Optimum Design of 2D Trusses Using Controlled Directional Movement of Nodes

Premanand Shenoy, K. S. Babu Narayan, Katta Venkataramana

Abstract— ***Optimization of structures has always been a subject of continuous interest in the field of structural engineering. The amount of research work and publications in this field show various mathematical approaches adopted to effectively use materials used for construction. A novel iterative Node Based Smoothing Method for the evolution of optimum design of trusses is presented.***

Index Terms— ***Structural Optimization, Topology, Sizing, Shape, Trusses, Iterative Process, nodal position.***

I. INTRODUCTION

Two dimensional trusses are basic and commonly used form of construction. Evolutionary ideas in optimizing the size of members, shape and topology lead to preservation of precious material. Various established methods are available for the Optimization of the Shape, Topology of trusses with minimum member sizing. Many of the methods involve high end mathematical formulations and manipulations. Iterative search methods , based on logical evolution of shape and topology, are another group where step by step refinement is done for the geometry of the truss, every time seeking a better optimal solution. These methods, though involve calculations many times repeated, prove to be easier to understand and to implement using simple algorithms.

II. OPTIMIZATION OF TRUSSES

A. Sizing Optimization

Sizing is mainly governed by the permissible stresses in the material under different conditions. The tensile stress limit could be a factor of the yield stress. But the slenderness ratio and Young's modulus play a critical role in compressive stress limit. If , in a member, the stress in the material exceeds the permissible stress, the member is unsafe and hence the design is unacceptable. The cross sectional areas of such members need upward revision. At the same time, some members may have been oversized initially. The cross sectional area of those members need downward revision. This procedure is called sizing optimization. An ideal situation is the one for which the material strength gets fully exploited for all the members, simultaneously.

B. Shape Optimization

It involves finding out a shape of a truss which gives minimum weight. This is achieved by moving the nodes of a truss in an efficient manner such that the shape evolved is the best to effectively transfer the loads expected on it, at various nodes.

C. Topology Optimization

In trying to find out the perfect shape of truss, we may find that some members may turn out to be totally inefficient, with the cross sectional areas demanded by size optimization reducing to minimum. Judicious removal of such members from the configuration of the truss, give rise to the best topology.

III. EVOLUTION OF CONTROLLED DIRECTIONAL NODE MOVEMENT METHOD

Motivation to develop the proposed method comes from the necessity of an algorithm that uses the basic principles of structural mechanics effectively, efficiently and intelligently to reduce material consumption in trusses in a systematic and sequential manner. Though sizing optimization is member based, the Method uses nodes as points of attraction in shape and topology optimization. The proposed method uses the Direct Stiffness Matrix Method for the analysis of the truss, with the given size of members, shape and topology of the structure, as starting values of variables. As usual, while assembling the Global Stiffness Matrix, the resolved components of the element stiffness values are algebraically added at the positions representing the end nodes of a member. Initial values of member properties like A (Cross sectional area) and **E** (Young's Modulus of the material) and **l** (Length of the member) are taken as inputs for the determination of element stiffness. Solution is obtained for forces and the actual axial stresses are determined. The ratio of actual stress to the permissible stress in the material at a cross section of a member is called utility ratio of that member , U. If U > 1 the member is unsafe. At a typical node, let us assume that there are '**m**' number of members connected to it. Let the cross sectional areas be $\mathbf{A_1, A_2, A_3, \ldots A_m}$ and lengths be $\mathbf{l_1, l_2, l_3, \ldots l_m}$. Performing the analysis and getting a safe design, need not lead to an optimum design. Let us assume the utility ratios of the members be $\mathbf{U_1, U_2, U_3, \ldots U_m}$, all of them being less than unity for a safe design. Utility ratios of members is the guiding premise for the formulation of Nodal Position Shift Method that leads to optimum design through a sequential movement of nodes. Sizing optimization is achieved by forcing change in cross sectional areas of members to reach values that give utility ratios close to unity. Achieving this condition for all the members , necessitates search for strategic node location, which has prompted the criterion given below. If a node can be moved to a new position such that all the members connected to it reach the utility ratios very close to unity simultaneously, then that position is deemed to be the

Revised Version Manuscript Received on June 23, 2015.

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perfect position for the node and hence the method. These conditions, when satisfied for all the nodes will result in all the members of the truss reaching utility ratios nearing unity. The cross sectional areas are derived from the forces in members in sizing optimization. Some of the members may end up with negligible cross sectional areas en-route the adaptive search for perfect nodal positions. This possibility is well addressed in the following section.

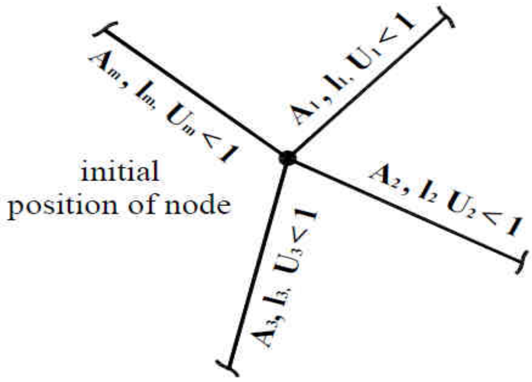

Fig. 1. Initial Position of Node

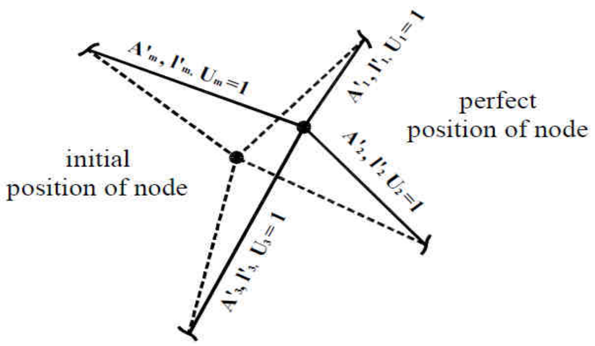

Fig. 2. Movement of Node

In the search process, if a member area demanded is miniscule (Negligible Cross sectional area) the indication is that such members are ineffective and the truss can perform without that member in question, as a part of the current configuration. Adaptive search may also encounter a situation where all the members at a node are ineffective. If all the members at a node are ineffective, then the truss can survive without that node, indicating it can be collapsed to any node it is linked to. This gives us an opportunity to remove such nodes from the truss and end up with a better shape.

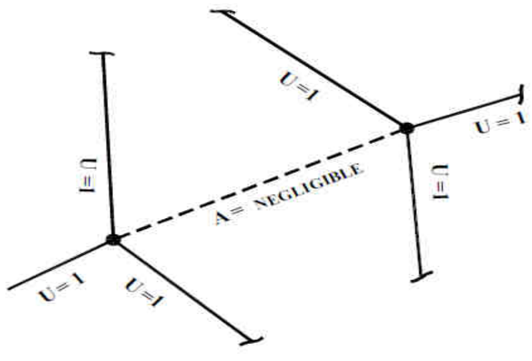

Fig. 3. Criterion for Removal a Member

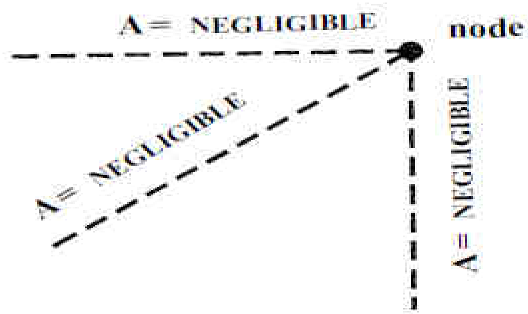

Fig. 4. Criterion for Removal of a Node

IV. MOVEMENT OF NODES

The multi objective optimization of trusses with same material can now be viewed as finding out the perfect nodal positions such that all the members of the truss, subjected to given set of constraints

a) have non negligible cross sectional areas
b) have utility ratios equal to unity (ideal situation)
c) have a set of lengths such that total weight of the truss W, is minimum

Where

W =Unit wt X Sum of (**Ai x** l_i) **---- Eq. 1**

i = 1 to m, where A_i is cross sectional area and l_i is the length of of the **i** th member Movement of node changes the lengths of members connected to the node. The reverse is also true. The change in lengths of members connected to a node, moves the node. To achieve the minimum, the problem is approached from three angles, simultaneously, for a member.

i. Reduce the length, if cross sectional area is to remain the same
ii. Reduce the cross sectional areas if the lengths is to remain the same.
iii. Increase the effectiveness

With these in mind, the lengths of members connected at a node are changed based on their relative qualifications. At a node , where **'m'** number of members are connected, for every member three factors are identified which are factors dependent on its length, cross sectional area and utility ratio.

Factor for weight consideration (C_1)

$C_{1i} = w_i / \text{sum of } (w_i)$ **---- Eq. 2**

i = 1 to m , where w_i is the weight of the **i** th member

Factor for area consideration (C_2)

$C_{2i} = A_i / \text{sum of } (A_i)$ **---- Eq. 3**

i = 1 to m, where A_i is sectional area of the **i** th member

Factor for inefficiency (C_3)

$C_{3i} = 1 - U_i$ **---- Eq. 4**

where U_i is utility ratio of the **i** th member Total forced change in length of ith member ,**dl** in the direction of member

$dl_i = MF \cdot C_{1i} \cdot C_{2i} \cdot C_{3i} \cdot l_i$ **---- Eq. 5**

where **MF** is the desired maximum percentage modification desired per iteration When the member length changes by **dl** , the node will be moved with respect to its original position by **dxl** in the global **X** axis and **dyl** in the Global **Y** axis.

$X_{new} = X_{old} + \text{sum of } (dxl)$ for all members **---- Eq. 6**

$Y_{new} = Y_{old} + \text{sum of } (dyl)$ for all member **---- Eq. 7**

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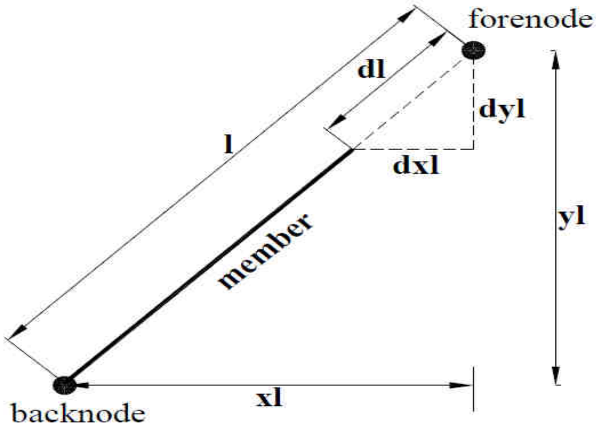


Fig. 5. Forced Change in Length of a Member

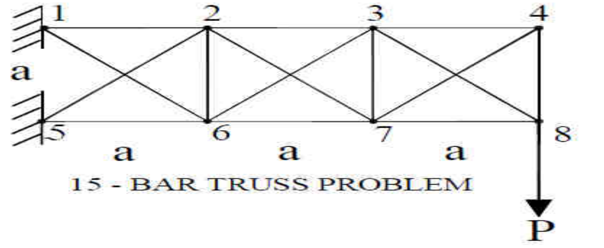
The factor **C2**, defined for the area of a member becomes negligible if the member is ineffective. The factor $\mathbf{C_3}$ which is meant for the inefficiency of a member reduces to zero, when utility ratio is unity. This means , the length of a member is not altered in both the cases. If all the members at a joint are efficient, the node is not moved. This ensures the convergence of the solution for optimization. On the other hand, the node will be forced to move relatively when every member is subjected to a forced change in length, every time controlling the movement in the direction of the members and checking whether it moves within the limits well defined in the beginning by a possible set of movement restrictions specified in the problem. This is repeated for all the nodes and the changed configuration of the truss is recorded.

V. ITERATIVE PROCEDURE

This Method is an iterative procedure with distinct loops for sizing and shape optimization. Topology optimization is achieved during the course of shape optimization. Starting with the initial geometry, member properties, loading and support conditions and constraints, solution is obtained for the stresses. Cross-sectional areas are increased or decreased iteratively to obtain a safe sizing optimization for the shape and topology, till the utility ratios stabilized. This is named as the sizing loop. Member lengths are modified as per **Eq. 5** to effect change in nodal positions and the whole procedure is repeated to get another set of stabilized utility ratios. This is named as the combined shape optimization loop. Size optimization loop is a part of shape optimization loop. The procedure is repeated till we get a stabilized set of utility ratios of all effective members equal to unity. While iterative loop in progress, some of the members of the truss are identified ineffective and Young's modulus values of such members are considered negligible for the consecutive loop. The final shape of the truss without these ineffective members is the optimum topology. **Fig. 8** shows the flowchart for the implementation of the Nodal Shift method.

VI. NUMERICAL EXAMPLE

The 15 bar truss problem, shown in Fig.6, solved by many researchers has been treated as benchmark to check the efficiency of the algorithm. The optimum design is to be achieved with the properties and movement restrictions stated in Table 1.



P = 10 kips (44.48 kN)
a = 120 in (3048 mm)
density = 0.1 lb/ cu. in (2768 kg/ Cu. m)
e = 10000 ksi (68947.6 N/ Sq. mm)
limit = 25 ksi (172.0 N / Sq. mm)

Fig. 6. Benchmark Problem

Table 1. Constraints for Nodal Movements

JOINT MOVEMENT CONSTRAINTS						
JOINT No.	CO-ORDINATES		Permissible Freedom			
	X	Y	Min X	Max X	Min Y	Max Y
1	0	120	0	0	120	120
2	120	120	100	140	100	140
3	240	120	220	260	100	140
4	360	120	360	360	50	90
5	0	0	0	0	0	0
6	120	0	100	140	-20	20
7	240	0	220	260	-20	20
8	360	0	360	360	20	60

Additional Conditions: $\mathbf{X_6 = X_2,\ X_7 = X_3,\ X_8 = X_4 = 360}$

VII. RESULTS AND DISCUSSION

The optimum topology evolved is shown in Fig. 7. Table 2 shows the set of Length, Area and Utility ratio for every member at the instance of optimum design. It is noted that the method clearly identifies member numbers 3,7,8,9 and 15 as ineffective. The positional changes of the nodes for the optimum configuration have been affected only on the foundation of utility ratio wherein permissible stresses both in tension and compression remain the same, as stated in the benchmark problem.

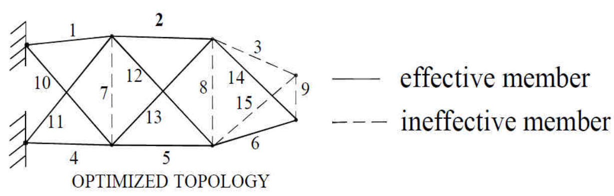


Fig. 7. Configuration for Minimum Weight - Topology

Table 3 shows the sizing and layout variables obtained by the method in comparison with the results given in references. It is seen that the results obtained are in agreement and showing a further improvement in optimum design. Fig. 9 shows the weight reduction of the truss corresponding to iterations performed with a typical Modification Factor 5%

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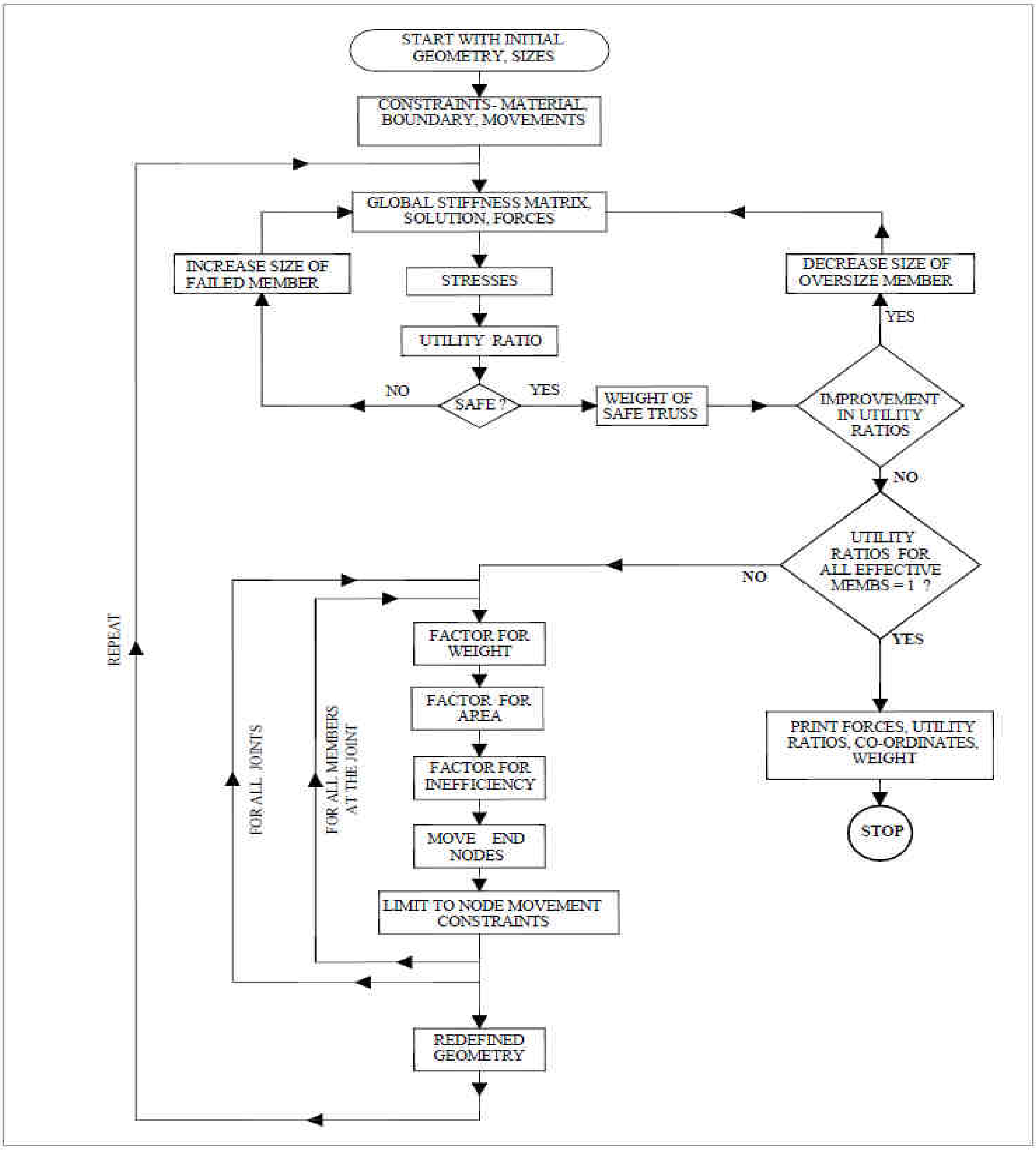

Fig. 8. Flow Chart for the Optimization Method

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Table 2. Final Cross Sectional Areas and Utility Ratios

RESULTS WITH REDUCTION FACTOR = 5 %								
15 - MEMBER TRUSS - OPTIMUM DESIGN - FORCES ON MEMBERS								
	Length	AREA	WT	FORCE	Type	STRESS	Allow. Stress	RATIO
Memb. No.	in	Sq. in	kips	kips		ksi	ksi	
1	120.0000	0.8671	0.0104	22.060	Tens	25.00	25.00	1.00
2	119.9030	0.7321	0.0088	17.750	Tens	25.00	25.00	1.00
3	123.8110	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
4	120.0000	1.1332	0.0136	-27.940	Comp	-25.00	25.00	1.00
5	119.9030	0.4670	0.0056	-11.910	Comp	-25.00	25.00	1.00
6	121.7860	0.4050	0.0049	-9.840	Comp	-25.00	25.00	1.00
7	120.0000	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
8	120.3080	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
9	70.0000	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
10	169.7060	0.4700	0.0080	11.230	Tens	25.00	25.00	1.00
11	169.7060	0.0951	0.0016	-2.910	Comp	-25.00	25.00	1.00
12	169.7860	0.0960	0.0016	3.280	Tens	25.00	25.00	1.00
13	169.7060	0.4710	0.0080	-11.660	Comp	-25.00	25.00	1.00
14	156.3420	0.5200	0.0081	12.580	Tens	25.00	25.00	1.00
15	150.2050	0.0000	0.0000	0.000	INEFFECT	0.00	25.00	0.00
TOTAL WEIGHT OF TRUSS			0.07066	kips				

Table 3. Comparison of Results for Benchmark Problem.

15 - MEMBER TRUSS - OPTIMUM DESIGN - COMPARISON OF RESULTS						
	EARLIER WORKS [1]					Present Work
MEMBER AREA	Gholizadeh et. al. [12]	Tang et al. [3]	Hwang and He [2]	Rahami et al. [4]	Kulkarni et al. [1]	
Sizing variables (in.^{2})						
A1	0.954	1.081	0.954	1.081	0.954	0.8671
A2	0.539	0.539	1.081	0.539	0.539	0.7321
A3	0.27	0.287	0.440	0.287	0.111	0.0000
A4	1.081	0.954	1.174	0.954	0.954	1.1332
A5	0.539	0.954	1.488	0.539	0.539	0.4670
A6	0.174	0.220	0.027	0.141	0.347	0.4050
A7	0.111	0.111	0.270	0.111	0.111	0.0000
A8	0.111	0.111	0.347	0.111	0.111	0.0000
A9	0.44	0.287	0.220	0.539	0.111	0.0000
A10	0.44	0.220	0.440	0.440	0.440	0.4700
A11	0.347	0.440	0.347	0.539	0.44	0.0951
A12	0.22	0.440	0.220	0.270	0.174	0.0960
A13	0.22	0.111	0.270	0.220	0.174	0.4710
A14	0.174	0.220	0.440	0.141	0.347	0.5200
A15	0.27	0.347	0.220	0.287	0.111	0.0000
Layout variables (in.)						
X2	113.65	133.612	118.346	101.5775	105.7835	120.000
X3	254.47	234.752	225.209	227.9112	258.5965	239.903
Y2	128.97	100.449	119.046	134.7986	133.6284	120.000
Y3	115.73	104.738	105.086	128.2206	105.0023	120.098
Y4	59.364	73.762	63.375	54.8630	54.4546	90.000
Y6	-12.733	-10.067	-20.000	-16.4484	-19.9290	0.000
Y7	3.5467	-1.339	-20.000	-13.3007	3.6223	-0.211
Y8	59.29	50.402	57.722	54.8572	54.4474	20.000
Weight (lbs)	73.93	79.820	104.573	76.6854	72.5152	70.660

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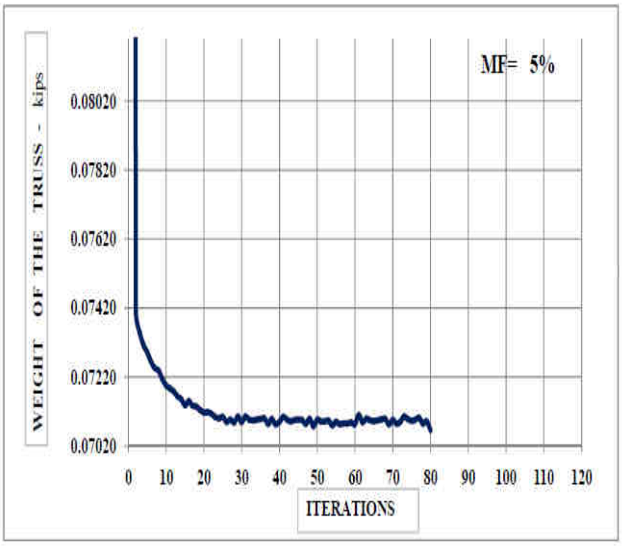

Fig. 9. Typical Graph Showing Weight reduction of Truss

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