta+2}}{(\beta+1)(\beta+2)}\}+\frac{a\alpha t_w{}^{\beta+2}}{\beta+2}-b\{\frac{t_w{}^3}{6}+\frac{\alpha t_w{}^{\beta+3}}{(\beta+2)(\beta+3)}\}+\frac{b\alpha t_w{}^{\beta+3}}{\beta+3}+ct_1(t_1-t_w)-\frac{c\alpha t_1(t_1{}^{\beta+1}-t_w{}^{\beta+1})}{\beta+1}-$$

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

$$\frac{c(t_1^{2}-t_w^{2})}{2}+\frac{c\alpha(t_1^{\beta+2}-t_w^{\beta+2})}{\beta+2}+\frac{c\alpha t_1^{\beta+1}(t_1-t_w)}{\beta+1}-\frac{c\alpha(t_1^{\beta+2}-t_w^{\beta+2})}{(\beta+1)(\beta+2)}-\frac{dt_1^{2}(t_1-t_w)}{2}+\frac{\alpha d t_1^{2}(t_1^{\beta+1}-t_w^{\beta+1})}{2(\beta+1)}+\frac{d(t_1^{3}-t_w^{3})}{6}-\frac{\alpha d(t_1^{\beta+3}-t_w^{\beta+3})}{2(\beta+3)}-$$
$$\frac{\alpha d t_1^{\beta+2}(t_1-t_w)}{\beta+2}+\frac{\alpha d(t_1^{\beta+3}-t_w^{\beta+3})}{(\beta+2)(\beta+3)}+\widetilde{c^{r}}_5\delta[cT(T-t_1)-\frac{c(T^2-T_1^{2})}{2}-\frac{dT(T^2-t_1^{2})}{2}+\frac{D(T^3-t_1^{3})}{3}]]$$

Where r=1,2,3

$$\widetilde{TAC(t_1)}=\frac{1}{6}[\widetilde{TAC_1}\,(t_w,t_1)+4\widetilde{TAC_2}\,(t_w,t_1)+\widetilde{TAC_3}\,(t_w,t_1)]$$

For minimum cost it should be,

$$\frac{\partial \widetilde{TAC(t_w,t_1)}}{\partial t_w}=0,\quad \frac{\partial \widetilde{TAC(t_w,t_1)}}{\partial t_1}=0$$

Provided it satisfies,

$$\frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_w^{2}}>0,\quad \frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_1^{2}}>0$$

And

$$\left[\frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_w^{2}}\right]\left[\frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_1^{2}}\right]-\left[\frac{\partial^2 \widetilde{TAC(t_w,t_1)}}{\partial t_w.\partial t_1}\right]>0.$$

IV. NUMERICAL SOLUTION

For crisp model: Let us take the in-put value:

C_1	C_2	C_3	C_4	C_5	α	β	δ	Q	a	b	c	d	T
100	20	10	4	8	0.2	0.5	0.1	100	20	10	25	1	1

And the out-put value:

t_w	t_1	$TAC(t_w,t_1)$
0.454	0.647	1542.370

For fuzzy model:

$\tilde{c_1}$=(90,100,110), $\tilde{c_2}$ =(15,20,25), $\tilde{c_3}$=(8,10,12), $\tilde{c_4}$ =(3,4,5), $\tilde{c_5}$=(6,8,10)

The solution of fuzzy model by graded mean representation is,

(1) When $\tilde{c_1},\tilde{c_2},\tilde{c_3},\tilde{c_4},\tilde{c_5}$ are all triangular fuzzy numbers then,
$TAC(t_1)$=1565.363, t_w=0.454, t_1=0.648

(2) When $\tilde{c_1},\tilde{c_2},\tilde{c_3},\tilde{c_4}$, are all triangular fuzzy numbers then
$TAC(t_1)$=1565.225, t_w=0.454, t_1=0.648

(3) When $\tilde{c_1},\tilde{c_2},\tilde{c_3}$,are triangular fuzzy numbers then,
$TAC(t_1)$=1542.362, t_w=0.454, t_1=0.645

(4) When $\tilde{c_1},\tilde{c_2}$,are triangular fuzzy numbers then,
$TAC(t_1)$=1542.1769, t_w=0.454, t_1=0.646

(5) When $\tilde{c_1}$,are triangular fuzzy numbers then,
$TAC(t_1)$=1542.370, t_w=0.454, t_1=0.647

V. SENSITIVITY ANALYSIS

We now examine to sensitivity analysis of the optimal solution of the model for change in I, keeping the other parameters unchanged. The initial data from the above numerical example.

Parameter	% of change	$TAC(t_w,t_1)$	t_w	t_1
C_1=50.0	-50	1492.370	0.454	0.647
C_1=75.0	-25	1517.370	0.454	0.647
C_1=100	0	1542.370	0.454	0.647
C_1=125	25	1567.370	0.454	0.647
C_1=150	50	1592.370	0.454	0.647
C_2=10	-50	1524.929	0.454	0.582
C_2=15	-25	1534.375	0.454	0.617
C_2=20	0	1542.370	0.454	0.647
C_2=25	25	1549.222	0.454	0.672
C_2=30	50	1555.158	0.454	0.694
C_3=5.0	-50	832.463	0.454	0.730
C_3=7.5	-25	1188.470	0.454	0.678
C_3=10	0	1542.370	0.454	0.647
C_3=12.5	25	1895.098	0.454	0.623
C_3=15	50	2247.108	0.454	0.606
C_4=2	-50	847.333	0.454	0.739
C_4=3	-25	1196.166	0.454	0.684
C_w=4	0	1542.370	0.454	0.647
C_4=5	25	1887.049	0.454	0.620
C_4=6	50	2230.766	0.454	0.599
C_5=4	-50	1539.653	0.454	0.638
C_5=6	-25	1541.025	0.454	0.643
C_5=8	0	1542.370	0.454	0.647
C_5=10	25	1543.689	0.454	0.651
C_5=12	50	1544.982	0.454	0.655

5.1(a)Effect, for increment parameters-

(1) $TAC(t_w,t_1)$ increase, for increase of c_1.
(2) $TAC(t_w,t_1)$ increase slowly, for increase of c_2.
(3) $TAC(t_w,t_1)$ increase rapidly, for increase of c_3.
(4) $TAC(t_w,t_1)$ increase rapidly, for increase of c_w.
(5) $TAC(t_w,t_1)$ increase slowly, for increase of c_0.

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

VI. CONCLUSON

In this paper, we have proposed a real life inventory problem in a fuzzy environment and presented solution along with sensitivity analysis approach. The inventory model developed with time depended demand, with shortages. Shortages have been allow partially backlogged in this model. Here demand rate considered as (a+bt) in $0 \leq t \leq t_w$, and it (c-dt) in $t_w \leq t \leq t_1$. This model has been developed for single item. In this paper, we have considered triangular fuzzy number and solved by graded mean integration method. In future, the other type of membership functions such as piecewise linear hyperbolic, L-R fuzzy number, trapezoidal fuzzy number, pentagonal fuzzy number etc can be considered to construct the membership function and then model can be easily solved.

REFERENCE

[1] BELLMAN, R.E, AND ZADEH (1970), Decision making in a fuzzy environment, Management Science 17, B141-B164
[2] CARLSSON, C. AND P. KORHONEN (1986), A parametric approach to fuzzy linear programming, Fuzzy sets and systems, 17-30.
[3] CLARK, A.J, (1992), An informal survey of multy-echelon inventory theory , naval research logistics Quarterly 19, 621-650.
[4] DUTTA, D.J.R. RAO, AND R.N TIWARY (1993), Effect of tolerance in fuzzy linear fractional programming, fuzzy sets and systems 55, 133-142.
[5] HAMACHER, H.LEBERLING AND H.J.ZIMMERMANN (1978), Sensitivity Analysis in fuzzy linear Programming Fuzzy sets and systems 1, 269-281.
[6] HADLEY, G. AND T.M. WHITE (1963),Analysis of inventory system, Prentice-Hall, ENGLEWOOD Cliffs, NJ.
[7] KHUN, H.W AND A.W. TUCKER (1951), Non-linear programming, proceeding second Berkeley symposium Mathematical Statistic and probability (ed) Nyman , J.University of California press 481-492.
[8] M.ROYCHOWDHURY, K.S CHAUDHURY (1983), An order level inventory model for deteriorating items items with Finite rate of replenishment, Opsearch, 20(1983), 99-106.
[9] RAYMOND, F.E (1931), Quantity and Economic in manufacturing, McGraw-Hill, New York.
[10] ZADEH, L.A (1965), Fuzzy sets, Information and Control, 8, 338-353.
[11] T.K DATTA , A.K PAL (1988), Order level inventory system with power demand pattern for items with variable rate of Deterioration , Indian journal of Pure and Applied Maths. 19(11), (1988), 1043-1053.
[12] ZIMMERMANN, H.J.(1985),Application of fuzzy set theory to mathematical programming, Information Science, 36, 29-58.

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)