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Maximization of Throughput for Gaussian Relay Channel with Hybrid Relaying: Decode-Amplify and Forward

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Abstract— For the classic three node Gaussian relay channel with hybrid relaying of both decode and forward (DF) and amplify and forward (AF) relaying, the throughput maximization over a finite horizon of N transmission block is presented . Here we are assuming the deterministic energy harvesting model in which the parameters such as energy arrival time and harvested amount are known prior to the transmission. Consider the two types of data traffic based on different delays i.e., delay constraint (DC) and no delay constraint (NDC) traffic cases.

Index Terms—amplify and forward, decode and forward, energy harvesting and Gaussian relay channel.

I. INTRODUCTION

Now a day, real time application of wireless sensor network uses conventional energy sources such as batteries. But it has disadvantages of limited operational time, difficulty of replacing or recharging the fixed supply batteries especially in toxic environment. Hence a much easier and safest way for energy source named ENERGY HARVESTING is considered here. EH is beneficial because it has unlimited energy supply and intermittent over time. Hence the energy harvested by any time always be greater than the total energy consumed up to that time. The energy harvesting nodes has recently got vital research attention in wireless communication. The power management strategies for WSNs is investigated with EH nodes by the authors in [1] and [2]. In [1], for the solar powered WSN ,mainly two modes of operation is considered which are sleep and active mode. Based upon the power available, the nodes switches between these two modes. The capacity of AWGN channel with EH constraints was studied in an independent and identically distributed (i.i.d) EH mode by the authors in [3]. And also it is showed that the EH model also can achieves the same capacity with the same total transmission energy consumed as that for the conventional case of constant power supply. The authors in[4] considered the throughput maximization problem for the Gaussian two hop relay channel but without considering the direct link between the source and the destination . Hence this direct link is considered as the special case in this paper. A half duplex Gaussian orthogonal relay channel is investigated by the authors in [5], where the source transmits to the relay and destination in channel 1, and the relay transmits to the destination in channel 2, with channels 1 and 2 being orthogonalized in the time–frequency plane.

The half duplex orthogonal Gaussian relay channel with EH source and relay nodes is considered here and it is shown in Fig.1. The orthogonal represents that the relay to destination link is orthogonal to both the source to relay and source to destination links, by assuming that the relay transmits and receives over two various frequency bands. Here we simply consider the deterministic source and relay energy profiles corresponding to practical scenarios so that the EH level can be predicted with negligible errors. The throughput maximization problem is examined over a finite horizon of N transmission blocks.

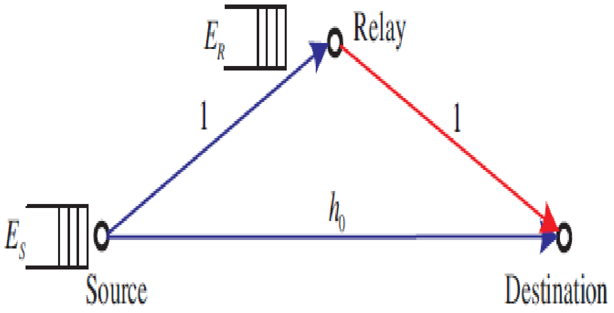

Fig. 1: Half Duplex Orthogonal Relay Channel

In each block the source transmit a new message which is received by the relay. Then the relay decode or amplify it based on the SNR factor and further it forward it to the destination in the subsequent one or more blocks. Our main objective is to maximize the total throughput by studying the optimal rate and power allocation of source and relay over different blocks. Specifically we consider the two types of data traffic based on different decoding delay are given below:

- **Delay constraint (DC) traffic**: In DC case , there is a limitation on delay i.e., the destination is needed to decode or amplify the i-th message from the source immediately after it receives the message from the source and from the relay. The source involves in i-th block while relay in the (i+1)-th block where i=1,2,….,N. so when the relay received the message from the source in one block , it needs to forward it to the destination in the next block immediately.
- **No delay constraint (NDC) traffic**: In the NDC case ,there is no limitation on decoding delay so that all the source messages are decoded at the end of each N-block transmission. The destination can tolerate the decoding delay. So even the relay receives the source message in one block, it can forward it to destination in any one of the remaining (i+1)-th to (N+1)-th blocks. The power allocation is also differing for both the cases. For the DC

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case the joint source and relay power allocation over time will be achieved. For the NDC case, the separation power allocation principle will be achieved for source and relay. From the explanation of the two cases, it is clearly noted that the NDC case allows more flexible relay operation when compared to the DC cases. It is expected that the NDC case achieves the larger throughput in general. And also the NDC case is able to exploit the “energy diversity” which will be shown in this paper.

Notation:

- log(.) and ln(.) stands for base 2 and the natural logarithm respectively;
- The AWGN channel capacity is denoted by

$$\boldsymbol{c}(x) = ½\log(1+x)$$

$(x)^+ = \max(0, x)$ where min(x, y) and max(x, y) are the minimum and the maximum between the two real numbers x and y respectively.

II. SYSTEM MODEL

Here we consider the classic three node relay channel in which there is one source to destination pair and one relay as shown in the Fig.1. We assume that the relay nodes operates in the half duplex over two different orthogonal frequency i.e., source-relay and the relay-destination while source operates in same band i.e., source-relay and the source-destination (direct link). We consider the hybrid relaying scheme, which requires the relay to successfully decode or amplify the source message. In hybrid relaying the selection of decode or amplify is based on the SNR value of source-relay link and given threshold SNR. The relay type selection process is given as follows:

- When $\gamma_{sr}(i) < \gamma_{th}$, then the relaying type is amplify and forward
- When $\gamma_{sr}(i) \geq \gamma_{th}$,then the relaying type is decode and forward

Where $\gamma_{sr}(i)$ represents the source-relay link SNR and γ_{th} represents the threshold SNR. Thus if $\gamma_{sr}(i)$ is less than γ_{th}, then the relay chooses the amplify to avoid the error propagation, otherwise it chooses the decode to avoid the noise amplification. During decode and forward relaying ,each N source transmission blocks ,let take i^{th} block,$1 \leq i \leq N$, the source transmit a new message w_i with rate R(i) and power $P_S(i)$. The relay upon receiving the source message, decodes w_i and generates a binning index for w_i based on ‘Random binning’ technique with rate $R_B(i+1)$. While in $(i+1)^{th}$,the relay transmit a message v_{i+1} with power $P_R(i+1)$ and rate $\boldsymbol{c}(P_R(i+1))$. While amplify and forward relaying the source transmit the signal to relay in one block since SNR of source to relay link is less than threshold SNR ,the relay then amplify the signal and forward it to the destination in the immediate next block. For the DC case ,v_{i+1} is the binning index of w_i only; while for NDC case ,v_{i+1} may contain the information of binning index for all message from source w_k where $k \leq i$. As we assumed the deterministic model, harvested energy amount $E_S(i)$ in the i^{th} block and $E_R(i+1)$ in the $(i+1)^{th}$ block are known prior to the transmission. In this paper we assume that there is small consumed energy at the source and relay other than transmission energy while the battery capacity is assumed to be infinite so consumed energy other than involved in transmission is negligible.

Thus the amount of energy available for each of the transmission block is constraint by the following EH constraints:

$$\sum_{i=1}^{K} P_s(i) \leq \frac{1}{B}\sum_{i=1}^{k} E_S(i), k=1,\ldots.N, \qquad (1)$$

$$\sum_{i=1}^{K} P_R(i+1) \leq \frac{1}{B}\sum_{i=1}^{k} E_R(i+1), k=1,\ldots.N, \qquad (2)$$

The channel input –output relationships for source and relay are given as

$$y_{sr}(i) = \sqrt{h}_{sr}\, x_s(i) + n_r(i), \qquad (3)$$

$$y_{sd}(i) = \sqrt{h}_{sd}\, x_s(i) + n_d(i), \qquad (4)$$

$$y_{rd}(i+1) = \sqrt{h}_{rd}\, x_r(i+1) + w_d(i+1), \qquad (5)$$

where $x_s(i)$ is the i^{th} source transmitted signal and $x_r(i+1)$ is the $(i+1)^{th}$ relay transmission blocks with corresponding powers as $P_S(i)$ and $P_R(i+1)$; $y_{sr}(i)$ is the received signal at the at the relay from the source; $y_{sd}(i)$ and $y_{rd}(i+1)$ are the received signals at the destination from the source and relay respectively; h_{sr}, $h_{rd,}$ and h_{sd} are the constant channel gain for source- relay, relay-destination and source – destination links respectively; $n_r(i)$, $n_d(i)$ and $w_d(i+1)$ are the additive white Gaussian noise (AWGN) with zero mean and unit variance. If the relay adopt the amplify protocol, then relay-destination link output signal (5) become modified as

$$y_{rd}(i+1) = \beta\sqrt{h}_{rd}\, x_r(i+1) + w_d(i+1),$$

where β is the amplification factor and it is given by

$$\beta = (P_R(i+1)/(P_S(i)h_{sr}+N_0))^{1/2}$$

with unit variance N_0.

The received signals to noise ratio (SNR) for the source –relay, relay-destination and source – destination are given as follows:

$$\gamma_{sr}(i) = P_S(i)h_{sr}$$

$$\gamma_{rd}(i) = P_R(i+1)h_{rd}$$

$$\gamma_{sd}(i) = P_S(i)h_{sd}$$

Defining new source and relay energy and power profiles as

$$\tilde{E}_S(i) = E_S(i)h_{sr}$$

$$\tilde{E}_R(i+1) = E_R(i+1)h_{rd}$$

$$\tilde{P}_S(i) = P_S(i)h_{sr}$$

$$\tilde{P}_R(i+1) = P_R(i+1)h_{rd}$$

With the new channel gains as $\tilde{h}_{sr} = \tilde{h}_{rd} = 1$ and $\tilde{h}_{sd} = \frac{hsd}{hsr} = h_0$. So without loss of generality, the equations in the (3),(4) and (5) are modified as

$$y_{sr}(i) = x_s(i) + n_r(i), \qquad (6)$$

$$y_{sd}(i) = \sqrt{h}_0 x_s(i) + n_d(i), \qquad (7)$$

$$y_{rd}(i+1) = x_r(i+1) + w_d(i+1), \qquad (8)$$

III. PROBLEM FORMULATION

A. Delay-Constrained Case

For the DC case , in the i^{th} transmission block ,the source transmits w_i message with power $P_S(i)$ and rate R(i) then the relay checks whether decodes or amplify if it is decode protocol then relay reliably decodes it only if

$$R(i) \leq \boldsymbol{c}(P_S(i))$$

In $(i+1)^{th}$ block the relay partitions w_i into number of bins with the equivalent rate $R_B(i+1)$ and transmit message v_{i+1} with the binning index to the destination with power $P_R(i+1)$ and at the destination it first decode v_{i+1} and then decodes the original message w_i. If it is amplify protocol, then the relay amplify the message w_i and then forward it to the destination. Considering the N block transmission, the average throughput in the unit of bits/sec/Hz (bps/Hz) is maximized

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by solving following equations:

$$\max_{\{P_S(i)\},\{P_R(i+1)\}} \frac{1}{2(N+1)} \sum_{i=1}^{N} \min\{\boldsymbol{c}(P_S(i)),\boldsymbol{c}(h_0 P_S(i))+\boldsymbol{c}(P_R(i+1))\} \qquad \text{(P1)}$$

s.t.(1),(2),$P_S(i) \geq 0, P_R(i+1) \geq 0, i=1,\ldots,N$

where the factor $\frac{1}{2}$ is due to the half duplex relaying and $\frac{1}{(N+1)}$ is due to the fact that each N block requires (N+1)-block duration.

B. No-Delay –Constrained Case

For the NDC case, the relay operates the same as in case of DC but only difference is that it is allowed to transmit the binning index for message w_i in message $v_{i+1},\ldots,v_{N+1}$ instead of v_{i+1} only as in the DC case. At the destination, the binning indices for all source messages can be successfully decoded if

$$\sum_{i=1}^{N} R_B(i+1)=\sum_{i=1}^{N} \boldsymbol{c}(P_R(i+1))$$

$$\sum_{i=k}^{N} R_B(i+1) \leq \sum_{i=k}^{N} \boldsymbol{c}(P_R(i+1)) , 2 \leq k \leq N$$

Where $R_B(i+1)=\min\{\boldsymbol{c}(P_R(i+1)), \boldsymbol{c}(P_S(i))-\boldsymbol{c}(h_0P_S(i))\}, i=1,..,N$
The average throughput for NDC case is solved by following equations:

$$\max_{\{P_S(i)\},\{P_R(i+1)\}} \frac{1}{2(N+1)} \sum_{i=1}^{N} \boldsymbol{c}(h_0P_S(i))+\boldsymbol{c}(P_R(i+1)) \qquad \textbf{(P2)}$$

s.t $\sum_{i=1}^{k} \boldsymbol{c}(h_0P_S(i))+\boldsymbol{c}(P_R(i+1)) \leq \sum_{i=1}^{k} \boldsymbol{c}(P_S(i))$, k=1,…,N

IV. OPTIMAL SOLUTION FOR DC CASE

Here we solve a problem P1 for DC case by presenting the solutions to develop the optimal power allocation algorithm.

A. The Case with Direct Link

By considering the lagrangian of the problem P1 given below as (9)

$$\text{(PI*)} \quad \max_{\{P_S(i)\},\{P_R(i+1)\}} \frac{1}{2(N+1)} \sum_{i=0}^{n} C\left(h_o P_S(i)\right) + C(P_R(i+1))$$

$$\text{s.t.} \quad C\left(h_o P_S(i)\right) + C\left(P_R(i+1)\right) \leq C\left(P_S(i)\right), \quad i = 1, \ldots, N, and\ (14)$$

$$\mathcal{L}(P_S(i), P_R(i+1), \mu_K, \lambda_k, \gamma_i, \eta_{i+1}) =$$

$$\frac{1}{2(N+1)} \sum_{i=1}^{N} \min\{\boldsymbol{c}(P_S(i)),\boldsymbol{c}(h_0P_S(i))+\boldsymbol{c}(P_R(i+1))-$$

$$\sum_{k=1}^{N} \mu_k \Big(\sum_{i=1}^{k} B P_S(i)-E_S(i)\Big)- \sum_{k=1}^{N} \lambda_k \Big(\sum_{i=1}^{k} B P_R(i+1)-E_R(i+1)$$

$$\Big)+\sum_{k=1}^{N} \gamma_i P_S(i) +\sum_{i=1}^{N} \eta_{i+1} P_R(i+1). \qquad (9)$$

where μ_k, λ_k, γ_i and η_{i+1} are the non negative lagrangian multipliers.
By taking derivative of (9) w.r.t $P_S(i)$ and $P_R(i+1)$ then equating it to 0,we get the optimal solution for P1 as follows,

1) Case I: if $P_R^*(i+1) \geq \frac{(1-h_o)P_S^*(i)}{1+h_oP_S^*(i)}$,

$$\begin{cases} P_S^*(i) = \left(\frac{1}{4(N+1)\sum_{k=i}^{N}\mu_k} - 1\right)^+ \\ P_R^*(i+1) = \frac{(1-h_{0)}P_S^*(i)}{1+h_oP_S^*(i)} \end{cases} ;$$

2) Case II: if $P_R^*(i+1) \leq \frac{(1-h_o)P_S^*(i)}{1+h_oP_S^*(i)}$,

$$\begin{cases} P_S^*(i) = \left(\frac{1}{4(N+1)\sum_{k=i}^{N}\mu_k} - \frac{1}{h_o}\right)^+ \\ P_R^*(i+1) = \left(\frac{1}{4(N+1)\sum_{k=i}^{N}\lambda_k} - 1\right)^+ \end{cases}$$

B. The Case without Direct Link

Similar to the case with direct link, we obtain the optimal power solutions for without direct link given as below:

1) Case I: if $P_R^*(i+1) \geq P_S^*(i)$,

$$\begin{cases} P_S^*(i) = \left(\frac{1}{4(N+1)\sum_{k=i}^{N}\mu_k} - 1\right)^+ \\ P_R^*(i+1) = P_S^*(i), \end{cases} ;$$

2) Case II: if $P_R^*(i+1) \leq P_S^*(i)$,

$$\begin{cases} P_S^*(i) = P_R^*(i+1) \\ P_R^*(i+1) = \left(\frac{1}{4(N+1)\sum_{k=i}^{N}\lambda_k} - 1\right)^+ \end{cases}$$

V. OPTIMAL SOLUTION FOR NDC CASE

In this section the problem P2 will be solved for NDC case. For that we prove a problem P2 will be solved by introducing the power allocation problem to the source and relay separately by two-stage strategy i.e,

1. Obtain the optimal source power allocation by ignoring the relay
2. Optimize the relay power allocation with the obtained source power solution.

Since for both cases with and without direct link, the separation principle can be applied, we analysis the both cases as unified one.

A. Optimal Source Power Allocation

First we consider the source power allocation by ignoring the relay given as follows:

$$\max_{p_S(i) \geq 0, \forall_i} \sum_{i=1}^{N} c(h\, p_S(i)) \qquad \text{(P3)}$$

s.t. $\sum_{i=1}^{K} P_s(i) \leq \frac{1}{B} \sum_{i=1}^{k} E_S(i)$,k=1,….N,
where h is a constant with $0<h \leq 1$.

$$P_S(i)=\frac{\sum_{k=1}^{is} Es(k)}{(is-i+1)B}$$

Where $i_s = \arg \min_{i \leq j \leq N} \{ \frac{\sum_{k=i}^{j} Es(k)}{(j-i+1)B} \}$, $i \leq N$.
The optimal source power profile is given as

$$P_S^*(n)=P_S(i),\ n=i,\ldots,i_s \text{ and } \{\text{set } i=i_s+1\}$$

B. Optimal Relay Power Allocation

The optimal relay power profile can be determined by using the optimal source power profile $P_S^*(i)$ as follows:

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$$\max_{p_R(i+1)\geq 0,\forall_i} \sum_{i=1}^{N} \boldsymbol{c}(P_R(i+1)) \qquad (P4)$$

$$\text{s.t. } \sum_{i=1}^{k} \boldsymbol{c}(P_R(i+1)) \leq \sum_{i=1}^{k} \boldsymbol{c}(P_S^*(i)) - \sum_{i=1}^{k} \boldsymbol{c}(h_0 P_S^*(i))$$
$k=1,..,N$.

VI. RESULT

The simulation result is given in fig 2 for the throughput comparison of delay constraint and no-delay constraint cases.

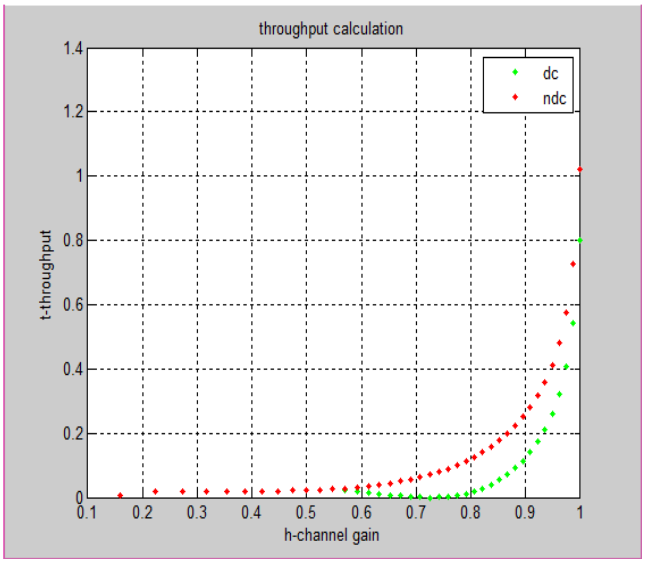

Fig. 2: Throughput Comparison of Various Power Allocation Schemes for Relay Channel

For the purpose of exposition, we assume a periodical energy profile for some predictable EH sources. Hence the source and relay energy profiles are given as

$$E_S(i)=A_S \sin\left(\frac{i-1}{N}2\pi+\frac{\pi}{2}\right)+A_S,$$
$$E_R(i+1)=A_R \sin\left(\frac{i-1}{N}2\pi+\theta\right)+A_R,\ 1\leq i\leq N$$

respectively, where $A_S, A_R > 0$ are the amplitudes of the sinusoidal energy profiles at the source and relay, respectively and θ is the phase shift between these two energy profiles.

Here we choose B=100, N=40, $\theta=\frac{5}{4}\pi$ and $A_S=A_R=200$.

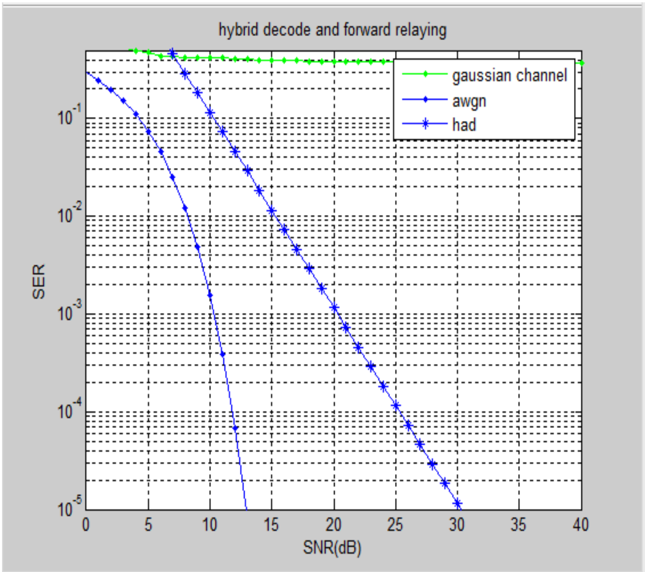

Fig. 3: SER vs. SNR for Gaussian Relay Channel

In fig 2, SER is plotted against SNR for the Gaussian relay fading channel and also for theoretical values of hybrid decode and amplify relay communication and with added theoretical AWGN noise over the channel. From the simulation results, it is observed that the NDC performs better than the DC. Since throughput increases with smaller value of channel gain for NDC but for DC it doesn't reach the throughput limit even when gain increases.

VII. CONCLUSION

In this paper, we obtain the throughput maximization problem for the orthogonal Gaussian relay channel by assuming it as deterministic model. Also the hybrid relaying is presented here i.e., the relay can choose any one from both decode and amplify protocol based on the threshold and source-relay link SNR values and forward it to the destination. In addition we efficiently compute the source and relay optimal power profiles.

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