Multi-Criteria Design Optimization of Control System Instrumentation using Principal Component Analysis (PCA) and Structural Modeling Approaches

Zine-Eddine Meguetta, Blaise Conrard, Mireille Bayart

Abstract—this article presents general approach of multi-criteria design of the control system instrumentation. The work reported here aims at defining that principal component analysis PCA can be used as method of design phase for non-linear system based on data measurements from the sensors and the available actuators for dynamical control system. The PCA consists to select inputs variables for quantifying the speed $v_{\text{ideal}}$ using structural modeling, despite the environmental disturbance is the slope of the road and uncertainties in measurements from the sensors and actuators implemented in the control system instrumentation in design phase.

Index Terms—Multi criteria design, principal component analysis, structural modeling, optimization.

I. INTRODUCTION

The complexity of designing using the multi criteria of performance for control system instrumentation has been emerging over the past few years, since the robust design is realized by data from several types of sensors and actuators.

The design of control system instrumentation has been examining over the past few years [1]-[6]. The interest of designing a control system is to build the optimal control instrumentation scheme, that is to say, a set of instruments (sensors and actuators) that allows the system to perform its missions despite the disturbance, taking into account multi-criteria of performance in design phase.

The design problem aims in determining the sensors and actuators and their redundancies to implement in order to obtain fault tolerant control system instrumentation in [2] using structural analysis for system of two electric vehicles; where, they can be exchanged by wireless communication.

This paper is an extension of results presented in [3] and [4]; the application used is an electric vehicle that moves on a road with a variable slope. The step of design concerns the ability to guarantee the performance and robustness of a system of the vehicle using multi criteria of performance (for instance: quality of control, quality of performance, fault tolerant level and reliability) in design phase; it essentially depends on the control algorithm which consists in finding all paths of the control $u(t)$ based on a structural model that link the physical variables for electric vehicle.

In this study, consider the three inputs ($c_{\text{v}}$ is the measured speed provided by speed sensor and $I_i$ is the measured slope provided by inclinometer and $u$ is the control signal of the motor) which are selected by the method of the Principal Component Analysis (PCA) and output is the speed $v_{\text{ideal}}$, taking advantage of the structural modeling. This study is based on the use of the structural modeling to quantize the speed $v_{\text{ideal}}$ where the value of desired speed $v_{\text{desired}}$ is known [4].

In this work, the optimization consists in finding the optimal set of instrumentation that satisfies the constraint of quality of control QoC, a lowest financial cost.

II. PRINCIPAL COMPONENT ANALYSIS PCA

PCA is attractive method where the available process measurements are highly correlated, the principal of this method is to reduce the dimensionality of a data set considering a large number of interrelated variables [7], [8]. It also use for large scale process of model based fault detection and it is method of multivariable analysis that has been often used for static treatment of multidimensional database [9], [10].

The principal components are determined by calculating the eigenvectors of the covariance matrix.

For a set of $N$ data vectors of the $m$ variables, $x=(x_1, x_2, \ldots, x_m)$, the PCA approach can be obtained by the calculating the covariance matrix $C$:

$$ C = \frac{1}{N} \sum_{i=1}^{N} X_i X_i^T $$

(1)

Where $X_i^T$ is the conjugate transpose operator, for find the eigenvalues $\lambda_j$ and eigenvectors $p_j$ of the covariance matrix $C$:

$$ V^{-1}CV = D $$

(2)

Where $D$ is the diagonal matrix of eigenvalues of $C$ and $V$ is the matrix of eigenvectors which diagonalizes the covariance matrix $C$. The use of PCA in this paper is to select the more influential inputs variables on the control system instrumentation.
III. MULTI-CRITERIA DESIGN OF CONTROL SYSTEM INSTRUMENTATION

In this work, the structural modeling is the tool of quantization unknown physical quantity based data selected by PCA. Our design methodology reside in two main aspects: the first concerns the dynamic of the system where we will identify all structural paths to access the physical quantity of the determined mission by the designer using structural modeling.

Also, the search algorithm of control $u_i$ (see flowchart in Figure 1), and the second aspect concerns the measurement chain of control system instrumentation in design phase.

![Flowchart Descriptive the Approach of Multi-Criteria Design](image-url)

**Figure 1: The flowchart descriptive the approach of multi-criteria design**
Considering the two types of failure modes for instruments implemented in the instrumentation as following:

$$\Delta G = |G_{\text{real}} - G_{\text{desired}}|$$

$$G = g \pm \varepsilon_G$$

Where:
- $G$ is the physical quantity,
- $G_{\text{real}}$ is the observed real quantity,
- $G_{\text{desired}}$ is the desired quantity fixed by the designer (reference),
- $\Delta G$ is the difference between the real and desired quantity,
- $g$ is the measurement, $\varepsilon_G$ is the uncertainty of measurement.

Specifically, we consider the chain of measures of all instrumentation can be described by the state space of a discrete-time dynamical system:

$$x(k+1) = A(k)x(k) + B(k)u(k)$$

and the elaboration of the data measurement:

$$y(k) = C(k)x(k) + D(k)f(k)$$

Where:
- $k$ is the index of time,
- $u(k)$ is control of the system, $x(k)$ is the state vector, $y(k)$ is the observed output signal $f(k)$ is the error vector (disturbance),
- $A(k), B(k), C(k), D(k)$ are the matrices associated with $x(k), u(k), y(k), f(k)$.

This design approach aims to achieve optimal architecture of a control system that meet the needs and specifications of the designer in Multi-criteria of performance: Fault Tolerant level (FTL), Reliability ($R(t)$), Quality of Control (QoC), a lower cost.

IV. PROCEDURE MULTI-CRITERIA DECISION: CHOICE OF THE OPTIMAL ARCHITECTURE INSTRUMENTATION

Multi-criteria decision procedure will then be used in this approach, in order to choose the best architecture of control system instrumentation using the criterion selected of performance and cost criteria.

The different sets of architecture instrumentations are grouped in $A$:

$$A = \{ \text{arch}_1, \text{arch}_2, ..., \text{arch}_n \}$$

Where $n$ is the total number of architectures.

Different criteria are grouped in the set $D$:

$$D = \{ \text{cri}_1, \text{cri}_2, ..., \text{cri}_m \}$$

Where $m$ is the total number of criteria, here the selected criterion included in $\{ \text{QoC}, \text{QoP}, \text{FTL}, R(t), ... \}$, the designer chooses the criteria of performance to be used in the design phase.

Each instrumentation architecture $\text{arch}_i$ can be evaluated for a criterion $\text{cri}_i$, and evaluation noted $\text{choice}_i(\text{arch}_i)$ if:

$$\exists \text{arch}_i \in A \cap \text{cri}_i \in \{ 1, 2, ..., N \} \cap \{ 1, 2, ..., m \}, \text{choice}_i(\text{arch}_i) \geq \text{choice}_i(\text{arch}_j)$$

The final optimization phase problem is obtained by integrate the cost criteria. It is the sum of the instruments costs $c_i, i=1, ..., K$. The optimization problem is:

$$\begin{bmatrix} \text{Min} \left( \sum_{i=1}^{K} c_i \right) \\
\text{QoC}(q_i) \leq n_{i1} \\
\text{QoP}(q_i) \leq n_{i2} \\
\text{FTL}(q_i) \leq n_{i3} \\
R(q_i) \leq r \end{bmatrix}$$

Where: $n_{i1}, n_{i2}, n_{i3}, r$ are respectively the quality of control, quality of performance, fault tolerant level and reliability desired by designer.

V. DESIGN APPROACH BASED PCA AND STRUCTURAL MODELING ON ELECTRIC VEHICLE DYNAMIC

A. Description of the process

Consider the following variables used in this electric vehicle moves on road slope, as shown in reference [3] and a structural analysis is used to describe this system in the reference [1]. The table 1 presents 13 set of instrumentation and parameters of variables using the equation (4): $u_t$ is the signal of motor, $I_i$ is the slope, $c_v$ is the speed, $Ch$ is the time, GPS latitude is GPS latitude, GPS longitude is GPS longitude and GPS elevation is GPS elevation. The costs associated to each sensor and actuators are: $C_{ch}$ a cost for a motor, $C_{ic}$ a cost for speed sensor, $C_i$ a cost for inclinometer, $C_{ch}$ a cost for chronometer, $C_{GPS}$ a cost for GPS.

B. Elaboration and selection of input variables using PCA:

The elaboration of data using principal component analysis (PCA) consists of collecting different information based sensors and actuators. The correlation circle is very interesting to see the correlation between variables and the principal components in the correlation circle shown in fig.2. The representation appears in the circle correlation three groups of behavior parameters with very close variation variables. In this study, thanks to PCA which allows to selected parameters that influence in the system of electric vehicle; we can keep the following three variables $c_v, u_t, I_i$. Where the control signal of the motor ($u$), measured speed provided by speed sensor ($c_v$), measured slope of the road provided by inclinometer ($I_i$). In addition, we have the value of the desired speed $v_{desired}$ to quantize the speed $v_{init}$.
Table 1: the measurements uncertainties from different instruments for 13 set of instrumentation

<table>
<thead>
<tr>
<th>Number of case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu$</td>
<td>210</td>
<td>240</td>
<td>250</td>
<td>150</td>
<td>170</td>
<td>200</td>
<td>240</td>
<td>270</td>
<td>200</td>
<td>240</td>
<td>230</td>
<td>240</td>
</tr>
<tr>
<td>GPS latitude</td>
<td></td>
<td>15.440</td>
<td>7.990</td>
<td>8.110</td>
<td>15.000</td>
<td>10.990</td>
<td>17.220</td>
<td>16.000</td>
<td>9.990</td>
<td>13.990</td>
<td>18.000</td>
<td>15.000</td>
<td>13.000</td>
</tr>
<tr>
<td>GPS elevation</td>
<td></td>
<td>2.199</td>
<td>5.850</td>
<td>6.240</td>
<td>1.290</td>
<td>2.890</td>
<td>3.332</td>
<td>8.990</td>
<td>0.580</td>
<td>2.980</td>
<td>2.350</td>
<td>1.740</td>
<td>1.870</td>
</tr>
</tbody>
</table>

Figure 2: The correlation circle using PCA, (1) is the signal of motor, (2) is the slope, (3) is the speed, (4) is the time, (5) is GPS latitude, (6) is GPS longitude and (7) is GPS elevation.

C. Algorithm of search the control $u_t$:
From the analytical and structural model of electric vehicle are presented in table 2. The results of search algorithm of the control $u_t$, which is presented in reference [5], from structural model gives in figure 3.
Table 2: Analytical and structural model of electric vehicle

<table>
<thead>
<tr>
<th>Analytical model</th>
<th>Structural model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_t = \frac{v_{t+\delta t} - v_t}{\delta t}$</td>
<td>$f_1(a_t, v_{t+\delta t}, v_t, \delta t) = 0$</td>
</tr>
<tr>
<td>$a_t = g \sin(\alpha_t) - \frac{F_r}{m_t} + \frac{u_t}{m_t}$</td>
<td>$f_2(a_t, \alpha_t, m_t, u_t) = 0$</td>
</tr>
<tr>
<td>$c_{v_t} = v_t + c_{v_t}^t$</td>
<td>$f_3(c_{v_t}^t, v_{t}, c_{v_t}^t) = 0$</td>
</tr>
<tr>
<td>$GPS = x_{t-\delta t} + c_{x_t}^t$</td>
<td>$f_4(GPS, x_{t-\delta t}, c_{x_t}^t) = 0$</td>
</tr>
<tr>
<td>$I_t = \alpha_t + c_{I_t}^t$</td>
<td>$f_5(I_t, \alpha_t, c_{I_t}^t) = 0$</td>
</tr>
<tr>
<td>$c_{v_t}^{t-\delta t} = v_{t-\delta t} + c_{v_t}^{t-\delta t}$</td>
<td>$f_6(c_{v_t}^{t-\delta t}, v_{t-\delta t}, c_{v_t}^{t-\delta t}) = 0$</td>
</tr>
<tr>
<td>$m_{t-\delta t} = M + \Delta m_{t-\delta t}$</td>
<td>$f_7(m_{t-\delta t}, M, \Delta m_{t-\delta t}) = 0$</td>
</tr>
<tr>
<td>$t_{\text{descent}} = \sqrt{\frac{2x_t}{a_t}}$</td>
<td>$f_8(t_{\text{descent}}, x_t, a_t) = 0$</td>
</tr>
<tr>
<td>$v_t = \frac{x_{t} - x_{t-\delta t}}{\delta t}$</td>
<td>$f_9(v_t, x_t, x_{t-\delta t}, \delta t) = 0$</td>
</tr>
<tr>
<td>$c_{h_t} = t_{\text{descent}} + c_{h_t}$</td>
<td>$f_{10}(c_{h_t}, t_{\text{descent}}, c_{h_t}) = 0$</td>
</tr>
<tr>
<td>$m_t = M + \Delta m_t$</td>
<td>$f_{11}(m_t, M, \Delta m_t) = 0$</td>
</tr>
<tr>
<td>$a_{t-\delta t} = \frac{v_t - v_{t-\delta t}}{\delta t}$</td>
<td>$f_{12}(a_{t-\delta t}, v_t - \delta t, v_t, \delta t) = 0$</td>
</tr>
<tr>
<td>$a_{t-\delta t} = g \sin(\alpha_{t-\delta t}) - \frac{F_r}{m_{t-\delta t}} + \frac{u_{t-\delta t}}{m_{t-\delta t}}$</td>
<td>$f_{13}(a_{t-\delta t}, \alpha_{t-\delta t}, m_{t-\delta t}, u_{t-\delta t}) = 0$</td>
</tr>
<tr>
<td>$\alpha_t = \alpha_{t-\delta t} + \Delta \alpha_t$</td>
<td>$f_{14}(\alpha_t, \alpha_{t-\delta t}, \Delta \alpha_t) = 0$</td>
</tr>
</tbody>
</table>
D. Results of Optimization problem:
The formalization of the optimization problem multi-criteria takes into account the costs of instruments and multi criteria of performance. Here, as an illustrative example, we chose the quality of control (QoC) as criteria of performance [4] for 13 set of admissible instrumentation (archi₁, archi₁₃).

Applying the optimization problem cited in equation (10), and knowing the value of the quality of control desired QoC equals 8. 10⁻², we obtain the equation (11):
VI. CONCLUSION

The use of PCA can help the designer to select the important measurements from the sensor and actuator implemented in the instrumentation. Moreover, the used method is a relatively easy to use and requires data information, that is to say, a correlation circle built by a PCA. The work presented in this paper aims at showing that the development of methodologies multi-criteria of performance (QoC, QoP, FTL, Reliability...) in design phase of control system instrumentation based PCA and structural modeling approaches and determines the optimal architecture instrumentation using the criteria of optimization of financial cost with QoC as illustrative example.

REFERENCES


International Conference on Control and Fault-Tolerant Systems, Nice, France, October 9-11, 2013.


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