Output Feedback Robust Stabilization of the Decoupled Multiple Model

Ahmed Tahar, Mohamed Naceur Abdelkrim

Abstract— This paper aims to design a controller to robustly stabilize uncertain nonlinear systems with norm bounded uncertainties and unmeasured state variables via decoupled multi-model. The stabilization conditions are given in the form of linear matrix inequalities. Sufficient conditions are derived for robust stabilization in the sense of Lyapunov asymptotic stability and are formulated in the format of linear matrix inequalities (LMIs). The effectiveness of the proposed decoupled multi-model controller and multi-observer design methodology is finally demonstrated through numerical simulations.

Index Terms—Decoupled multiple model, LMI, Multi-observer, robust control.

I. INTRODUCTION

The control of dynamic systems in the presence of severe nonlinearities and uncertainties is of great attention for several researchers. In order to overcome this kind of problems, it is necessary to develop an intelligent modelling and control approaches. Indeed, multiple model and multi-control approaches are considered to be useful for industrial processes which are, often, complex, uncertain, ill-define, and have available qualitative knowledge from domain experts for their controller design. There have been many successful applications in the industry to date ([1]–[3], [19], [20], [23]). Two basic structures of multiple model can be illustrated for aggregating the local models between them ([4], [5], [22]). In the first structure, the local models have a common state vector (Takagi-Sugeno multiple model); in the second one, the local models are decoupled and their state vectors are different (decoupled multiple model). The decoupled state multiple model has been unfortunately poorly studied in the literature. However, it represents an increasing relevance to Takagi-Sugeno multiple model. Indeed, the usefulness of this multiple model has been clearly employed for the control and the modelling of nonlinear systems ([4], [6], [7], [21]). More recently, the state estimation problem has been considered in [6] and [7]. The main aspect of the decoupled multiple model is that submodels of different order (e.g. number of states) can be used.

This fact introduces some adaptability degrees in the modelling step because the dimensions of the submodels can be well adapted to the complexity of the system inside each operating zone and consequently the total number of parameters necessary for describing the system can be reduced. As well as stability, robustness is another performance to be examined in the study of uncertain nonlinear control systems. The parametric uncertainty is a principal cause responsible for the degraded stability and the performance of an uncertain nonlinear control system. In fact, in many cases it is very difficult, if not impossible, to obtain the correct values of some system parameters. This is due to the imprecise measurement, unmeasurable system parameters, or on-line variation of the parameters. Therefore, this has improved some active research in the last few years ([8], [9]). Robustness in multiple model-based control has been largely studied in the past, such as the stability robustness against modelling errors ([10], [11]), some control techniques for T-S multiple models ([12], [13]). Motivated by the aforementioned concerns, this paper deals with the parametric uncertainties issue in a nonlinear system with decoupled multiple model. It is well recognized that the observer design is a very important complication in control systems. Since in many practical nonlinear control systems, state variables are often inaccessible, output feedback or observer-based control is required and has caused some interest. Tanaka et al. [14], Ma et al. [15] and Jun Yoneyama et al. [16] studied fuzzy observer designs for T-S fuzzy control systems, and they proved that a state feedback controller and an observer always yields a stabilizing output feedback controller provided that the stabilizing property of the control and asymptotic convergence of the observer are guaranteed by the Lyapunov method. However, in the above output feedback fuzzy controllers, the parametric uncertainties for T-S fuzzy control system were not considered. So the robustness of the closed-loop system cannot be guaranteed. Tong et al. [12] studied the robust fuzzy control problem for nonlinear systems in the presence of parametric uncertainties and the state variables unavailable for measurement. The Takagi-Sugeno (T-S) fuzzy model system with parametric uncertainties is adopted for modelling the nonlinear system and establishing fuzzy state observer. This paper is dedicated to the design of an output feedback robust control strategy for nonlinear systems described by decoupled multi-model. Some sufficient conditions in the LMI format and systematic design procedures for both controller and observer designs for general nonlinear systems with parametric uncertainties are proposed. The stability conditions for nonlinear systems with parametric uncertainties given in [12] are extended to nonlinear systems represented by decoupled multi-model, which are formulated in the LMI format.

The paper is organized as follows: Section 2 reviews

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decoupled multi-model and decoupled observers. The output feedback controller design for robust stabilization of decoupled multi-model systems with parametric uncertainties are presented in Section 3. Section 4 shows a design example and simulation results. Finally, conclusion is given in Section 5.

II. Decoupled Multiple Model and Multi-Observer

The multiple model strategy is based on the basic idea that complex dynamic behaviours can be accurately represented with the help of an interpolation of simple submodels. In this paper, heterogeneous multiple model will be employed [17]. The state space representation of this multiple model is:

$$\begin{aligned} &\dot{x}_i(t) = A_i x_i(t) + B_i u(t), \\ &y_i(t) = C_i x_i(t), \\ &y(t) = \sum_{i=1}^{L} \mu_i(\xi(t)) y_i(t), \end{aligned} \quad (1)$$

Where L is the number of the submodels, $x_i \in \mathbb{R}^{n_i}$ and $y_i \in \mathbb{R}^{p}$ are respectively the state vector and the output of the i[th] submodel; $u \in \mathbb{R}^{m}$ is the input and $y \in \mathbb{R}^{p}$ the measured output. The matrices $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m}$, $C_i \in \mathbb{R}^{p \times n_i}$ are known and appropriately dimensioned.

The complete partition of the operating space of the system is performed using a decision variable $\xi(t)$ that is assumed to be known and real-time available (e.g. the inputs and/or exogenous signals). Notice that the contribution of each submodel is quantified by the weighting functions $\mu_i(\xi(t))$ that satisfy the following convex sum constraints:

$$\sum_{i=1}^{q} \mu_i(\xi(t)) = 1 \text{ and } 0 \le \mu_i(\xi(t)) \le 1, \quad \forall i = 1 \dots q, \ \forall t \quad (2)$$

Let us notice that in this multiple model no blend between the parameters of the submodels is performed. Indeed, the submodel contribution is taken into account via a weighted sum between the submodel outputs and consequently the submodels do not share the same state space. Thanks to this fact, the dynamics of the submodels are completely decoupled and consequently the dimension of the state vector x_i of each submodel can be different (of course the output vectors dimensions must be identical). Therefore, this structure is well adapted for modelling strongly nonlinear systems whose structure varies with the operating regime, for example when the complexity of the dynamic behaviour is not uniform in the operating range.

Remark 1: The outputs $y_i(t)$ of each submodel must be considered as intermediary modelling signals only used in order to provide a representation of the nonlinear system. Hence, they cannot be employed for driving an observer because they are not physically available and consequently no measurement is possible. Only the global output $y(t)$ of the multiple model can be used for this purpose. In this paper, an uncertain nonlinear system described by a decoupled multiple model is considered. The state space representation of this multiple model is given by:

$$\begin{cases} \dot{x}_i(t) = (A_i + \Delta A_i) x_i(t) + (B_i + \Delta B_i) u(t) \\ y_i(t) = C_i x_i(t) \\ y(t) = \sum_{i=1}^{q} \mu_i(\xi(t)) y_i(t) \end{cases} \quad (3)$$

The parametric uncertainties in the system are represented by matrices ΔA_i and ΔB_i

Assumption 1. The parameter uncertainties considered here are norm-bounded, in the form:

$$[\Delta A_i, \Delta B_i] = D_i F_i(t) [E_{1i}, E_{2i}]$$

$$F_i^T(t) F_i(t) \le I$$

Where D_i ,E_{1i} and E_{2i} are known real constant matrices of appropriate dimension, and $F_i(t)$ is an unknown matrix function with Lebesgue-measurable elements, I is the identity matrix of appropriate dimension.

A. *Augmented form of the decoupled multiple model*

Consider the following augmented state vector:

$$x(t) = \begin{bmatrix} x_1^T & \cdots & x_i^T & \cdots & x_q^T \end{bmatrix}^T \in \mathbb{R}^n, \ n = \sum_{i=1}^{q} n_i$$

The decoupled multiple model (3) may be rewritten in the following compact form:

$$\begin{aligned} &\dot{x}(t) = (\tilde{A} + \Delta\tilde{A}) x(t) + (\tilde{B} + \Delta\tilde{B}) u(t) \\ &y(t) = \tilde{C}(t) x(t) \end{aligned} \quad (4)$$

Where

$$\tilde{A} = diag\{A_1 \ \cdots \ A_i \ \cdots \ A_q\} , \quad (5)$$

$$\tilde{B} = \begin{bmatrix} B_1^T & \cdots & B_i^T & \cdots & B_q^T \end{bmatrix}^T , \quad (6)$$

$$\tilde{C}(t) = \begin{bmatrix} \mu_1(t) C_1 & \cdots & \mu_i(t) C_i & \cdots & \mu_L(t) C_q \end{bmatrix} \quad (7)$$

$$\Delta\tilde{A}(t) = \tilde{D}\tilde{F}(t)\tilde{E}_1 \quad (8)$$

$$\Delta\tilde{B}(t) = \tilde{D}\tilde{F}(t)\tilde{E}_2 \quad (9)$$

$$\tilde{D} = diag\{D_1 \ \cdots \ D_i \ \cdots \ D_q\} \quad (10)$$

$$\tilde{E}_1 = diag\{E_{11} \ \cdots \ E_{1i} \ \cdots \ E_{1q}\} \quad (11)$$

$$\tilde{E}_2 = \begin{bmatrix} E_{21}^T & \cdots & E_{2i}^T & \cdots & E_{2q}^T \end{bmatrix}^T \quad (12)$$

$$\tilde{F} = diag\{F_1(t) \ \cdots \ F_i(t) \ \cdots \ F_q(t)\} \quad (13)$$

Remark 2. The matrix $\tilde{C}(t)$ can be rewritten as a weighted sum of matrices as follows:

$$\tilde{C}(t) = \sum_{i=1}^{q} \mu_i(t) \tilde{C}_i , \quad (14)$$

Where $\tilde{C}_i$ is a constant block matrix given by:

$$\tilde{C}_i = [0 \ \cdots \ C_i \ \cdots \ 0] \quad (15)$$

Such that the term C_i is found on the ith block column of $\tilde{C}_i$

B. *Decoupled multi-observer*

The proposed observer based on the decoupled multiple model has the following form:

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$$\begin{cases} \dot{\hat{x}}_i(t) = A_i \hat{x}_i(t) + B_i u(t) + K_i\left(y(t) - \hat{y}(t)\right) \\ \hat{y}_i(t) = C_i \hat{x}_i(t) \\ \hat{y}(t) = \sum_{i=1}^{q} \mu_i(\xi(t)) \hat{y}_i(t) \end{cases} \quad (16)$$

Where $K_i \in \mathbb{R}^{n_i \times p}$ is the gain associated to the i[th] observer.

The observer may be rewritten in the following compact form:

$$\begin{cases} \dot{\hat{x}}(t) = \tilde{A}\hat{x}(t) + \tilde{B}u(t) + \tilde{K}\left(y(t) - \hat{y}(t)\right) \\ \hat{y}(t) = \tilde{C}(t)\hat{x}(t) = \sum_{i=1}^{q} \mu_i(\xi(t)) C_i \hat{x}(t) \end{cases} \quad (17)$$

Where $\hat{x}(t) = \left[\hat{x}_1^T(t) \cdots \hat{x}_i^T(t) \cdots \hat{x}_q^T(t)\right]^T$ is the state estimation and $\hat{y}(t)$ the output estimation and $\tilde{K} = \left[K_1^T \quad \cdots \quad K_i^T \quad \cdots \quad K_q^T\right]^T$ is the augmented gain of the observer to be determined such as the exponential convergence of $\hat{x}(t)$ towards $V(x(t), e(t))$ is guaranteed.

III. OUTPUT FEEDBACK ROBUST STABILIZATION OF THE DECOUPLED MULTIPLE MODEL

Define observation error as

$$e(t) = x(t) - \hat{x}(t) \quad (18)$$

The objective is to design a decoupled multimodel output feedback controller based on the observer for robust stabilization of system (3) in the form

$$u_i(t) = -L_i \hat{x}_i(t) \quad (19)$$

$$u(t) = \sum_{i=1}^{q} \mu_i(\xi(t)) u_i(t) = -\sum_{i=1}^{q} \mu_i(\xi(t)) L_i \hat{x}_i(t) = -\sum_{i=1}^{q} \mu_i(\xi(t)) L_i \hat{x}(t) = -\tilde{L}(t)\hat{x}(t) \quad (20)$$

$$\tilde{L}(t) = \left[\mu_1(t) L_1^T \quad \cdots \quad \mu_i(t) L_i^T \quad \cdots \quad \mu_q(t) L_q^T\right]^T \quad (21)$$

Remark 3. The time-varying matrix $\tilde{L}(t)$ can be also rewritten, using the weighting functions properties, as the following weighted sum of constants matrices:

$$\tilde{L}(t) = \sum_{i=1}^{q} \mu_i(t) \tilde{L}_i \quad (22)$$

Where $\tilde{L}_i$ is a constant block matrix given by:

$$\tilde{L}_i = [0 \quad \cdots \quad L_i \quad \cdots \quad 0], \quad (23)$$

From systems (3)-(18) and (22), we have

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(\xi(t)) \begin{bmatrix} \left(\tilde{A} + \Delta\tilde{A}(t) - \left(\tilde{B} + \Delta\tilde{B}(t)\right)\tilde{L}_i\right) x(t) \\ + \left(\tilde{B} + \Delta\tilde{B}(t)\right)\tilde{L}_i e(t) \end{bmatrix} \quad (24)$$

$$\dot{\hat{x}}(t) = \sum_{i=1}^{q} \mu_i(\xi(t)) \left[\left(\tilde{A} - \tilde{B}\tilde{L}_i\right)\hat{x}(t) + \tilde{K}\tilde{C}_i e(t)\right] \quad (25)$$

$$\dot{e}(t) = \sum_{i=1}^{q} \mu_i(\xi(t)) \begin{bmatrix} \left(\tilde{A} + \Delta\tilde{B}(t)\tilde{L}_i - \tilde{K}\tilde{C}_i\right) e(t) \\ + \left(\Delta\tilde{A}(t) - \Delta\tilde{B}(t)\tilde{L}_i\right) x(t) \end{bmatrix} \quad (26)$$

The main result for the global asymptotic stability of decoupled multimodel, with parametric uncertainties and unavailable state variables, are summarized in the following theorem:

Theorem:

If there exist symmetric and positive definite matrices $\tilde{P}_1 > 0$ and $\tilde{P}_2 > 0$, some matrices $\tilde{M}_i$ and $\tilde{N}$, and scalars ε_i, $(i = 1, \ldots, q)$, such that the following LMIs are satisfied, then the decoupled multimodel system (3) is asymptotically stabilizable via the decoupled multimodel-based output-feedback controller (20)

$$\text{a)} \begin{bmatrix} \Xi_i & * & * \\ \tilde{E}_1 Q - \tilde{E}_2 \tilde{M}_i & -\left(\varepsilon_i^{-1} + 1\right)^{-1} I & * \\ \tilde{D}^T & 0 & -\left(\varepsilon_i + 1\right)^{-1} I \end{bmatrix} < 0 \quad (27)$$

$$\text{b)} \begin{bmatrix} T_i & * & * \\ \tilde{E}_2 \tilde{L}_i & -\left(\varepsilon_i^{-1} + 1\right)^{-1} I & * \\ \tilde{D}^T \tilde{P}_2 & 0 & -\left(\varepsilon_i + 1\right)^{-1} I \end{bmatrix} < 0 \quad (28)$$

where

$$\Xi_i = Q\tilde{A}^T + \tilde{A}Q - \tilde{M}_i^T \tilde{B}^T - \tilde{B}\tilde{M}_i + I \quad (27a)$$

$$T_i = \tilde{L}_i^T \tilde{B}^T \tilde{B} \tilde{L}_i + \tilde{A}^T \tilde{P}_2 + \tilde{P}_2 \tilde{A} - \tilde{C}_i^T \tilde{N}^T - \tilde{N}\tilde{C}_i \quad (28a)$$

and $Q = \tilde{P}_1^{-1}$ (27b)

$$\tilde{L}_i = \tilde{M}_i Q^{-1} \quad (27c)$$

$$\tilde{K} = \tilde{P}_2^{-1}\tilde{N}, \quad (28b)$$

where * denotes the transposed elements in the symmetric positions.

Proof. Before proceeding, we recall the following matrix inequality, which will be needed throughout the proof of Theorem.

Lemma 1 (Lee et al. [18]). Given constant matrices *D* and *F*, symmetric constant matrix *S* and unknown constant matrix F_i of appropriate dimension satisfying the constraint $F_i^T F_i < R$. The following two propositions are equivalent:

$$S + DFE + E^T F^T D^T < 0$$

$$S + \begin{pmatrix} E^T & D \end{pmatrix} \begin{pmatrix} \varepsilon^{-1} R & 0 \\ 0 & \varepsilon I \end{pmatrix} \begin{pmatrix} E \\ D^T \end{pmatrix} < 0 \text{ for some } \varepsilon > 0$$

Consider the Lyapunov function candidate

$$V(x(t), e(t)) = V_1(x(t)) + V_2(e(t)) \quad (29)$$

With

$$V_1(x(t)) = x^T(t)\tilde{P}_1 x(t) \quad ; \tilde{P}_1 = Q^{-1} > 0, \quad (30)$$

$$V_2(e(t)) = e^T(t)\tilde{P}_2 e(t) \quad ; \tilde{P}_2 > 0 \quad (31)$$

$$\tilde{P}_1 = diag\{P_{11} \quad \cdots \quad P_{1i} \quad \cdots \quad P_{1q}\} \quad (32)$$

$$\tilde{P}_2 = diag\{P_{21} \quad \cdots \quad P_{2i} \quad \cdots \quad P_{2q}\} \quad (33)$$

The time derivative of $V_1(x(t))$ along the trajectory of (4) is

$$\dot{V}_1(x(t)) = \dot{x}^T(t)\tilde{P}_1 x(t) + x^T(t)\tilde{P}_1 \dot{x}(t) = \sum_{i=1}^{q} \mu_i(\xi(t)) \begin{bmatrix} x^T(t)\left(\tilde{H}_i^T \tilde{P}_1 + \tilde{P}_1 \tilde{H}_i\right) x(t) + \\ 2x^T(t)\tilde{P}_1\left(\tilde{B} + \Delta\tilde{B}(t)\right)\tilde{L}_i e(t) \end{bmatrix} \quad (34)$$

With $\tilde{H}_i = \tilde{A} + \Delta\tilde{A}(t) - \left(\tilde{B} + \Delta\tilde{B}(t)\right)\tilde{L}_i$ (35)

Using Lemme 1 and Assumption 1, the following inequality holds:

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$$2x^T(t)\tilde{P}_1\left(\tilde{B}+\Delta\tilde{B}(t)\right)\tilde{L}_i e(t)\le$$
$$x^T(t)\left(\tilde{P}_1^2+\tilde{P}_1\tilde{D}\tilde{D}^T\tilde{P}_1\right)x(t)+e^T(t)\tilde{L}_i^T\left(\tilde{B}^T\tilde{B}+\tilde{E}_2^T\tilde{E}_2\right)\tilde{L}_i e(t) \quad (36)$$

Taking account of (36) in (34), the time derivative of $V_1(x(t))$ is as follows:

$$\dot{V}_1(x(t))\le$$
$$\sum_{i=1}^{q}\mu_i(\xi(t))\begin{bmatrix} x^T(t)\left(\tilde{H}_i^T\tilde{P}_1+\tilde{P}_1\tilde{H}_i+\tilde{P}_1^2+\tilde{P}_1\tilde{D}\tilde{D}^T\tilde{P}_1\right)x(t) \\ +e^T(t)\tilde{L}_i^T(t)\left(\tilde{B}^T\tilde{B}+\tilde{E}_2^T\tilde{E}_2\right)\tilde{L}_i e(t)\end{bmatrix} \quad (37)$$

The time of $V_2(e(t))$ along the trajectory of (26) is

$$\dot{V}_2(e(t))=\dot{e}^T(t)\tilde{P}_2 e(t)+e^T(t)\tilde{P}_2\dot{e}(t)=$$
$$\sum_{i=1}^{q}\mu_i(\xi(t))\begin{bmatrix} e^T(t)\left(\Sigma_i^T\tilde{P}_2+\tilde{P}_2\Sigma_i\right)e(t) \\ +2e^T(t)\tilde{P}_2\left(\Delta\tilde{A}(t)-\Delta\tilde{B}(t)\tilde{L}_i\right)x(t)\end{bmatrix} \quad (38)$$

With $\Sigma_i=\tilde{A}-\tilde{K}\tilde{C}_i+\Delta\tilde{B}(t)\tilde{L}_i$ (39)

Using Lemme 1 and Assumption 1, the following inequality holds:

$$2e^T(t)\tilde{P}_2\left(\Delta\tilde{A}(t)-\Delta\tilde{B}(t)\tilde{L}_i\right)x(t)\le$$
$$e^T(t)\left(\tilde{P}_2\tilde{D}\tilde{D}^T\tilde{P}_2\right)e(t)+ \quad (40)$$
$$x^T(t)\left(\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)^T\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)\right)x(t)$$

The time-derivative of $V_2(e(t))$ is then as follows:

$$\dot{V}_2(e(t))\le$$
$$\sum_{i=1}^{q}\mu_i(\xi(t))\begin{bmatrix} e^T(t)\left(\left(\Sigma^T\tilde{P}_2+\tilde{P}_2\Sigma\right)+\tilde{P}_2\tilde{D}\tilde{D}^T\tilde{P}_2\right)e(t) \\ +x^T(t)\left(\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)^T\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)\right)x(t)\end{bmatrix} \quad (41)$$

Finally, by combining (34) and (41), the time derivative of $V(x(t),e(t))$ can be written as follows:

$$\dot{V}(x(t),e(t))=$$
$$\sum_{i=1}^{q}\mu_i(\xi(t))\begin{bmatrix} x^T(t)\left(\tilde{H}_i^T\tilde{P}_1+\tilde{P}_1\tilde{H}_i+\tilde{P}_1^2+\tilde{P}_1\tilde{D}\tilde{D}^T\tilde{P}_1+\tilde{R}_i\right)x(t) \\ +e^T(t)\left(\Sigma_i^T\tilde{P}_2+\tilde{P}_2\Sigma_i+\tilde{P}_2\tilde{D}\tilde{D}^T\tilde{P}_2+\tilde{S}_i\right)e(t)\end{bmatrix} \quad (42)$$

With

$$\tilde{R}_i=\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)^T\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right) \quad (43)$$
$$\tilde{S}_i=\tilde{L}_i^T\left(\tilde{B}^T\tilde{B}+\tilde{E}_2^T\tilde{E}_2\right)\tilde{L}_i \quad (44)$$

Assuming that the first sum is negative definit, i.e.

$$\tilde{H}_i^T\tilde{P}_1+\tilde{P}_1\tilde{H}_i+\tilde{P}_1^2+\tilde{P}_1\tilde{D}\tilde{D}^T\tilde{P}_1+\tilde{R}_i<0 \quad (45)$$

Or equivalently

$$\tilde{P}_1^2+\tilde{P}_1\tilde{D}\tilde{D}^T\tilde{P}_1+\tilde{R}_i+$$
$$\left(\tilde{A}+\Delta\tilde{A}(t)-\left(\tilde{B}+\Delta\tilde{B}(t)\right)\tilde{L}_i\right)^T\tilde{P}_1+ \quad (46)$$
$$\tilde{P}_1\left(\tilde{A}+\Delta\tilde{A}(t)-\left(\tilde{B}+\Delta\tilde{B}(t)\right)\tilde{L}_i\right)<0$$

Then, applying Assumption 1 to (42) yields

$$\Upsilon_i+\tilde{P}_1\tilde{D}\tilde{F}(t)\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)+\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)^T\tilde{F}^T(t)\tilde{D}^T\tilde{P}_1<0 \quad (47)$$

Where

$$\Upsilon_i=\tilde{P}_1^2+\tilde{P}_1\tilde{D}\tilde{D}^T\tilde{P}_1+\tilde{R}_i+\tilde{A}^T\tilde{P}_1+\tilde{P}_1\tilde{A}-\tilde{L}_i^T\tilde{B}^T\tilde{P}_1-\tilde{P}_1\tilde{B}\tilde{L}_i \quad (48)$$

According to Lemma1, the above matrix inequality (49) holds for all $F_i(t)$ satisfying $F_i(t)^T F_i(t)\le I$ if and only if there exists a constant $\varepsilon_i^{1/2}$ such that:

$$\Upsilon_i+\begin{bmatrix}\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)^T & \tilde{P}_1\tilde{D}\end{bmatrix}\begin{bmatrix}\varepsilon_i^{-1}I & 0 \\ 0 & \varepsilon_i I\end{bmatrix}\begin{bmatrix}\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right) \\ \left(\tilde{P}_1\tilde{D}\right)^T\end{bmatrix}<0 \quad (49)$$

By rewriting $\tilde{P}_1\tilde{D}\tilde{D}^T\tilde{P}_1+\tilde{R}_i$ in the following form :

$$\begin{bmatrix}\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)^T & \tilde{P}_1\tilde{D}\end{bmatrix}\begin{bmatrix}I & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix}\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right) \\ \left(\tilde{P}_1\tilde{D}\right)^T\end{bmatrix} \quad (50)$$

Then (49) becomes

$$\Gamma_i+\begin{bmatrix}\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right)^T & \tilde{P}_1\tilde{D}\end{bmatrix}\begin{bmatrix}\left(\varepsilon_i^{-1}+1\right)I & 0 \\ 0 & \left(\varepsilon_i+1\right)I\end{bmatrix}\begin{bmatrix}\left(\tilde{E}_1-\tilde{E}_2\tilde{L}_i\right) \\ \left(\tilde{P}_1\tilde{D}\right)^T\end{bmatrix}<0 \quad (51)$$

Where

$$\Gamma_i=\tilde{P}_1^2+\tilde{A}^T\tilde{P}_1+\tilde{P}_1\tilde{A}-\tilde{L}_i^T\tilde{B}^T\tilde{P}_1-\tilde{P}_1\tilde{B}\tilde{L}_i \quad (52)$$

Let $Q=\tilde{P}_1^{-1}$, $\tilde{M}_i=\tilde{L}_i Q$

Pro-and-post multiplying both sides of (51) by $Q=\tilde{P}_1^{-1}$ results in

$$\Xi_i+\begin{bmatrix}\left(\tilde{E}_1 Q-\tilde{E}_2\tilde{M}_i\right)^T & \tilde{D}\end{bmatrix}\begin{bmatrix}\left(\varepsilon_i^{-1}+1\right)I & 0 \\ 0 & \left(\varepsilon_i+1\right)I\end{bmatrix}\begin{bmatrix}\left(\tilde{E}_1 Q-\tilde{E}_2\tilde{M}_i\right) \\ \tilde{D}^T\end{bmatrix}<0 \quad (53)$$

Where

$$\Xi_i=Q\tilde{A}^T+\tilde{A}Q-\tilde{M}_i^T\tilde{B}^T-\tilde{B}\tilde{M}_i+I \quad (54)$$

Applying Schur complement to (53) results in the first LMI, (27), in Theorem.

The second LMI (28) can be established through a similar procedure. Assume that the second sum in Eq. (42) is negative definite:

$$\Sigma_i^T\tilde{P}_2+\tilde{P}_2\Sigma_i+\tilde{P}_2\tilde{D}\tilde{D}^T\tilde{P}_2+\tilde{S}_i<0 \quad (55)$$

Or equivalently

$$\tilde{P}_2\tilde{D}\tilde{D}^T\tilde{P}_2+\tilde{S}_i+\left(\tilde{A}-\tilde{K}\tilde{C}_i+\Delta\tilde{B}(t)\tilde{L}_i\right)^T\tilde{P}_2+$$
$$\tilde{P}_2\left(\tilde{A}-\tilde{K}\tilde{C}_i+\Delta\tilde{B}(t)\tilde{L}_i\right)<0 \quad (56)$$

then, applying Assumption 1 to (56) yields

$$\Psi_i+\tilde{P}_2\tilde{D}\tilde{F}(t)\left(\tilde{E}_2\tilde{L}_i\right)+\left(\tilde{E}_2\tilde{L}_i\right)^T\tilde{F}^T(t)\tilde{D}^T\tilde{P}_2<0 \quad (57)$$

Where

$$\Psi_i=\tilde{P}_2\tilde{D}\tilde{D}^T\tilde{P}_2+\tilde{S}_i+\left(\tilde{A}-\tilde{K}\tilde{C}_i\right)^T\tilde{P}_2+\tilde{P}_2\left(\tilde{A}-\tilde{K}\tilde{C}_i\right)<0 \quad (58)$$

According to lemme2, the above matrix inequality (58) holds :

$$\Psi_i+\begin{bmatrix}\left(\tilde{E}_2\tilde{L}_i\right)^T & \tilde{P}_2\tilde{D}\end{bmatrix}\begin{bmatrix}\varepsilon_i^{-1}I & 0 \\ 0 & \varepsilon_i I\end{bmatrix}\begin{bmatrix}\tilde{E}_2\tilde{L}_i \\ \left(\tilde{P}_2\tilde{D}\right)^T\end{bmatrix}<0 \quad (59)$$

By rewriting $\tilde{P}_2\tilde{D}\tilde{D}^T\tilde{P}_2+\tilde{L}_i^T\tilde{E}_2{}^T\tilde{E}_2\tilde{L}_i$ in the following form

$$\tilde{P}_2\tilde{D}\tilde{D}^T\tilde{P}_2+\tilde{L}_i^T\tilde{E}_2{}^T\tilde{E}_2\tilde{L}_i=\begin{bmatrix}\left(\tilde{E}_2\tilde{L}_i\right)^T & \tilde{P}_2\tilde{D}\end{bmatrix}\begin{bmatrix}I & 0 \\ 0 & I\end{bmatrix}\begin{bmatrix}\tilde{E}_2\tilde{L}_i \\ \left(\tilde{P}_2\tilde{D}\right)^T\end{bmatrix} \quad (60)$$

Then (59) becomes

$$\Pi_i+\begin{bmatrix}\left(\tilde{E}_2\tilde{L}_i\right)^T & \tilde{P}_2\tilde{D}\end{bmatrix}\begin{bmatrix}\left(\varepsilon_i^{-1}+1\right)I & 0 \\ 0 & \left(\varepsilon_i+1\right)I\end{bmatrix}\begin{bmatrix}\tilde{E}_2\tilde{L}_i \\ \left(\tilde{P}_2\tilde{D}\right)^T\end{bmatrix}<0$$

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(61)

Where

$$\Pi_i = \tilde{L}_i^T \tilde{B}^T \tilde{B} \tilde{L}_i + \tilde{A}^T \tilde{P}_2 + \tilde{P}_2 \tilde{A} - \tilde{C}_i^T \tilde{K}^T \tilde{P}_2 - \tilde{P}_2 \tilde{K} \tilde{C}_i \quad (62)$$

Let $\tilde{N} = \tilde{P}_2 \tilde{K}$

We can rewrite (61) in the following form:

$$T_i + \left[\left(\tilde{E}_2 \tilde{L}_i \right)^T \quad \tilde{P}_2 \tilde{D} \right] \begin{bmatrix} \left(\varepsilon_i^{-1} + 1 \right) I & 0 \\ 0 & \left(\varepsilon_i + 1 \right) I \end{bmatrix} \begin{bmatrix} \tilde{E}_2 \tilde{L}_i \\ \left(\tilde{P}_2 \tilde{D} \right)^T \end{bmatrix} < 0 \quad (63)$$

Where

$$T_i = \tilde{L}_i^T \tilde{B}^T \tilde{B} \tilde{L}_i + \tilde{A}^T \tilde{P}_2 + \tilde{P}_2 \tilde{A} - \tilde{C}_i^T \tilde{N}^T - \tilde{N} \tilde{C}_i \quad (64)$$

Applying Schur complement to (63) results in the second LMI, (28), in Theorem .

In order to show clearly implementation of the observer-based decouple multi-model controller, the design procedures are given as follows:

- First, we solve LMI (27) in the variables Q and $\tilde{M}_i$.
- Once gains $\tilde{L}_i$ have been calculated from (27b), conditions (28) become linear in $\tilde{P}_2$ and $\tilde{N}$, and can be easily resolved using the LMI tool to determine gains $\tilde{K}$ from (28b).

IV. Computer Simulation

Consider the decoupled multiple model with L = 2 submodels with different dimensions (n1 = 3 and n2 = 2), given by:

$$A_1 = \begin{bmatrix} -0.1 & 0.5 & 0.2 \\ -0.3 & -0.4 & 0.1 \\ -0.1 & 0.2 & -0.3 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -0.3 & -0.1 \\ 0.4 & -0.2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.3 & 0.5 & 0.6 \end{bmatrix}^T, \qquad B_2 = \begin{bmatrix} 0.4 & 0.3 \end{bmatrix}^T,$$

$$C_1 = \begin{bmatrix} -0.5 & 0.3 & 0.5 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.4 & -0.3 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \end{bmatrix}^T, \qquad D_2 = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}^T,$$

$$E_{11} = \begin{bmatrix} 0.2 & -0.1 & 0.2 \end{bmatrix}, \qquad E_{12} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$

$$E_{21} = -0.2, \qquad E_{22} = -0.3,$$

Here, the decision variable $\xi(t)$ is the input signal $u(t)$ .The weighting functions are obtained from normalized Gaussian functions:

$$\mu_i\left(\xi(t)\right) = \eta_i\left(\xi(t)\right) \Big/ \sum_{j=1}^{L} \eta_j\left(\xi(t)\right),$$

$$\eta_i\left(\xi(t)\right) = \exp\left(-\left(\xi(t) - c_i\right)^2 / \sigma^2 \right),$$

With the standard deviation $\sigma = 0.6$ and the centers $c_1 = -0.3$ and $c_2 = 0.3$.

Using LMI optimization algorithm [16] to solve LMIs (31)-(34), feedback gain and observer gain matrices can be obtained as

$$L_1 = \begin{bmatrix} 1.3591 & 1.1193 & 1.0416 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.5883 & 0.599 \end{bmatrix}$$

$$\tilde{K} = \begin{bmatrix} -0.5838 & -0.0925 & -0.2567 & 0.0969 & -0.0422 \end{bmatrix}^T$$

The initial values of states are chosen

$$x(0) = \begin{bmatrix} 0.9 & 0.7 & 0.1 & -0.7 & 0.4 \end{bmatrix}^T, \hat{x}(0) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

. Figs. 1-2 illustrate the closed-loop system behaviours. The simulation results show that the decoupled multimodel based controller through multi-observer is robust against norm-bounded parametric uncertainties. The state estimation errors are given by Fig. 3. Fig. 4 allows the comparison of the nominal model states and estimated model states. From the latter, one can see that the synthesized observer and decoupled multi-model controller showed their effectiveness. The evolution of the control law is given in Fig. 5.

V. Conclusion

In this paper, we have developed a new robust decoupled multi-model controller design methodology for non linear systems represented by multiple models with parameter uncertainties under the conditions that the state variables are unavailable for measurement. The basic approach is based on the rigorous Lyapunov stability theory, and the basic tool is linear matrix inequality (LMI). Some sufficient conditions for robust stabilization of the decoupled multi-model are formulated in the LMIs format. The simulation results have shown the effectiveness of the proposed control design method.

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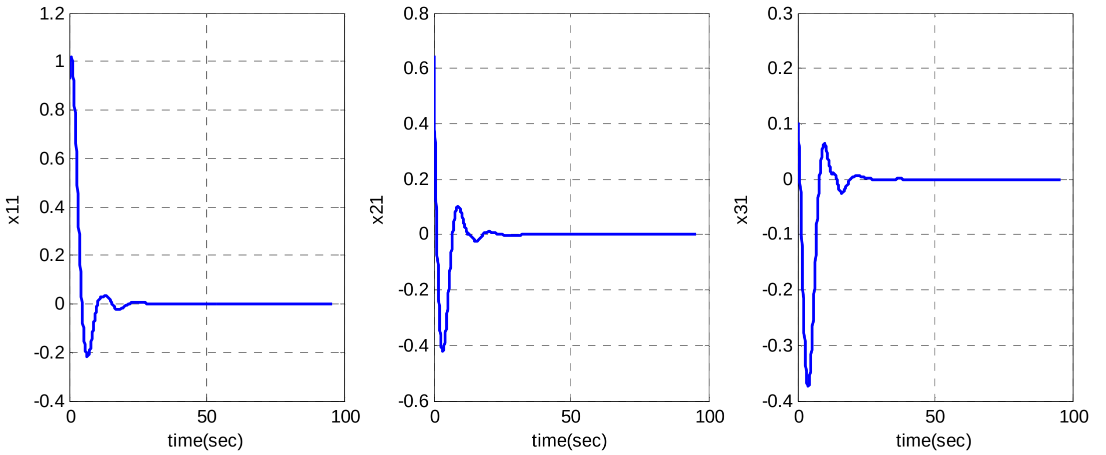

Fig. 1. Closed-loop response of submodel 1

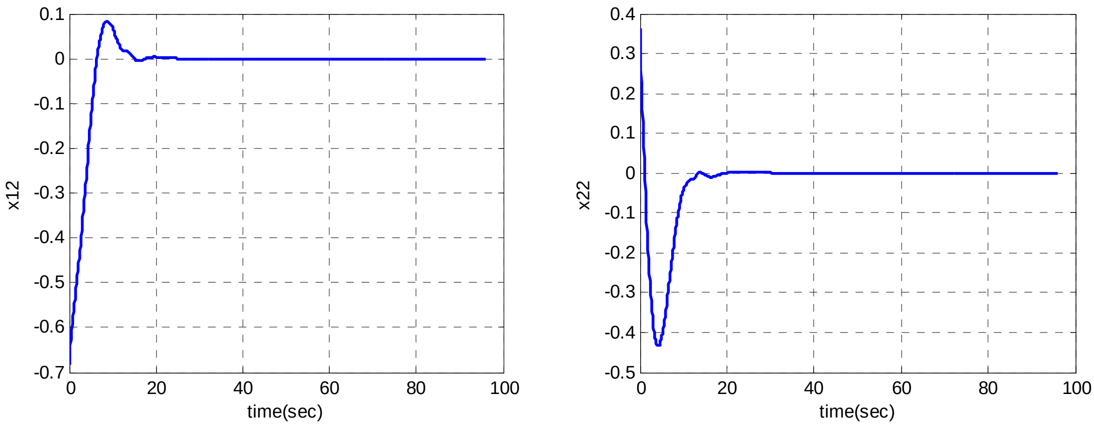

Fig. 2. Closed-loop response of submodel 2

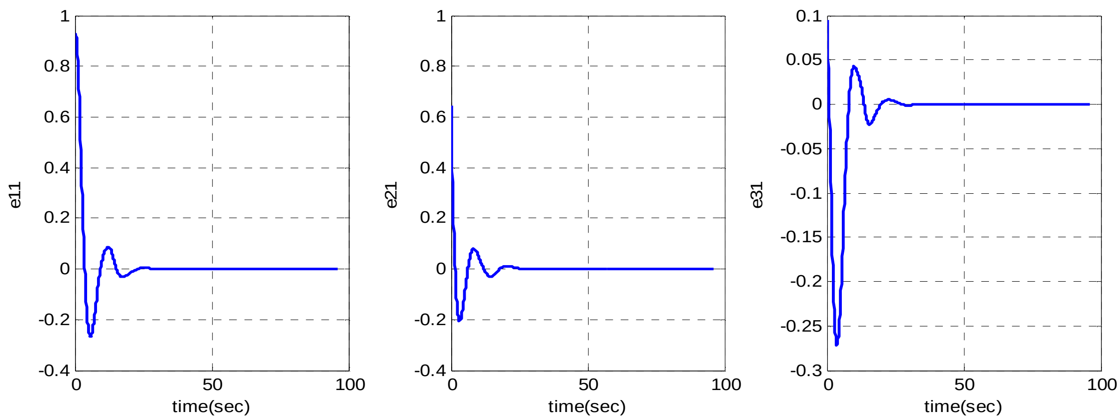

Fig. 3. State estimation errors of submodel 1

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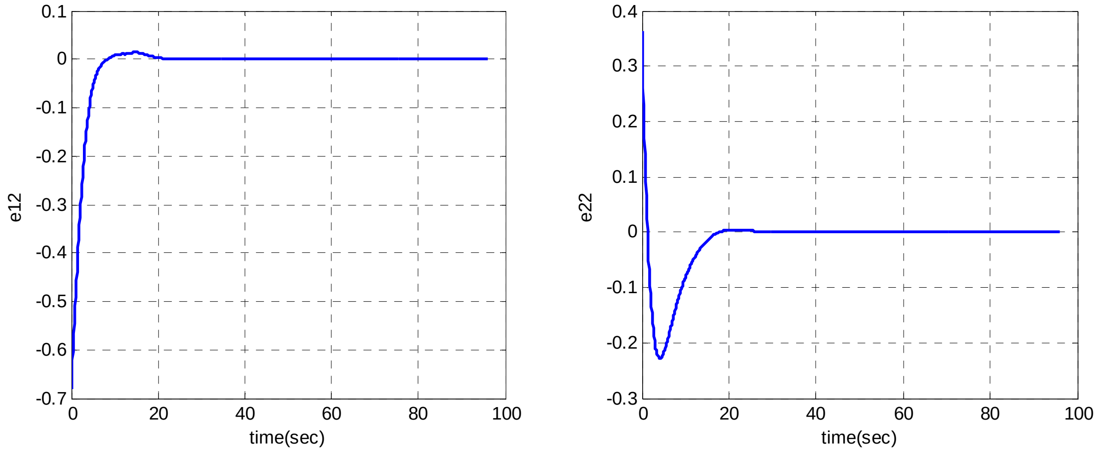

Fig. 4. State estimation errors of submodel 2

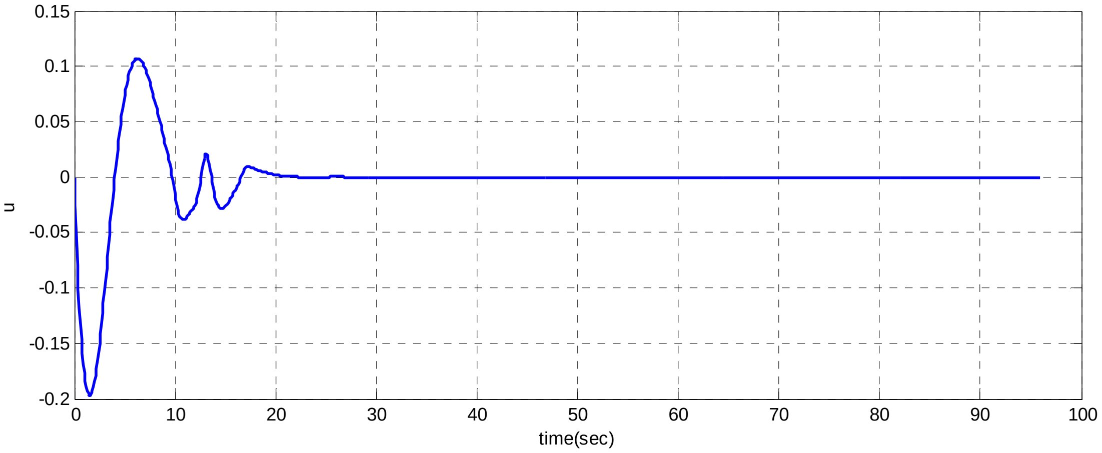

Fig. 5. Control input

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