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State Feedback Observer Design for a Three Phase Induction Machine using Singular Value Decomposition Method of Pole Placement

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Abstract— This paper discusses the tracking of the desired poles by designing a state feedback controller and observer using Singular Value Decomposition method of pole placement for time varying systems. As the fluxes are taken as state variables, the measurement of these variables might become tedious in some cases. Hence, the state variables are fed back to realize control over the system. The accuracy of the values obtained from the controller may not be precise owing to the price, placing and disturbances induced by sensors. Hence an observer comes in handy and the characteristics for different torque conditions are observed.

Index Terms— matrix concatenate, pole placement, singular value decomposition, state feedback.

I. INTRODUCTION

The poles of a time varying controllable and observable system can be randomly assigned by State feedback [1, 2]. In order to study the poles that are moving with respect to time, Singular Value Decomposition (SVD) method [3- 5] of pole placement is carried out. An induction machine on d-q reference frame [6, 7] is taken in order to understand the theory. Matrix Concatenate is used to build a state model of an induction machine and is considered to be the faulty system. This paper presents the tracking of the poles of the faulty system to the desired system by using singular value decomposition [8- 10] method.

A. Nomenclature

ψqs, ψds = stator flux linkages of q and d axis,
ψqr, ψdr = rotor flux linkages of q and d axis,
ωe = stator angular frequency,
ωr = rotor angular speed,
ωb = base angular frequency,
Vqs, Vds = stator voltages of q and d axis,
Vqr, Vdr = rotor voltages of q and d axis,
J = moment of inertia,
P = Number of poles,
Xls = stator leakage reactance,
Xlr = rotor leakage reactance,
Te = electrical output torque,
TL = load torque

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II. MATHEMATICAL MODEL OF INDUCTION MACHINE

The general state space mathematical model of time varying MIMO system is given below

$$\dot{x}(t) = A(t)\,x(t) + B(t)\,u(t) \quad (1)$$

$$y(t) = C(t)\,x(t) + D(t)\,u(t) \quad (2)$$

Using (1) & (2), the linearized time varying state model of squirrel cage induction machine in terms of flux linkages derived from Krause's model with d-q [11, 12] reference frame is given as,

$$\begin{bmatrix} \dot{\psi}qs \\ \dot{\psi}ds \\ \dot{\psi}qr \\ \dot{\psi}dr \end{bmatrix} = \begin{bmatrix} a & -\omega e & b & 0 \\ \omega e & a & 0 & b \\ c & 0 & d & \omega r-\omega e \\ 0 & c & \omega e-\omega r & d \end{bmatrix} \begin{bmatrix} \psi qs \\ \psi ds \\ \psi qr \\ \psi dr \end{bmatrix} + \begin{bmatrix} \omega b & 0 & 0 & 0 \\ 0 & \omega b & 0 & 0 \\ 0 & 0 & \omega b & 0 \\ 0 & 0 & 0 & \omega b \end{bmatrix} \begin{bmatrix} Vqs \\ Vds \\ Vqr \\ Vdr \end{bmatrix} \quad (3)$$

From the above state equation (3), four elements of the system matrix depends on ωe, ωr are time varying and the other elements (a, b, c, d) are assumed as constants. The state variables are the flux linkages (ψqs, ψds, ψqr, ψdr).

$$Te = \frac{3}{4}\frac{p}{\omega b}(\psi ds\ iqs - \psi qs\ ids) \quad (4)$$

$$iqs = \frac{1}{xls}(\psi qs - \psi mq) \quad (5)$$

$$ids = \frac{1}{xls}(\psi ds - \psi md) \quad (6)$$

$$\psi mq = xml\left(\frac{\psi qs}{xls} + \frac{\psi qr}{xlr}\right) \quad (7)$$

$$\psi md = xml\left(\frac{\psi ds}{xls} + \frac{\psi dr}{xlr}\right) \quad (8)$$

$$Te - Tl = \frac{2J}{p}\frac{d\omega r}{dt} \quad (9)$$

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$$a = \frac{\omega b\, Rs}{Xls}\left(\frac{Xml}{Xls} - 1\right) \quad (10)$$

$$b = \frac{Rs\, Xml}{Xls\, Xlr} . \omega b \quad (11)$$

$$c = \frac{Rr\, Xml}{Xls\, Xlr} . \omega b \quad (12)$$

$$d = \frac{\omega b\, Rr}{Xlr}\left(\frac{Xml}{Xlr} - 1\right) \quad (13)$$

$$X_{ml} = {}^{1}\Big/\left(\frac{1}{Xm} + \frac{1}{Xls} + \frac{1}{Xlr}\right) \quad (14)$$

A 30kW induction machine of 220V, 60 Hz three phase ac supply is taken for consideration. For the same, two cases of load torque conditions are considered namely zero load torque and varying load torque.

III. Direct AC Start-Up of Induction Machine for Zero Load Torque

The output of the induction machine is given in fig.1 where it is seen that the torque oscillates for a time of 0.1 sec and reaches a steady state of zero. Let this be the reference system.

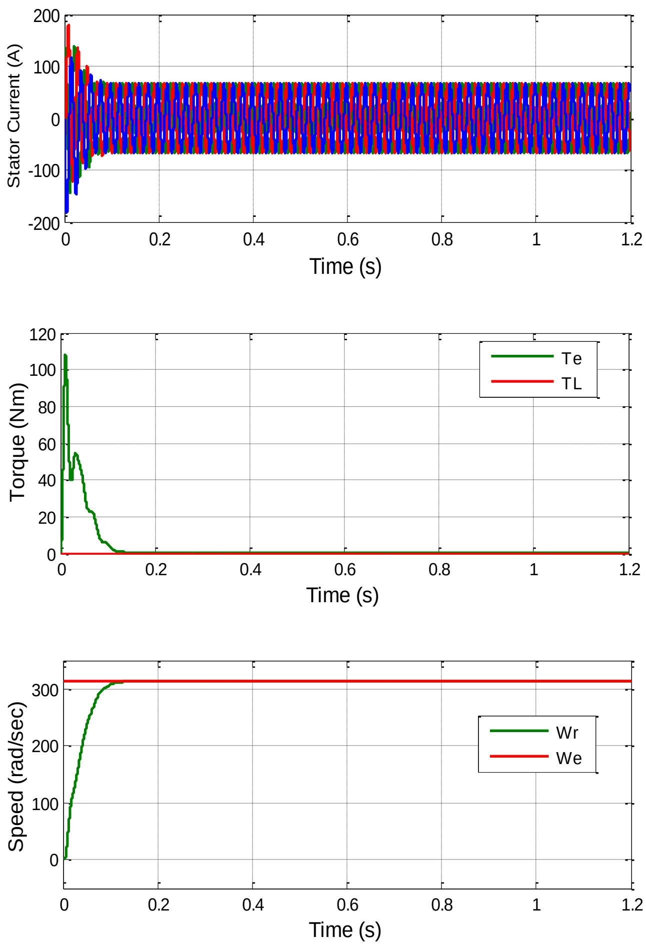


Fig. 1 Outputs of Three phase induction machine during direct startup

The fig.2 shows the elements of system matrix A in which, the variables depending on rotor speed are varying and the others are constants. The time varying desired poles of induction machine for zero load torque is shown in fig.3 where the poles' magnitude vary from 300 to 100 and 300 to 200 for the other pair.

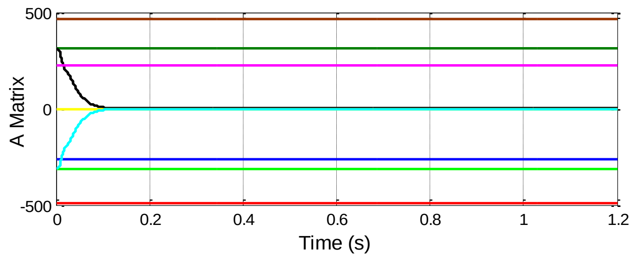


Fig. 2 Elements of system matrix A

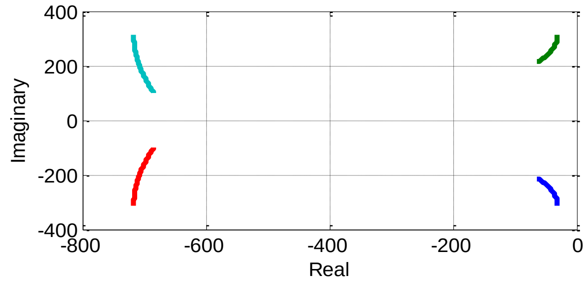


Fig. 3 Locus of Eigen values of system matrix A

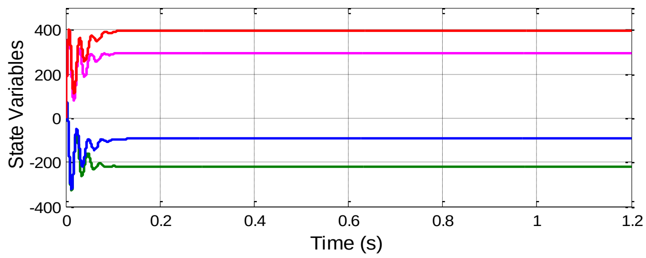
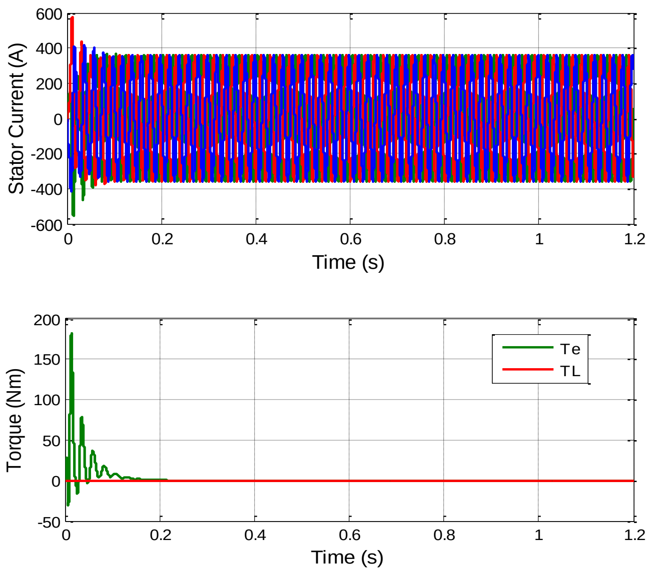


Fig. 4 State variables of reference model

IV. Direct State Model of Induction Machine for Zero Load Torque

With the help of design equations (1) and (2), the state model of induction machine for a faulty system can be modeled by using matrix concatenate in matlab. The resulting output thus obtained is shown in fig.5.



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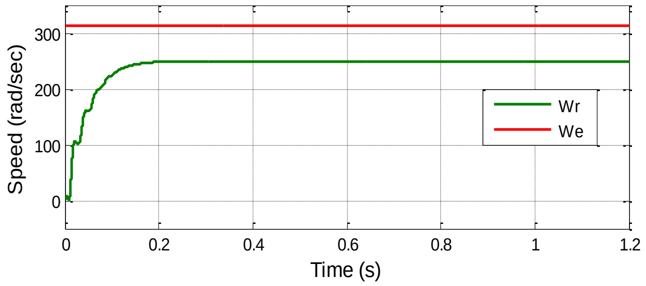


Fig. 5 Outputs of the Induction machine using faulty system

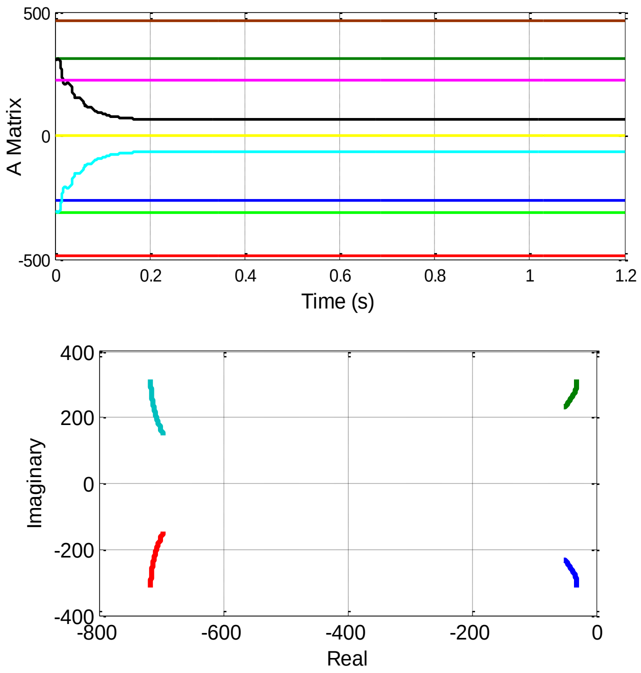
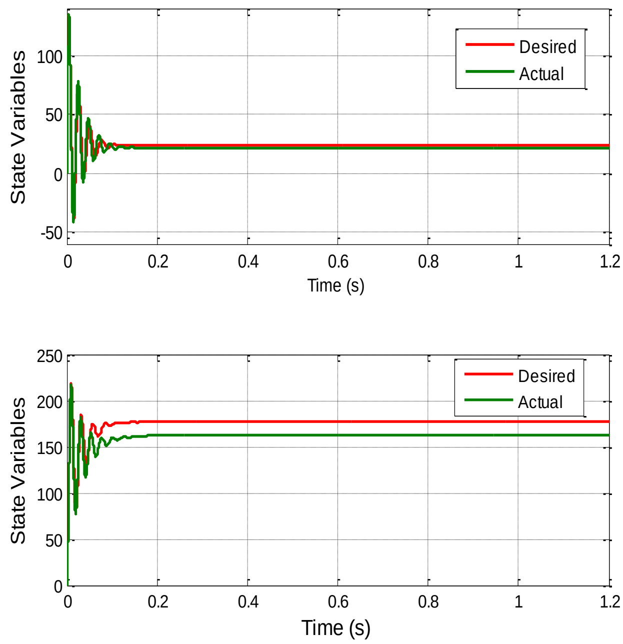


Fig. 6 Elements of System matrix A and Locus of Eigen Values

The extensive variations in the current, torque and speed is due to the differences prevailing between the state variables of faulty system and the reference model as shown in fig.7.



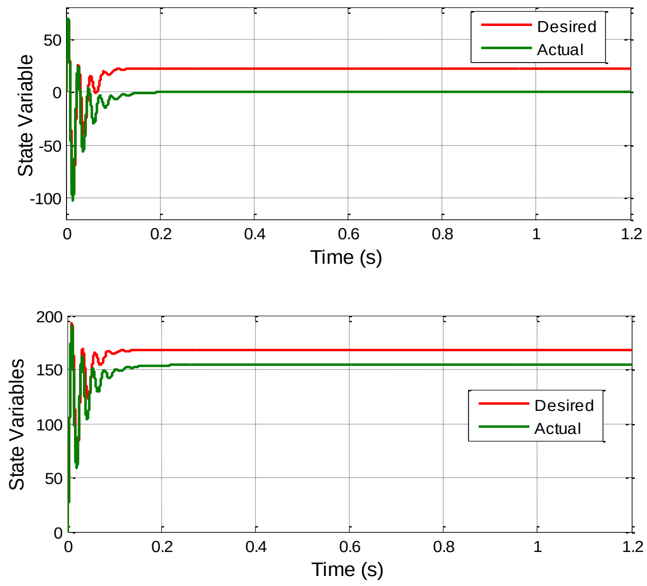


Fig. 7 State variable comparison between reference model and faulty system

V. SINGULAR VALUE DECOMPOSITION BASED STATE FEEDBACK CONTROLLER FOR ZERO LOAD TORQUE

In order to evade the difficulties involved in shaping the preliminary conditions and removing the noise prevailing in the system, we go for State Feedback [13, 14]. Full state feedback is a method employed in feedback control system to place the closed loop poles [15, 16] of a plant in predetermined locations in the s- plane. Initially, the state feedback gain matrix 'K' is found. The resulting poles from the 'K' matrix are checked whether it matches with the reference system by finding Eigen values [17, 18] of $|A\text{-}BK|$. Generally, the desired poles are constant whereas in this system the desired poles which are the Eigen values of matrix A are also varying with time. Singular Value Decomposition (SVD) is one of the pole placement [19, 20] approaches used to calculate the state feedback gain matrix 'K'. As it is a state feedback controller, it is assumed that all state variables are available for measurement. The steps involved in determining the gain matrix are:

- The matrix $S_i = [\lambda_i\text{-}A\ B]$ is formed for all Eigen values. The MATLAB function *svd* is used to implement the SVD method.
- The syntax for this function is,
  [u, sig, V] = svd(S)
- The Gain Matrix can be found by the relation

$$K_{svd} = Q_{svd}\, M_{svd}^{-1}$$

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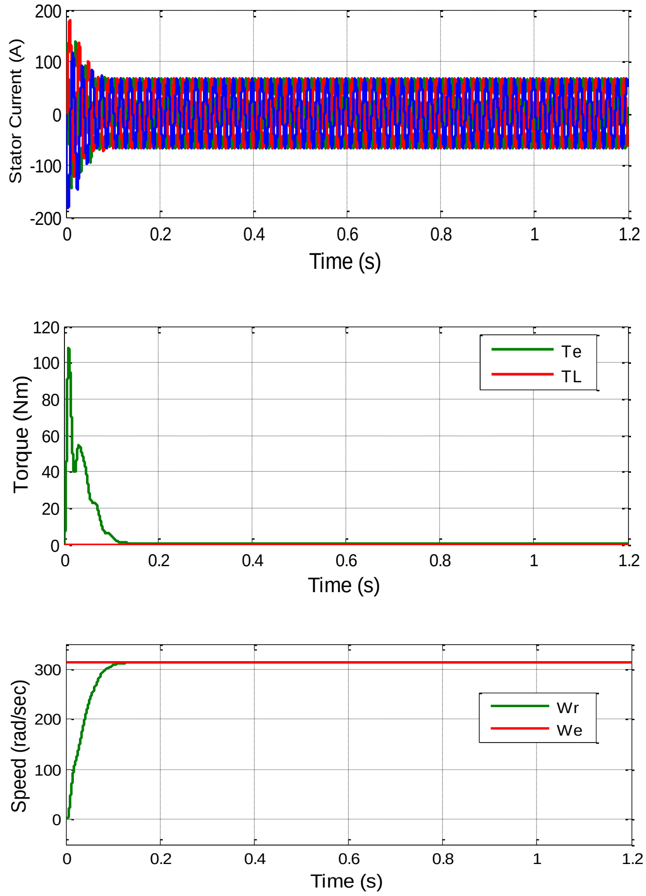

Fig. 8 Outputs based on state feedback controller by singular value decomposition

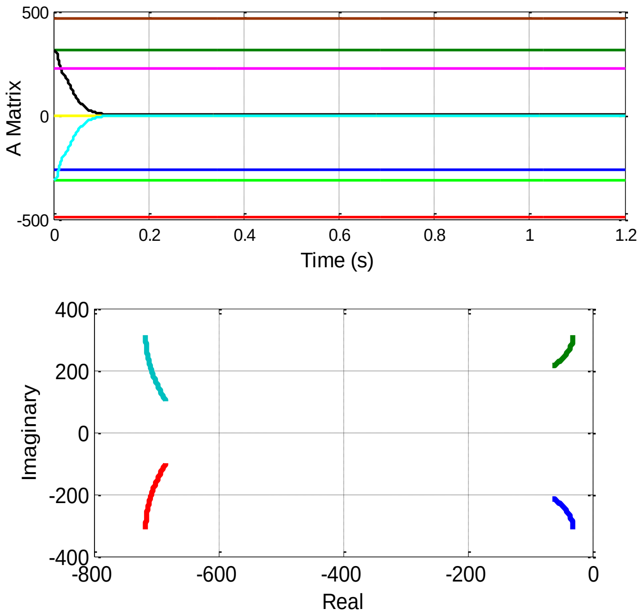

Fig. 9 Elements of System matrix A and Locus of Eigen Values

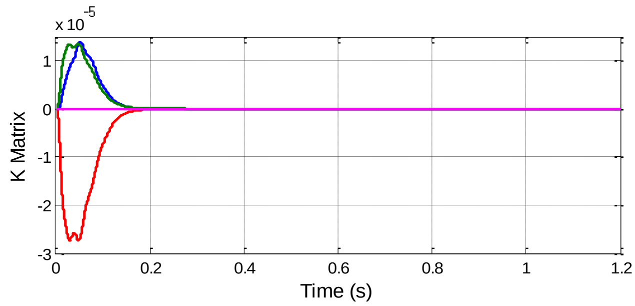

Fig. 10 Controller Gain Matrix, K

All the elements of matrix A and the locus of the Eigen values also trace the reference model as in fig.9. There is no difference between the reference state variables and that obtained with state feedback model as seen in fig.11. This is due to the State Feedback Gain Matrix ‘K’ shown in fig.10.

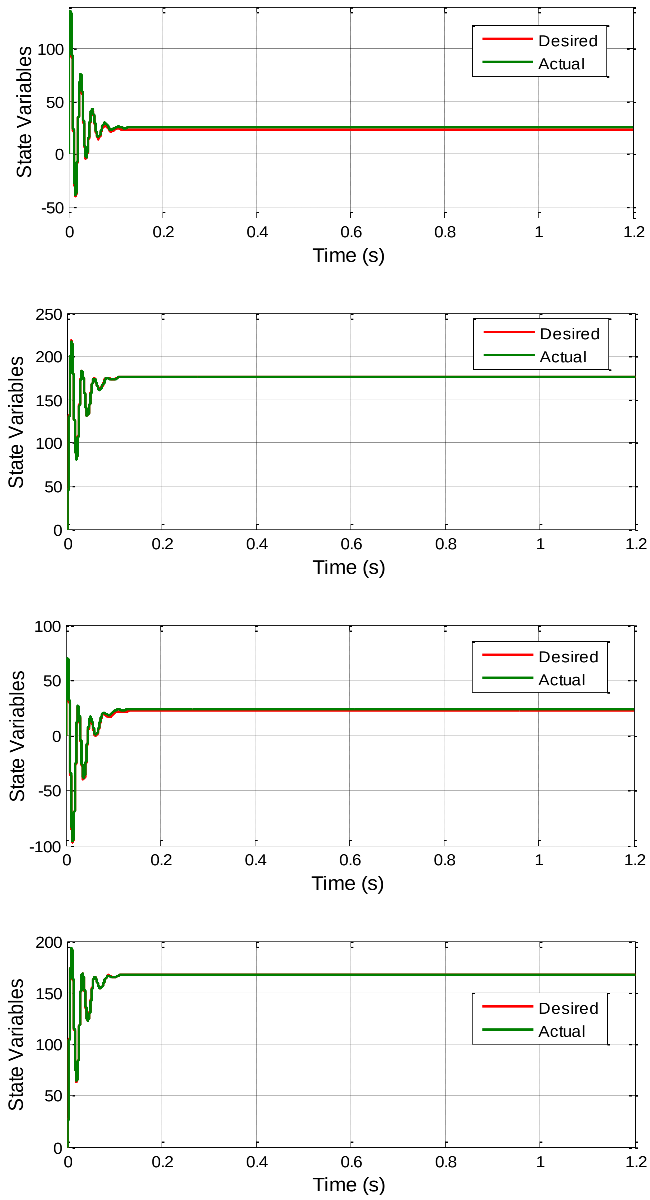

Fig. 11 State variable comparison between reference model and controller model

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VI. Singular Value Decomposition Based State Observer For Zero Load Torque

The state variables which cannot be measured by state feedback controller can be calculated from the output through state observers [19, 20]. Here, the observer gain matrix 'Ke' is found in order to make the system to function like the reference system using Singular Value Decomposition Method of Pole Placement. The corresponding results obtained are shown in fig.12.

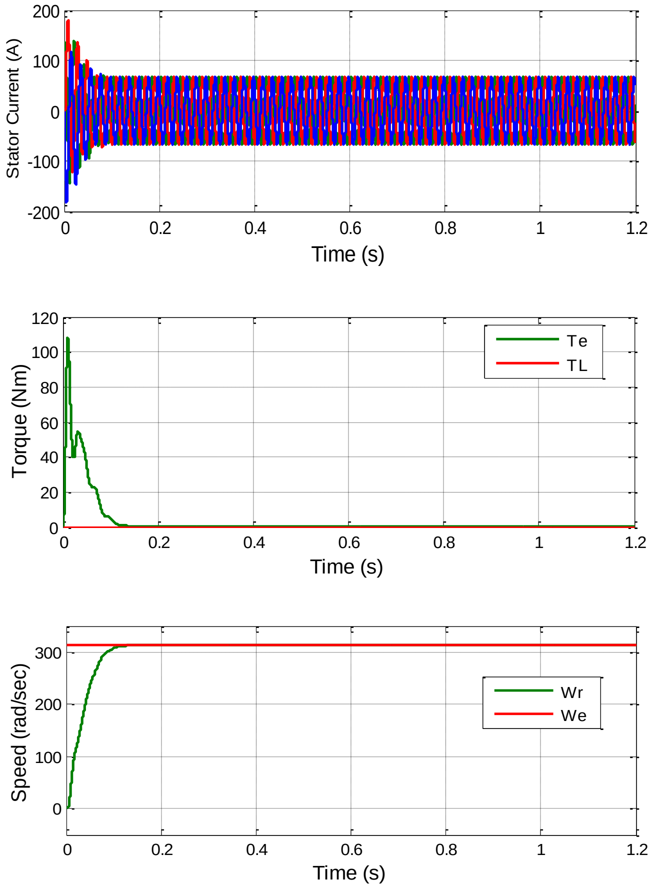


Fig. 12 Outputs based on state feedback observer by singular value decomposition

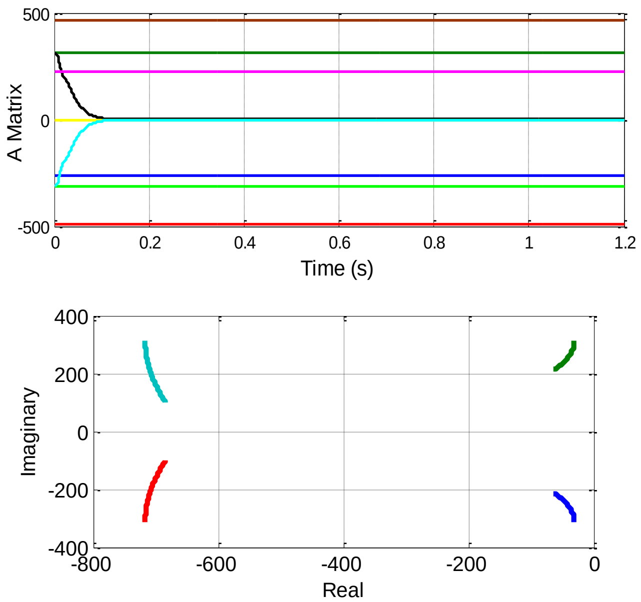


Fig. 13 Elements of System matrix A and Locus of Eigen Values

It is observed that there is no deviation of the time varying elements of A matrix from the reference system as seen in fig.13. Also, from fig.15, it is seen that, there is very less or no variation in the state variable comparison. This is due to the state observer gain matrix 'Ke' as seen in fig.14.

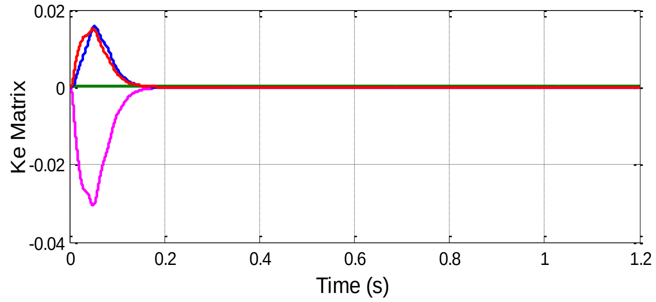
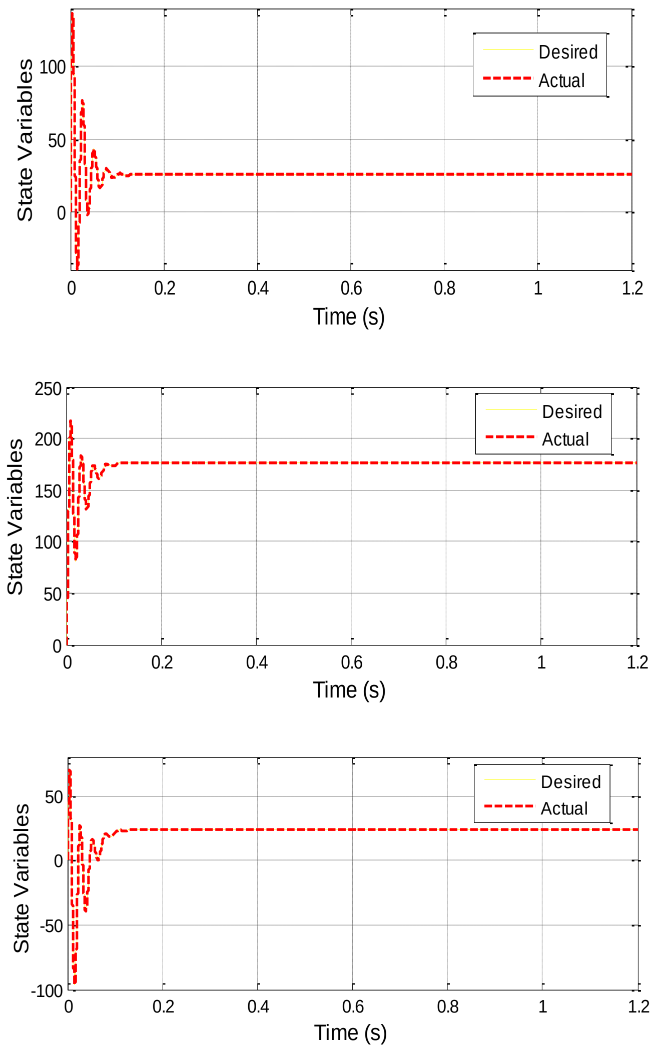


Fig. 14 Observer Gain Matrix, Ke



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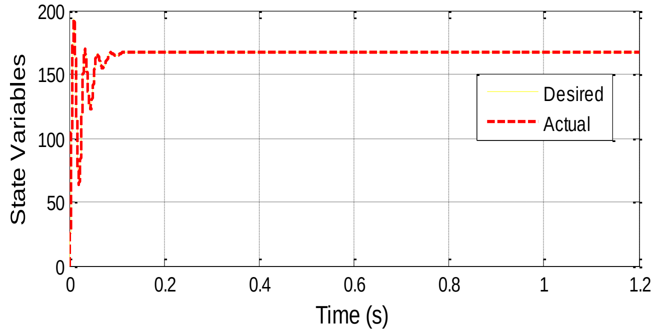

Fig. 15 State variable comparison between reference model and observer model

VII. DIRECT AC START-UP OF INDUCTION MACHINE FOR VARYING LOAD TORQUE

The same system can be executed for varying load torque where it is seen from fig.16 that, as TL is kept zero between 0-0.4 secs, the speed slowly increases to synchronous speed. Likewise, as TL is increased positively between 0.4-0.8 secs, the speed of rotor falls below synchronous speed or runs at sub-synchronous speed while the vice-versa or super-synchronous operation happens when TL is decreased to the negative axis between 0.8-1.2 secs and operates as a generator.

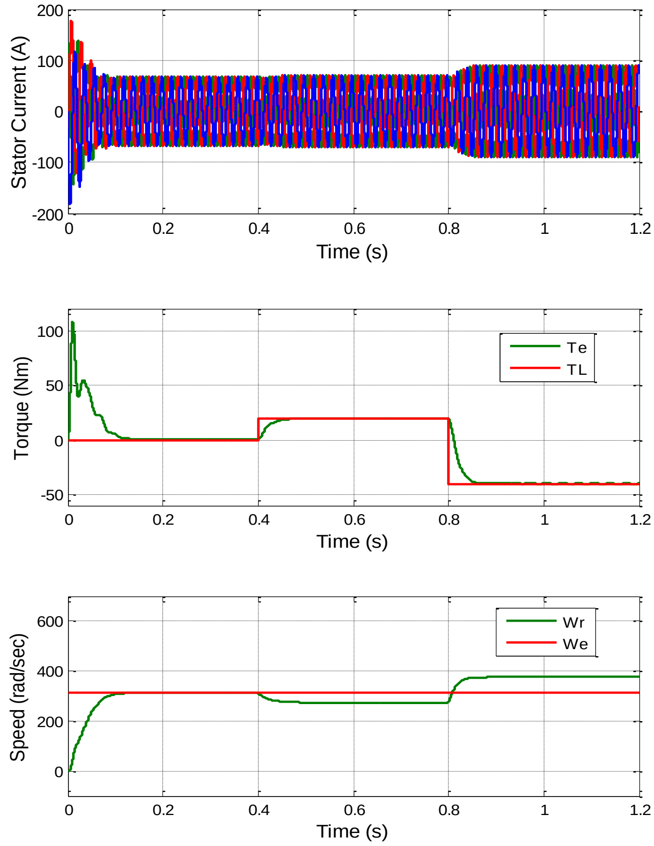

Fig. 17 Output of three phase induction machine during direct startup

The fig.18 shows the elements of system matrix A in which, the variables depending on rotor speed are varying and the others are constants. The time varying desired poles of induction machine for varying load torque is shown in fig.18 where the poles' magnitude vary from 300 to 50 and 300 to 200 for the other pair.

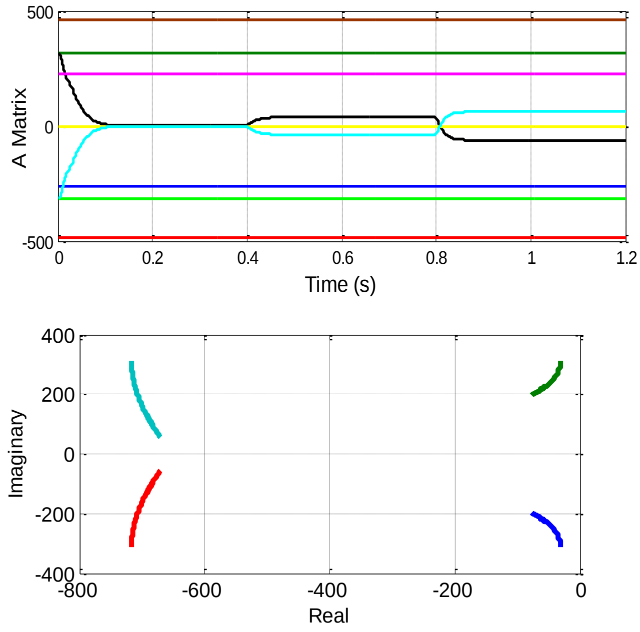

Fig. 18 Elements of system matrix A and Locus of Eigen Values

The state variables which are the fluxes are obtained as in fig.19.

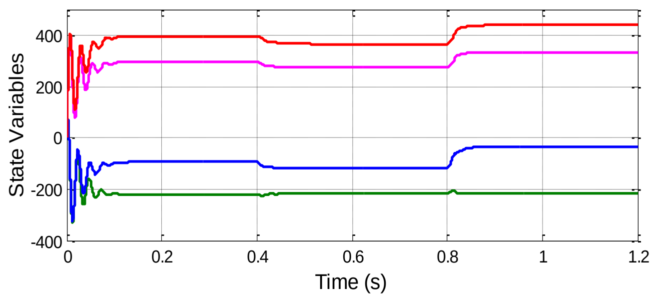

Fig. 19 State variables of the reference model

VIII. DIRECT STATE MODEL OF INDUCTION MACHINE FOR VARYING LOAD TORQUE

From the results obtained from faulty system, it is seen from fig.20 that, the rotor speed instead of reaching synchronous speed, it operates in sub-synchronous speed during 0-0.2 sec. Similar changes are observed for the remaining time period where the rotor does not track the original reference speed. The magnitude of stator current is also large.

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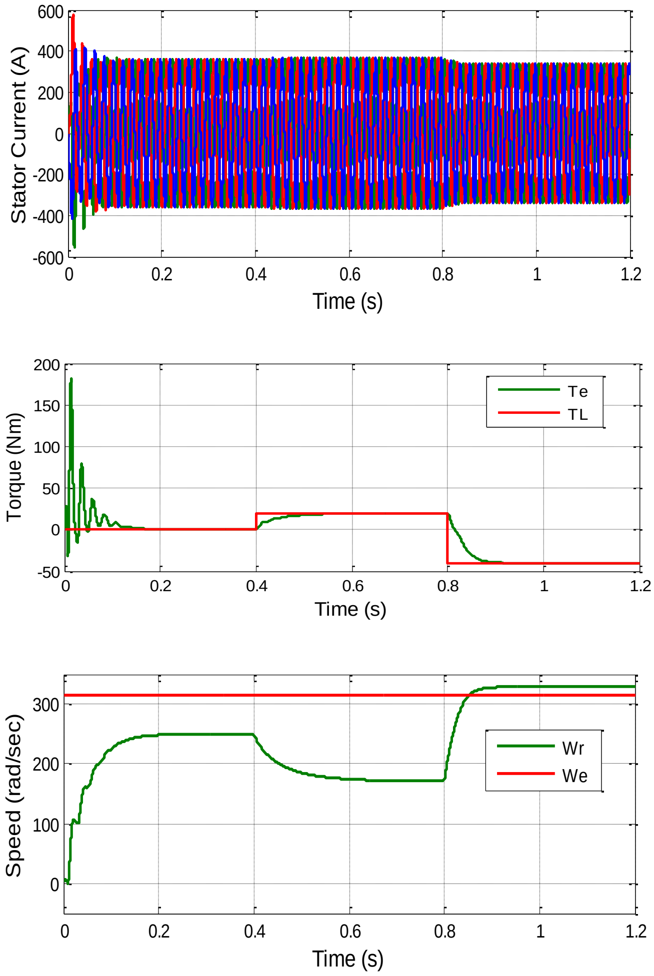
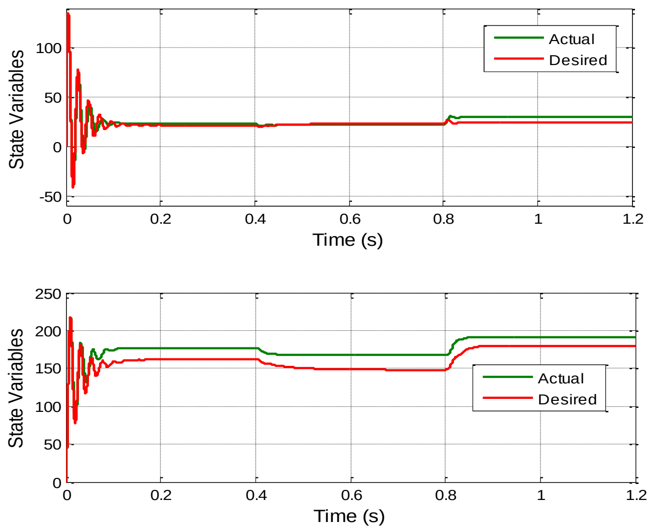


Fig. 20 Outputs of three phase induction machine using faulty system

The reason for these deviations is due to the variations prevailing between the state variables with respect to the reference state variables which are shown in fig.21.



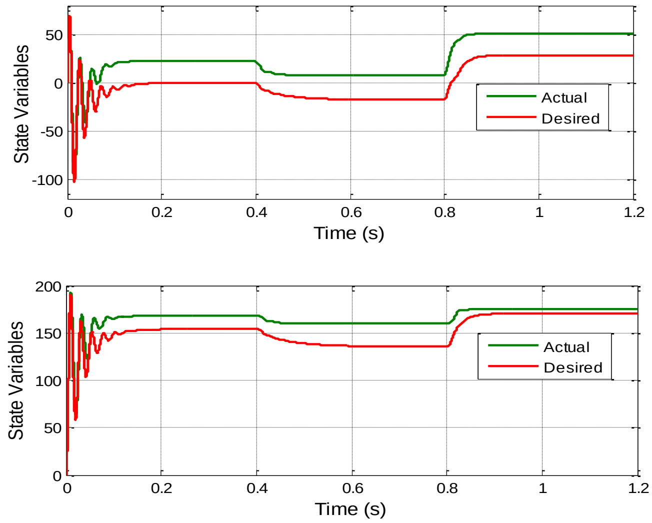


Fig. 21 State variable comparison between reference model and faulty system

IX. SINGULAR VALUE DECOMPOSITION BASED STATE CONTROLLER FOR VARYING LOAD TORQUE

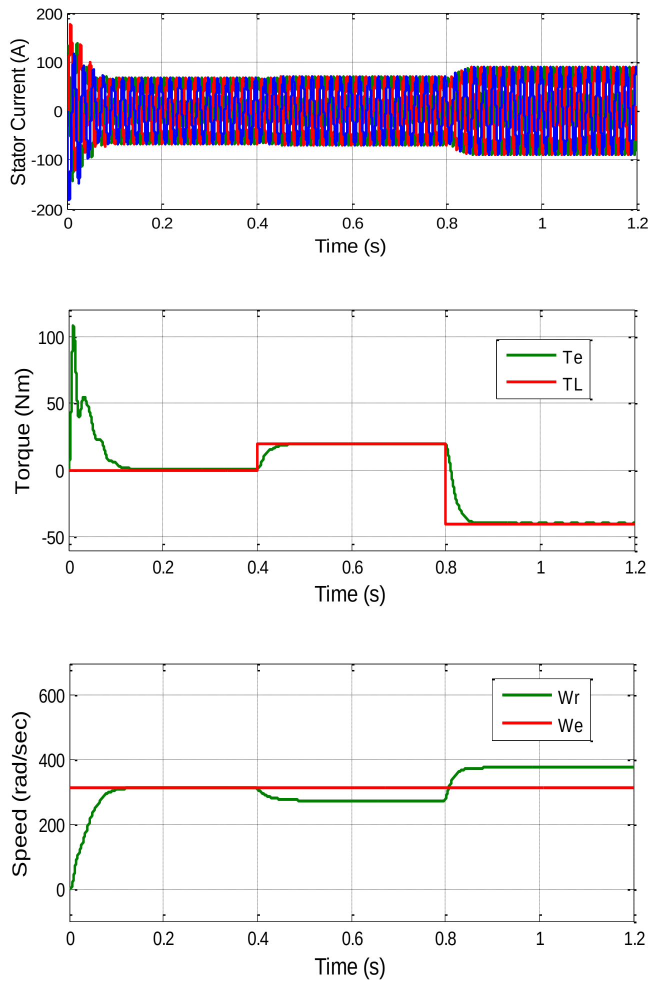


Fig. 23 Outputs based on state feedback controller by singular value decomposition

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The fig.23 clearly states that, the state feedback controller system model follows the reference system exactly as seen from the results of the current, torque and speed.

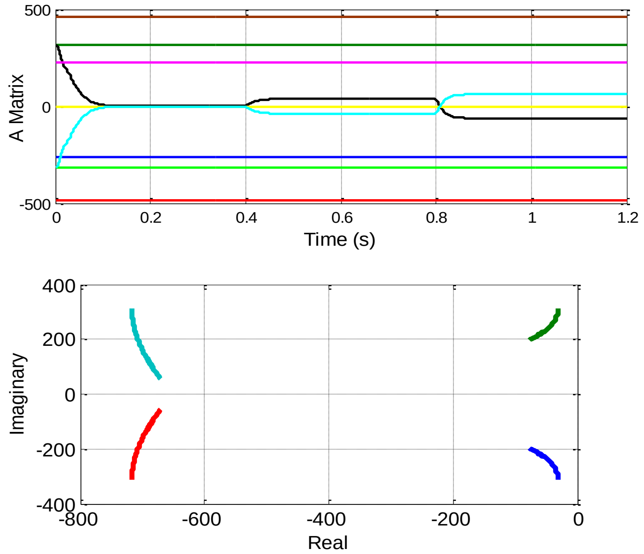


Fig. 24 Elements of system matrix A and Locus of Eigen Values

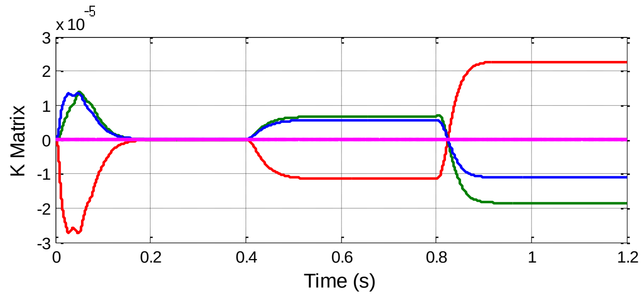
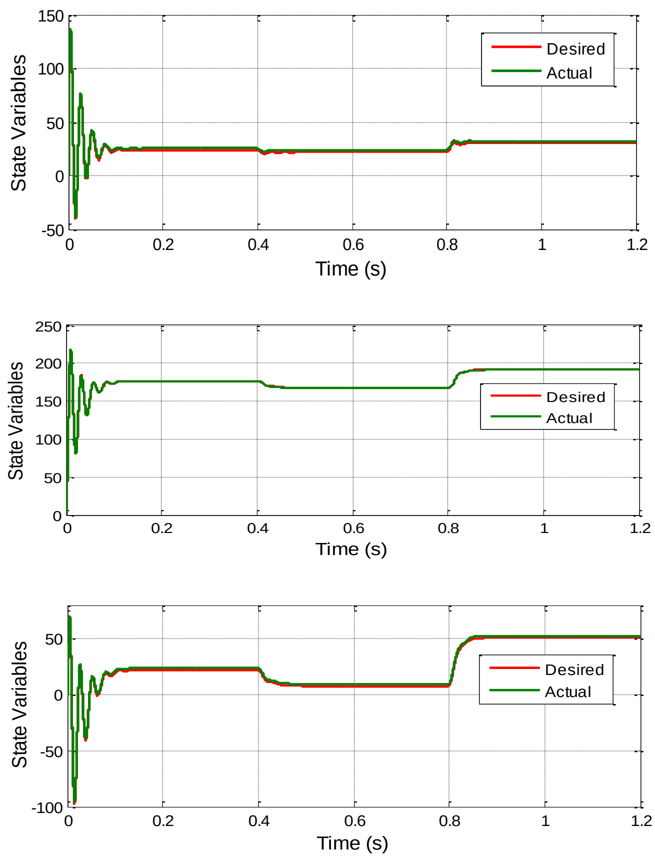


Fig. 25 Controller gain Matrix, K



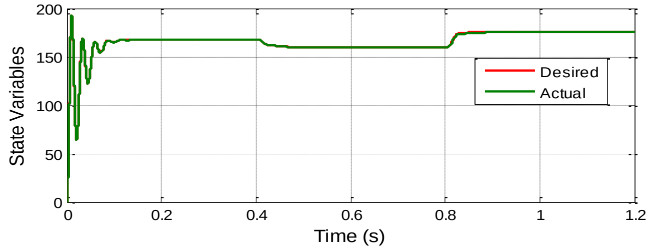


Fig. 26 State variable comparison between reference model and controller model

From the observations seen in fig.26, all the state variables are forced to follow the desired value which in other words can be said that the faulty system is made to trace the reference system. This is achieved by the state feedback gain matrix ‘K’ seen in fig.24.

X. Singular Value Decomposition Based State Observer For Varying Load Torque

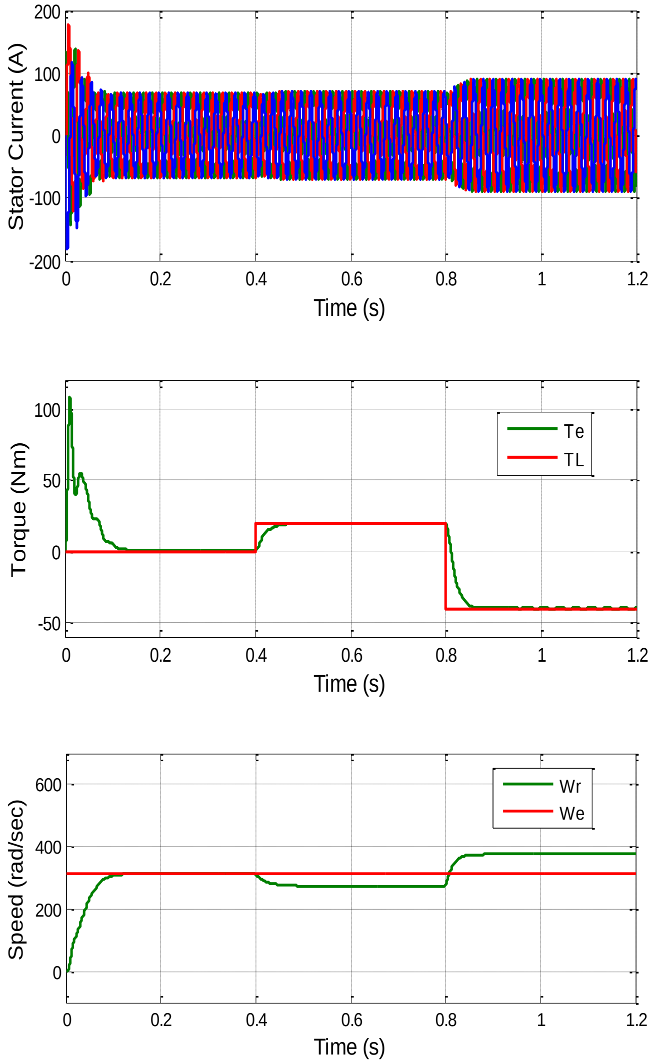


Fig. 27 Outputs based on state feedback observer by singular value decomposition

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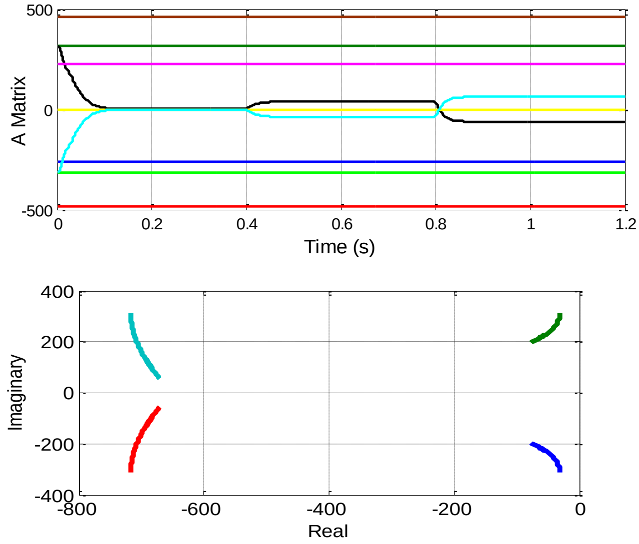


Fig. 28 Elements of system matrix A and Locus of Eigen Values

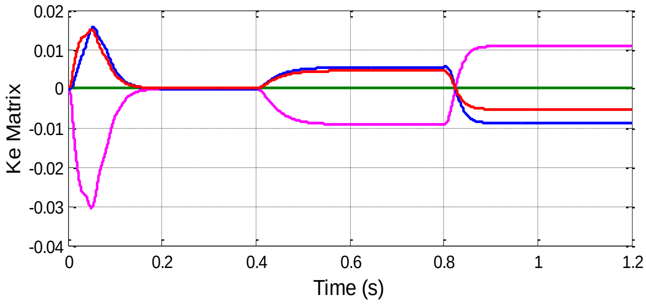
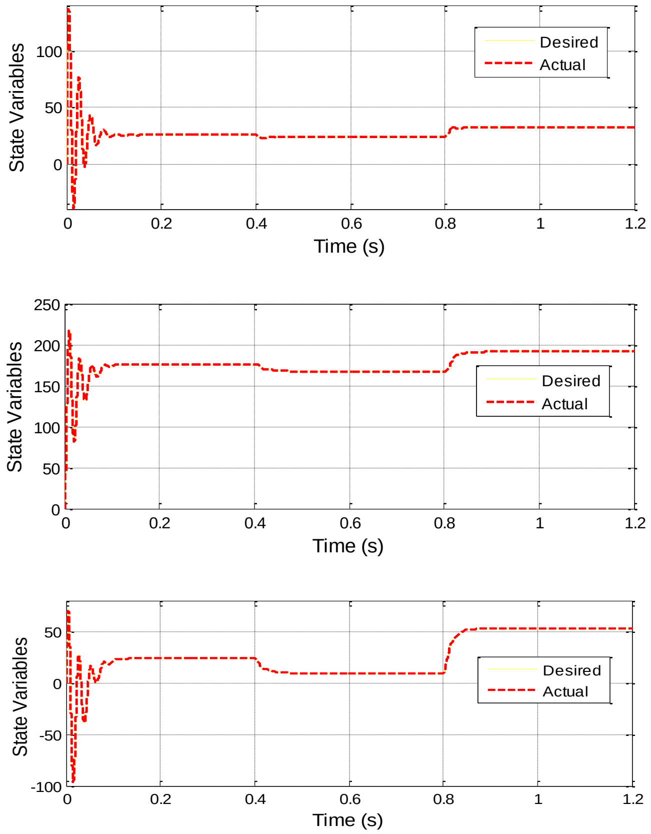


Fig. 29 Observer gain Matrix, Ke



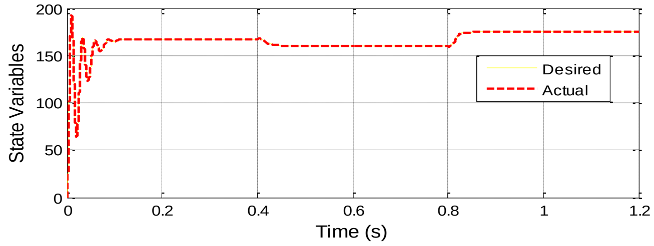


Fig. 30 State variable comparison between reference model and observer model

XI. CONCLUSION

The results discussed above shows that the given system is made to follow the reference system by using Singular Value Decomposition method using state feedback controller and observer for both zero load torque and varying load torque conditions.

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