Numerical Study of Natural Convection of Nanofluid in a Square Enclosure in the Presence of the Magnetic Field

Youness El Hammami, Mohamed El Hattab, Rachid Mir, Touria Mediouni

Abstract—.This article reports a numerical study on natural convection in an enclosure that is filled with a water–Cu nanofluid and is influenced by a magnetic field. Side walls are the heated surfaces (hot and cold walls). Top and bottom walls of the cavity are assumed to be adiabatic. Theoretical models of Maxwell–Garnetts (MG) and Brinkman are employed to predict the thermal conductivity and viscosity of the nanofluid respectively. The transport equations for continuity, momentum and energy are solved with finite volume approach using the SIMPLE algorithm. This study is carried out to predict the effect of Rayleigh number, Hartmann number and the solid volume fraction of nanoparticles on the flow and heat transfer rate. Results show, There is an opposite effect of Ra and Ha on flow regime, by increasing the magnetic force (higher Hartmann number), the conduction heat transfer becomes the dominate mechanism in heat transfer; this increment causes thermal dissipation in the flow of nanofluid to be enhanced. When the Rayleigh number is high and the Hartmann number is low, the streamlines and isotherms are distributed strongly in the enclosure domain and the heat is transferred due to convection.

Keywords— Copper-water nanofluid; Magnetic field; Heat transfer; Natural convection; Numerical study

I. INTRODUCTION

The natural convection heat transfer is an important phenomenon in the engineering systems, due to its wide applications in electronic cooling, heat exchangers, the built-in-storage solar industry and thermal insulation systems, the food storge industry and the geophysical fluid mechanics. Improvement in the heat transfer performance of these systems is an essential topic from an energy saving perspective. A new innovative technique to enhance the heat transfer is made by using solid particles (smaller than 100 nm) in the base fluid. This technique was developed by Choi [1], who was the first to introduce the term nanofluid to refer to a fluid in which nanoparticles are suspended. Much work has been conducted to study the physical properties of nanoparticles, but fairly, little attention has been dedicated to the natural convection of nanofluids in the enclosures [2]–[5].

Jou and Tzeng [6] used nanofluids to enhance natural convection heat transfer in a rectangular enclosure, using Khanafer’s model. They indicated that increasing the volume fraction of nanofluids causes an increase in the average heat transfer coefficient. Guiet et al. [7] performed a numerical study to investigate the natural convection in a square enclosure with a protruding heater located at its bottom wall and filled with copper-water nanofluid. Numerical study of the natural convection in partially heated rectangular enclosures, has been performed by Oztop and Abu–Nada [8]. Different types of nanoparticles were tested. It was found that the heater location affects the flow, and temperature fields when using nanofluids. Recently, El Hattab et al. [9], [10] have studied numerically the natural convection heat transfer in a cavity with a protruding heater. They found that type of nanofluid is a key factor for heat transfer enhancement. It is also observed that, at a given Rayleigh number and definite heater geometry, the heat transfer is enhanced with increasing the volume fraction of nanoparticles and the heat transfer increased with an increase in the width and the length of the heat source.

Some practical cases such as the crystal growth in fluids, the metal casting, fusion reactors and the geothermal energy extractions, natural convection is under the influence of a magnetic field [11]–[15]. Pirmohammadi and Ghassemi [16] study steady, laminar, natural-convection flow in the presence of a magnetic field in a tilted enclosure heated from below and cooled from top and filled with liquid gallium. Ece and Buyuk [17] examined the steady and laminar natural convection flow in the presence of a magnetic field in an inclined rectangular enclosure heated and cooled on adjacent walls. They found that the magnetic field suppressed the convective flow and the heat transfer rate. Sivasankaran and Ho [18] studied numerically the effects of temperature dependent properties of the natural convection of water in a cavity under the influence of a magnetic field. They showed that the heat transfer ratewas influenced by the direction of the external magnetic field and was decreased with an increase of the magnetic field. Noghrehabadi and Samimi [19] investigated the effect of both thermophoresis and Brownian motion on natural convection of Cuo–Water nanofluid. They observed that an improvement in heat transfer rate was registered for the whole range of Rayleigh numbers when Brownian and thermophoresis effects are considered. Nithyadevi et al. [20] studied the magneto convection in a square cavity with partially active vertical walls with a time periodic boundary condition. It was found that, the heat transfer rate is maximum for the middle–middle thermally active locations while it is poor for the top heating

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and bottom cooling active locations. Also, the average Nusselt number decreases with an increase of the Hartmann number and increases with an increase of Prandtl number and Grashof number.

Therefore, studying the performance of the nanofluid in cavities influenced by a magnetic field needs to be investigate more. The aim of the present study is to investigate numerically the heat transfer rate in a square enclosure filled with a water–Cu nanofluid and influenced by magnetic field. This investigation has been accomplished using Brinkman's model for calculating the effective viscosity of nanofluids and Maxwell-Garnetts's model for the effective thermal conductivity. This study is carried out to predict the effect of Rayleigh number, Hartmann number and volume fraction of nanoparticles, on the heat transfer rate.

II. Mathematical Model

This study investigates a two-dimensional problem of natural convection in a cavity of side height (*H*), as shown in fig. 1. Side walls are the heated surfaces (hot and cold walls). Top and bottom walls of the cavity are assumed to be adiabatic. A uniform magnetic field strength of B_0 is considered in the horizontal direction. The cavity is filled with Cu-water and the thermophysical properties of Cu nanoparticles and water (*Pr=6.2*) as base fluid are listed in Table I. The nanofluid is considered to be Newtonian, and the nanofluid flow is assumed to be laminar and incompressible. It is also assumed that, both the nanoparticles and the base fluid are in thermal equilibrium and no slip occurs between them. The Boussinesq approximation is valid for the buoyancy term. The viscous dissipation and joule heating are also neglected.

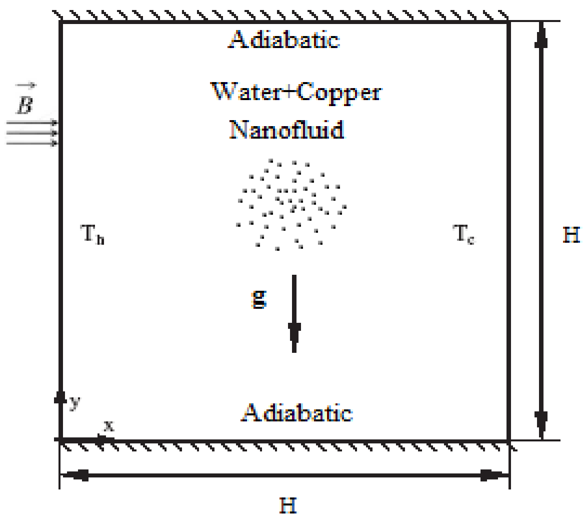

Fig. 1. Physical Model

Therefore, governing equations (continuity, momentum and energy equations) can be written in dimensional form as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\frac{(\rho\beta)_{nf}\,g}{\rho_{nf}}(T-T_c)-\frac{\sigma_{nf}}{\rho_{nf}}B_0^2 \upsilon \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

The properties of nanofluid are obtained as following [21]:

$$\sigma_{nf}=(1-\Phi)\sigma_f+\Phi\sigma_p \tag{5}$$

$$\rho_{nf}=(1-\Phi)\rho_f+\Phi\rho_p \tag{6}$$

$$\alpha_{nf}=k_{eff}/(\rho C_P)_{nf} \tag{7}$$

$$(\rho C_P)_{nf}=(1-\Phi)(\rho C_P)_f+\Phi(\rho C_P)_p \tag{8}$$

$$(\rho\beta)_{nf}=(1-\Phi)(\rho\beta)_f+\Phi(\rho\beta)_p \tag{9}$$

The effective dynamic viscosity of the nanofluid is calculated using Brinkman's model [22] as follows:

$$\mu_{nf}=\frac{\mu_f}{(1-\Phi)^{2.5}} \tag{10}$$

The thermal conductivity of the nanofluid is evaluated from the Maxwell formula [23] as

$$k_{nf}=k_f\left[\frac{(k_p+2k_f)-2\Phi(k_f-k_p)}{(k_p+2k_f)+\Phi(k_f-k_p)}\right] \tag{11}$$

Where Φ is the solid volume fraction, ρ_{nf} is the density, $C_{P\,nf}$ is the heat capacity, β_{nf} is the thermal expansion coefficient, σ_{nf} is the electrical conductivity of the nanofluid, α_{nf} is the thermal diffusivity of nanofluid, and k_{nf} is the thermal conductivity of nanofluid. Here, subscript np and f indicate physical properties of solid nanoparticle and base fluid, respectively. The effective dynamic viscosity, μ_{nf} of the Cu-water nanofluid is calculated according to the Brinkman model:

Table I Thermophysical properties of water and copper [24], [25]

Physical properties	Water	Copper(Cu)
C_p(J/kgK)	4179	385
ρ(kg/m^3)	997.1	8933
μ(kg/ms)	9.09×10^{-4}	–
k(W/mk)	0.613	401
β(K^{-1})	21×10^{-5}	1.67×10^{-5}
σ(S/m)	0.05	5.96×10^{7}

By using the following dimensionless parameters:

$$X=\frac{x}{H},\quad Y=\frac{y}{H},\quad U=\frac{uH}{\alpha_f},\quad V=\frac{vH}{\alpha_f},\quad P=\frac{(p+\rho_c g y)H^2}{\rho_{nf}\alpha_f^{\,2}},\quad \theta=\frac{T-T_c}{T_h-T_c} \tag{12}$$

The governing equations (1) – (4) may be written in the dimensionless form as:

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$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{13}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{14}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}Ra\,\mathrm{Pr}\,\theta \tag{15}$$

$$-Ha^2\,\mathrm{Pr}V$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{16}$$

Where U and V fluid velocity in X-direction and Y-direction, respectively, P is pressure, B_0 is the magnetic field strength, θ is the temperature. Rayleigh number Ra, Prandtl number Pr and Hartman number Ha are defined as

$$Ra=\frac{g\beta_f H^3(T_h-T_c)}{\nu_f\alpha_f},\ \mathrm{Pr}=\frac{\nu_f}{\alpha_f},\ Ha=B_0L\sqrt{\frac{\sigma_{Nf}}{\rho_{nf}\upsilon_f}} \tag{17}$$

The dimensionless boundary conditions are written as:

On the hot wall: $X=0,\ U=V=0,\ \theta=1$ (18)

On the cold wall: $X=1,\ U=V=0,\ \theta=0$ (19)

On the bottom wall: $Y=0,\ U=V=\frac{\partial\theta}{\partial Y}=0$ (20)

On the top wall: $Y=1,\ U=V=\frac{\partial\theta}{\partial Y}=0$ (21)

In order to evaluate the heat transfer enhancement in the cavity, the local Nusselt number on the left hot wall can be defined as:

$$Nu_Y=-\frac{k_{eff}}{k_f}\left.\frac{\partial\theta}{\partial X}\right|_{X=0} \tag{22}$$

Average Nusselt number along the hot wall of the cavity is considered to evaluate the overall heat transfer rate and is defined as:

$$Nu_{avr}=\int_0^1 Nu_Y\,dY \tag{23}$$

The stream function is calculated using $U=\frac{\partial\psi}{\partial Y}$ and

$$V=-\frac{\partial\psi}{\partial X} \tag{24}$$

III. NUMERICAL METHOD AND VALIDATION

A Fortran computer code is developed to model the considered problem of natural convection in a enclosure filled with Cu–water nanofluid in a presence of magnetic field. The non-dimensional governing equations (13)–(16), with the stated boundary conditions are solved numerically by the control volume method [26]. The power-law scheme is used for the convection and diffusion terms. The velocity and pressure are coupled by SIMPLE algorithm [26]. The sweeping method line by line, with the algorithm of Thomas was used for the iterative resolution of the systems of equations. The convergence criterion is that the maximal residual is less than 10^{-6}.

The average Nusselt number (Nu_{avr}) of the hot wall of the cavity for five different uniform grids is examined to study the independence of the solution to the grid size. The results for Ra=10^5, Φ=0.03 and Ha=30 are presented in table II. As it is evident from the table, a uniform 101 × 101 grid is sufficiently fine for the numerical calculation.

Table II Result of the average Nusselt number (Nu_{avr}) and the maximum stream function ($|\Psi|_{max}$) for different grids (Φ=0.03, Ra=10^5, Ha= 30).

Grid size (X×Y)	Nu_{avr}	Error%	$\lvert\Psi\rvert_{max}$	Error%
41×41	3.118	–	5.310	–
61×61	3.073	1.443	5.328	0.338
81×81	3.056	0.553	5.341	0.243
101×101	3.048	0.261	5.348	0.131
121×121	3.043	0.164	5.353	0.093

The numerical procedure is validated by comparison between the predicted results with obtained in the literature for the natural convection of air inside a square cavity whose two sides are set to differential temperatures while keeping the top and bottom surfaces at adiabatic condition is a classic case for validation. Average Nusselt number for the hot wall calculated from the present model is compared with the data available in the literature. The comparison is shown in table III. The results are good agreement with the solutions available in the literature.

Table III Averge Nusselt number for the air filled square cavity obtained by various studies are shown and compared with present work for different Rayleigh values

Rayleigh number (Ra)	Present work	Vahl Davis [27]	Fusegi et al.[28]	Khanafer et al. [29]	Barakos et al. [30]
10^4	2.217	2.243	2.302	2.245	2.245
10^5	4.475	4.519	4.646	4.522	4.510
10^6	8.727	8.799	9.012	8.826	8.806

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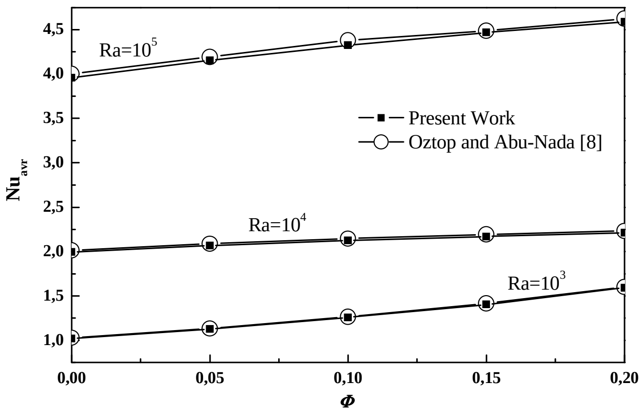

Fig. 2. Comparison between the present work and numerical results by Oztop and Abu-Nada [8] for natural convection in partially heated enclosure filled with water–Al_2O_3 nanofluid.

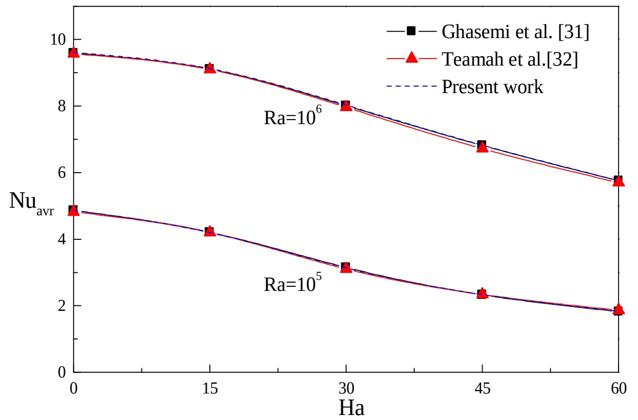

Fig. 3. Comparison of the average Nusselt number versus Hartmann number at different values of Rayleigh number with a water-Al_2O_3 nanofluid (Φ= 0.03) between the present work and the results of Ghasemi et al.[31] and Teamah et al[32].

In order to check on the accuracy of the numerical technique employed for the solution of the problem considered in the present study, it was validated with Oztop and Abu-Nada [8] for the average Nusselt number without the presence of the magnetic field for the range of $10^3 \leq Ra \leq 10^5$ as shown in Fig 2. Another validation with Ghasemi et al. [31] and Teamah et al [32] for the average Nusselt number at the Hartmann number ranging 0 to 60 in the presence of water-Al_2O_3 nanofluid (Φ= 0.03) is represented in Fig 3. Based on the comparisons made, it is clear that the present work is in good agreement with other work reported in literature as shown in figures 2 and 3.

IV. RESULTS AND DISCUSSION

A numerical study has been performed to analyze the convective laminar flow and heat transfer in the presence of vertical magnetic field using Cu-water nanofluid. The obtained numerical results are discussed in this section. This physical phenomenon is investigated for a wide range of the controlling parameter as shown in Table IV.

Table IV the ranges of the physical parameters

Rayleigh number, Ra	10^3	to	10^6
solid volume fraction, Φ	0	to	10%
Hartmann number, Ha	0	to	60

The results depicted in Figs .4 and 5 demonstrate the influence of magnetic field (Ha=0, 30 and 60) and different Rayleigh numbers (Ra=10^3, 10^4, 10^5 and 10^6) on the fluid flow and the temperature distribution in the enclosure. In this party the solid volume fraction is fixed at Φ=0.05. For low Rayleigh number Ra =10^3, the conduction regime is dominant, weak clockwise circulation in the cavity is found. As Hartmann number increases circulation strength is decreased. On the other hand the isotherms are parallel lines for Ha=30 and 60 (conduction domain) but for Ra = 10^4 this change can be seen at Ha = 60. As the Rayleigh number increases Ra =10^6, the convection mode is pronounced, the flow cell becomes stronger. Since, the cell is coming to the hot wall from the cavity bottom and departs from it at the cavity top.

Therefore, the intensity of thermal gradient is important near the vertical walls, this is leads to high heat transfer is occurred at the cavity. At a high Rayleigh number, Ra=10^6 the convection is dominant the circulating cell is very strong. The streamlines are crowded near the cavity wall and the cavity core is empty. As well as isothermals are stratified in vertical direction of the enclosure and appear as horizontal lines. The isotherms are affected by variations in the Hartmann number, these effects are more noticeable at Ra =10^5, where an increase in the Hartmann number results in the isotherms changing they go from horizontal to vertical. This is an indication of weaker convection flows at higher Hartmann numbers. There is an opposite effect of Ra and Ha on flow regime, this is can be explained by the sign of Ha is opposite to the sign of Ra in source term equation (15).

Fig. 6 shows the effects of the Hartmann number on the vertical velocity (dimensionless Y–velocity V) and the dimensionless temperature (θ), along the horizontal mid–span of the enclosure at different values of the Rayleigh number (Ra =10^3, 10^4, 10^5 and 10^6) and for a solid volume fraction *Φ* = 0.05. These graphs provide a better understanding of the flow behaviour within the enclosure for the water–Cu nanofluid in presence of magnetic field. The velocity profiles verify the existence of clockwise circulating cell within the enclosure. The maximum vertical velocity increases when the Rayleigh number increases due to strong buoyant flows inside the enclosure and decreases when the Hartmann number increases due to the influence of the magnetic field on the convective flows.

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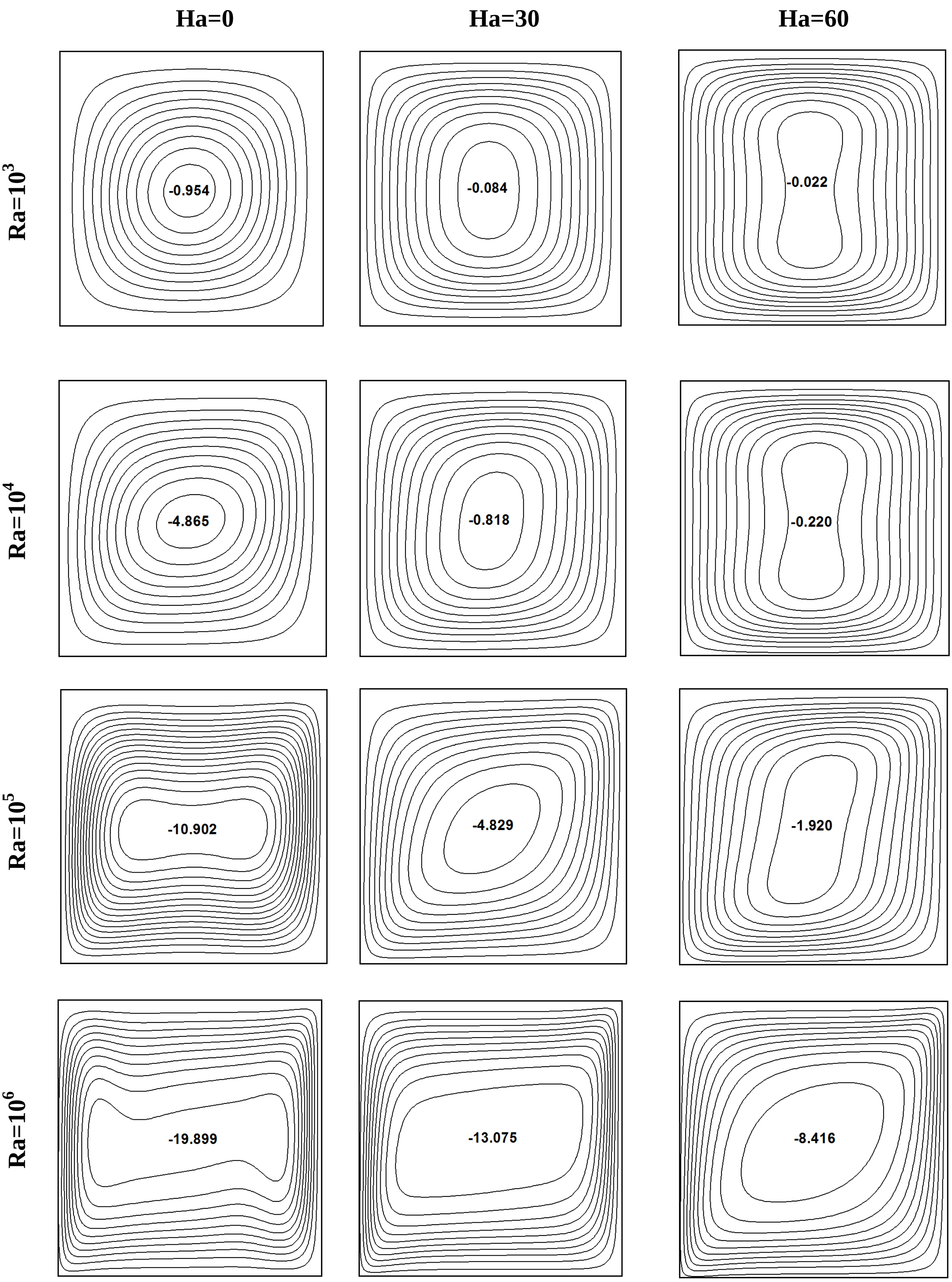

Fig. 4: Streamlines for different Rayleigh and Hartmann numbers for Φ=0.05

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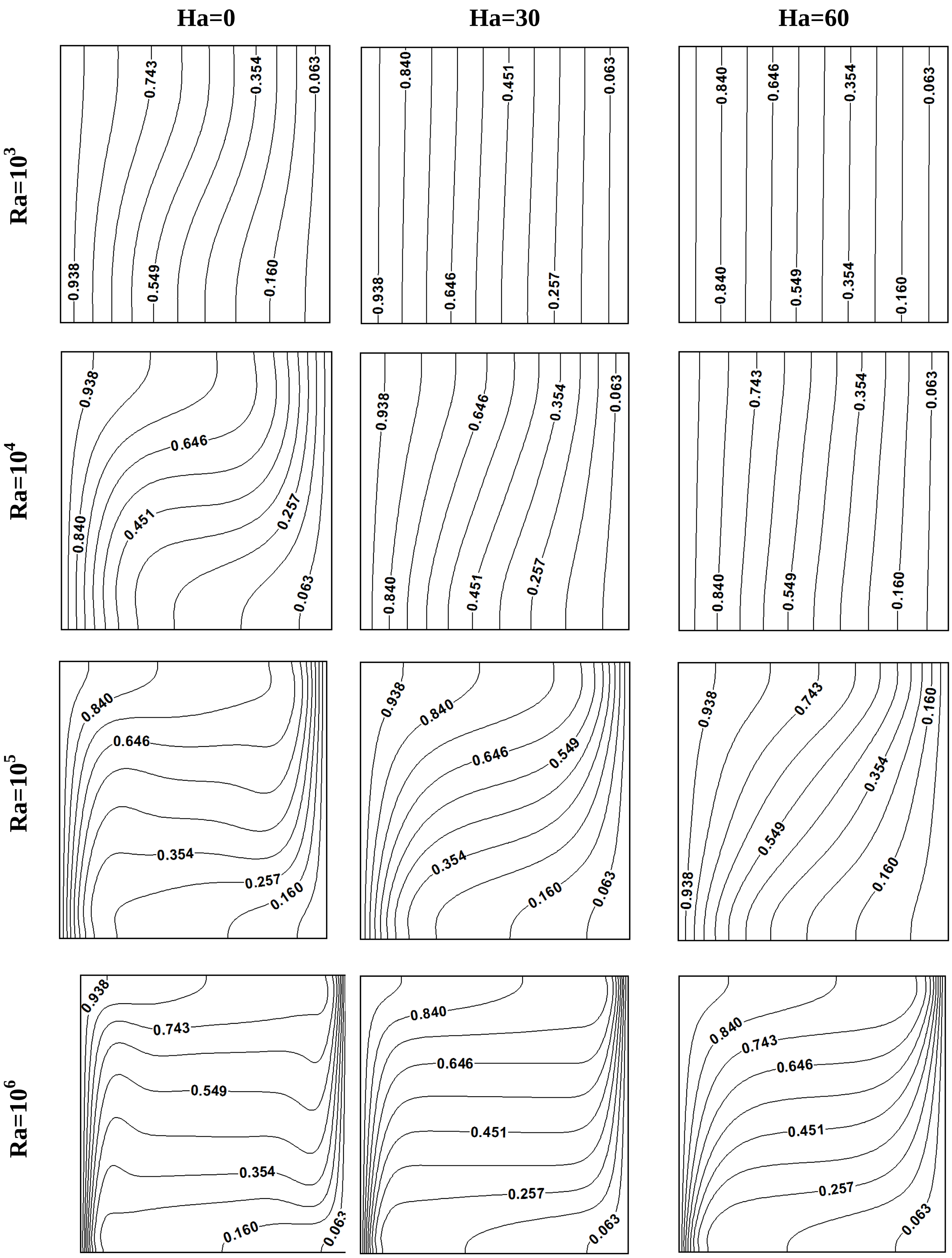

Fig. 5. Isotherms for different Rayleigh and Hartmann numbers for Φ=0.05.

The flow velocity is almost zero around X=0.5 for the Rayleigh number (Ra=10^3 and 10^4) as well as in the interval X [0.3–0.7] for Rayleigh number upper 10^4. The influence of the Hartmann number on the y-velocity profile is more significant at Ra=10^3 and 10^4, where the convective flow field is not very strong and can be influenced by the magnetic field. The irregular and random movement of particles decreases thereafter the energy exchange rates in the fluid. The velocity shows a parabolic variation near the isothermal walls. This influence can also be inferred from the temperature profiles. The temperature profiles show the maximum influence of the Hartmann number at Ra= 10^4 and 10^5. On the other hand, at Ra= 10^3, the heat transfer is mainly due to conduction; it is observed that the Hartmann number has an insignificant influence on the temperature profile. However, at Rayleigh number upper than Ra=10^4, where strong convection flows occur within the enclosure, it is observed that the Hartmann number has a major influence on the temperature profile. This increment causes thermal dissipation in the flow of nanofluid to be enhanced for

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Rayleigh number Ra=10^3 and 10^4. However, the increase in Ha number dramatically decreases the amount of vertical velocity.

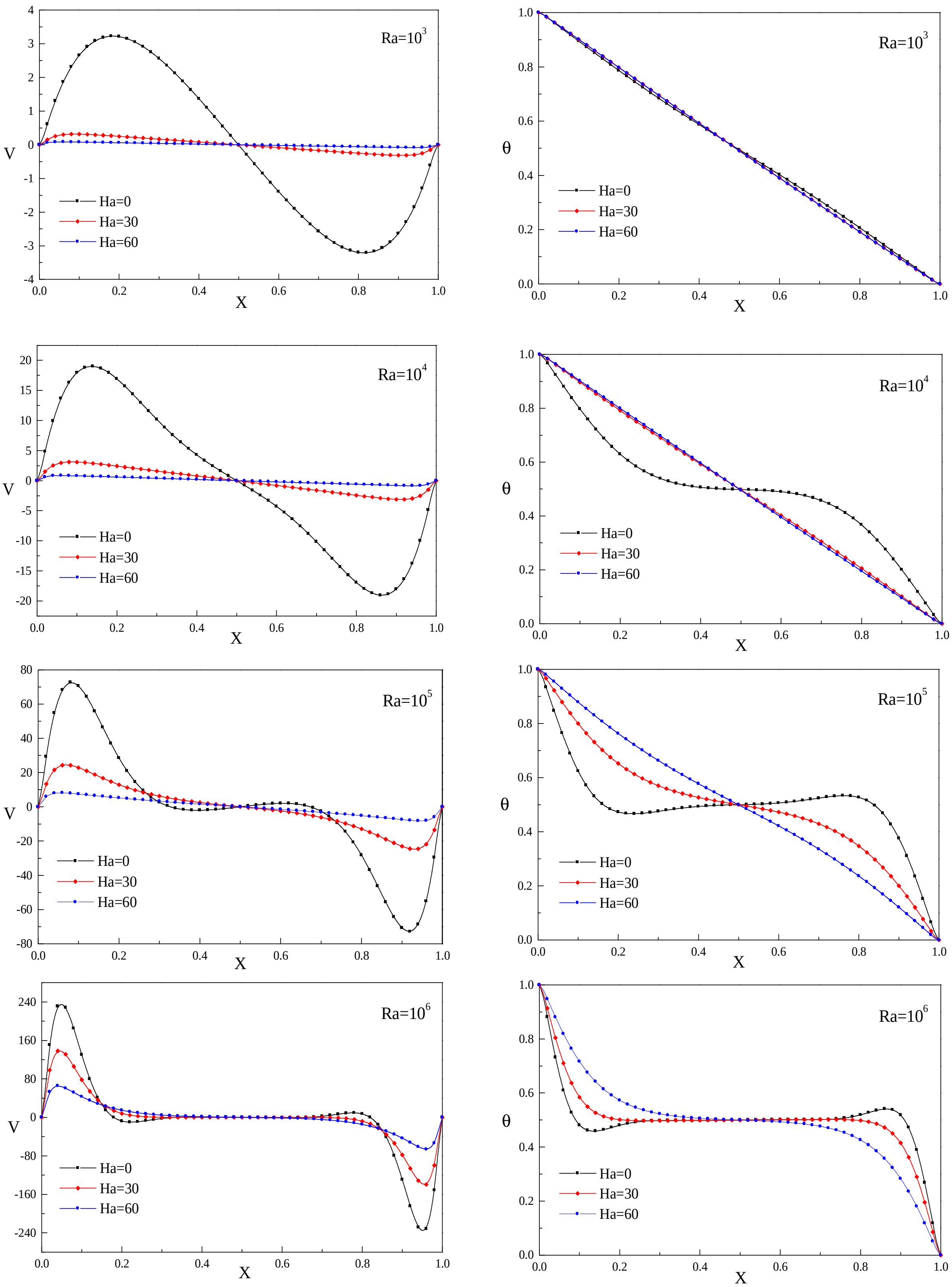

Fig. 6. Vertical velocity profile (left) and temperature profile (right) at Y=0.5 for Φ = 0.05.

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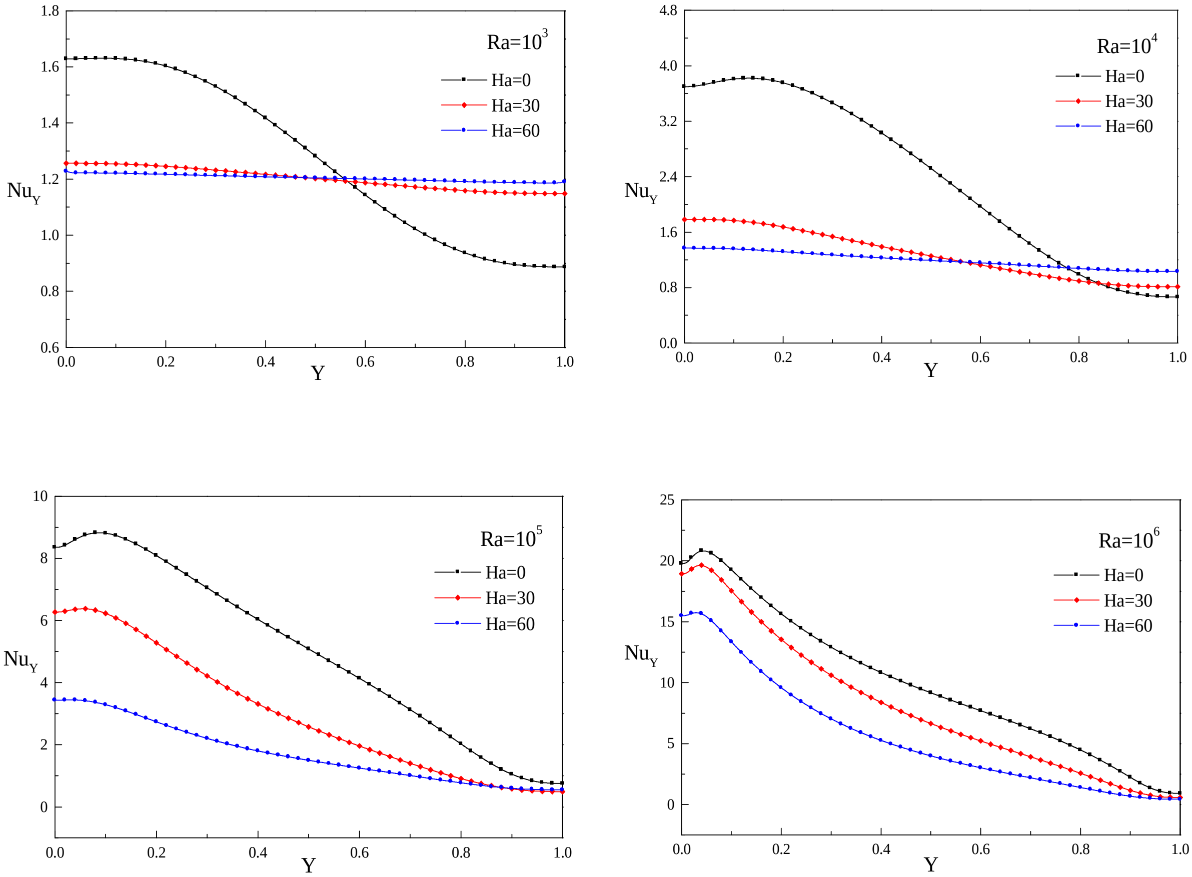

Fig. 7. Local Nusselt number along the left hot wall of the cavity for various Rayleigh number at Φ = 0.05.

Fig.7 shows how the local Nusselt number along the hot wall for various Hartmann number (Ha=0, 30 and 60) at different values of the Rayleigh number and solid volume fractions (Φ = 0.05). For Ra= 10^3, where the heat transfer is partially only due to conduction, the magnetic field does not have a considerable effect on the heat transfer performance, the local Nusselt number remains unchanged when the Hartmann number increases(Ha=30 and 60). The Hartmann number has a more apparent effect on the local Nusselt number at Ra=10^4, where the buoyant flows are significantly influenced by the magnetic field. For high Ha numbers and at Ra=10^5 and 10^6, it is obvious that the Nusselt number along the hot wall decreases, because, by increasing the Ha number, the effect of convection reduces and the dominant heat transfer mechanism is conduction. For higher values of the Rayleigh number, where the heat transfer is partly or mainly due to convection and the magnetic field can suppress the convection flows.

Fig.8 shows the ratio between the average Nusselt number and average Nusselt number without magnetic field on the heat transfer performance of the enclosure ($Nu_{avr}/Nu_{avr,Ha=0}$) for different Rayleigh number at various values of the Hartmann number. As Ra increases the $Nu_{avr}/Nu_{avr,Ha=0}$ decreases due to effect of magnetic field to minimum values around critical Rayleigh number Ra[10^4-10^5] depending on the Hartmann number value and then increases due to the strong effect of the natural convection with respect to the effect of magnetic field

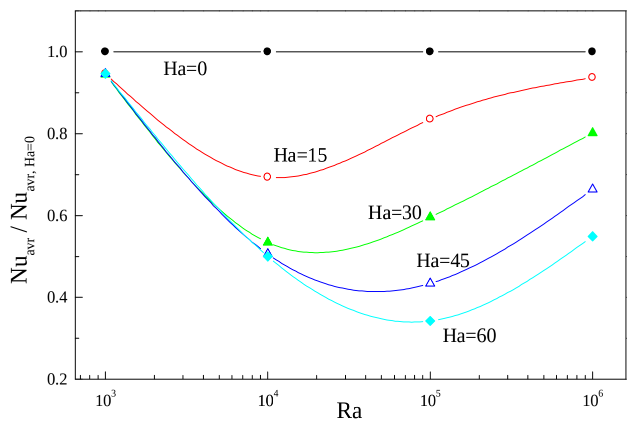

Fig. 8. Variation of average Nusselt number ratio with Rayleigh number at different Hartmann numbers (Φ = 0.05).

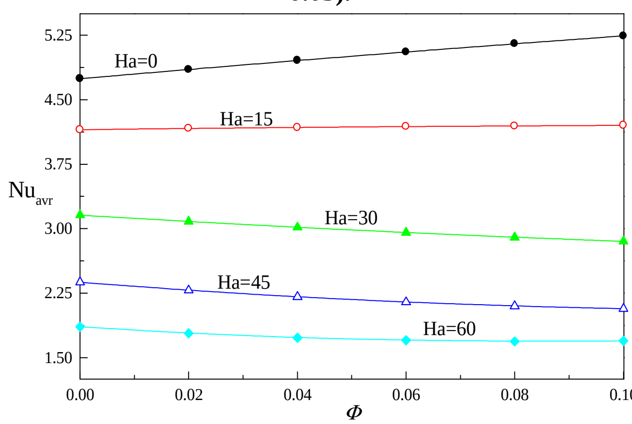

Fig. 9. Variation of average Nusselt number with solid volume fraction at various Hartmann numbers (Ra = 10^5).

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The average Nusselt number ratio at Ha =0 is the reference value and the solid volume fraction is assumed to be constant (Φ = 0.05) for this part of the study. As seen earlier, the influence of the Hartmann number on the average Nusselt number ratio is insignificant at $Ra=10^3$, where conduction is the main heat transfer system. At $Ra = 10^6$, where the convection flows are very strong, the Hartmann number also has a slight effect on $Nu_{avr}/Nu_{avr,Ha=0}$. Though, at other values of the Rayleigh number, where the magnetic field suppresses the convection flows, the Hartmann number can be seen to have a significant effect on the average Nusselt number ratio.

Fig. 9 shows the average Nusselt number at $Ra = 10^5$ and for different volume fractions and Ha number. From this figure, it can be found that, as the volume fraction increases from 0% to 10%, the average Nusselt number along the heated surface increases by about 11%, and 2% for Ha = 0 and Ha=15 respectively. As we can see, the concentration of nanoparticles has a negligible effect for Hartmann number Ha = 15. For Ha = 30, 45 and 45 the average Nusselt number along the heated surface decreases by about 10%, 13%,and 9%. When the Hartmann number becomes higher in value of Ha = 30, this effect of solid volume fractions becomes inversely. By increasing the magnetic field, flow field is affected and, subsequently, the dominant mechanism of heat transfer changes. The magnetic field has a negative effect on buoyancy force and decreases the flow motion.

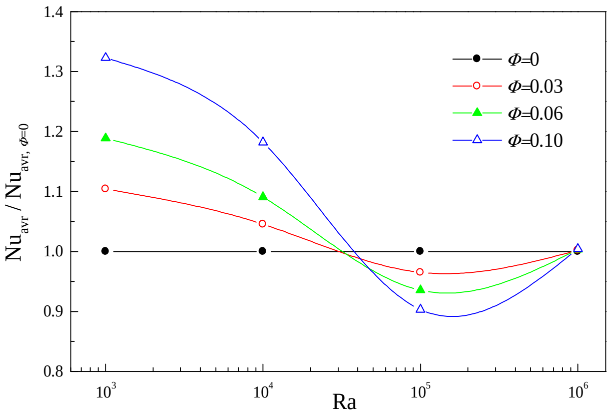

Fig. 10. Variation of average Nusselt number ratio with Rayleigh number at different solid volume fractions (Ha =30).

Fig. 10 presents the variations in the average Nusselt number ratio ($Nu_{avr}/Nu_{avr,\,\Phi=0}$) with the Rayleigh number at different values of the solid volume fraction Φ (0, 0.03, 0.06 and 0.1). The average Nusselt number ratio at Φ=0 is the reference value and the Hartmann number is assumed to be constant (Ha=30) for this part of the study. The results show that for the nanofluid, which is influenced by a magnetic field, there is a critical Rayleigh number approx the value of $Ra = 5.10^4$ that minimises the average Nusselt number ratio regardless of the value of the solid volume fraction. At values around critical Rayleigh number ($Ra < 5.10^4$), the addition of nanoparticles augments the heat transfer proportionally to the solid volume fraction, but if the Rayleigh number increases in this value ($Ra > 5.10^4$) the heat transfer is obviously reduced as the solid volume fraction increases. The addition of nanoparticles is necessary to enhance the heat transfer for weak magnetic field applications and for Rayleigh number below 5.10^4, but for strong magnetic field applications there is no need for nanoparticles because the heat transfer will decrease.

V. CONCLUSION

This paper presents the results of a numerical study of natural convection heat transfer in a square enclosure filled with a water–Cu nanofluid (water with copper nanoparticles) and the cavity is under the influence of a magnetic field. This study is carried out to predict the effect of Rayleigh number, Hartmann number and the volume fraction of nanoparticles on the flow and heat transfer rate. The influence of the magnetic field was studied to 5 percent of nanoparticle and deduced that it has a remarkable effect on the heat transfer and flow field in the cavity. The study leads to the following conclusions:

- There is an opposite effect of Ra and Ha on flow regime, by increasing the magnetic force (higher Hartmann number), the conduction heat transfer becomes the dominate mechanism in heat transfer; this increment causes thermal dissipation in the flow of nanofluid to be enhanced. When the Rayleigh number is high and the Hartmann number is low, the streamlines and isotherms are distributed strongly in the enclosure domain and the heat is transferred due to convection.
- The Hartmann number has a more apparent effect on the average Nusselt number at $Ra=10^4$, where the buoyant flows are significantly influenced by the magnetic field
- As Ra increases the $Nu_{avr}/Nu_{avr,Ha=0}$ decreases due to effect of magnetic field to minimum values around critical Rayleigh number Ra[10^4–10^5] depending on the Hartmann number value and then increases due to the strong effect of the natural convection with respect to the effect of magnetic field.
- The addition of nanoparticles augments the heat transfer proportionally to the solid volume fraction but there is a critical Rayleigh number approx the value of $Ra = 5.10^4$ that minimises the average Nusselt number ratio regardless of the value of the solid volume fraction.

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