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# A Simple Adaptive PD Control Scheme for Underactuated Mechanical Manipulators

**Yang. Jung-Hua, Yu. Shih-Shien**

***Abstract**—Robot arms have been widely used in the industry for many decades. They have played a very important role in factory automation. However, actuators failure might occur due to unfrequent maintenance or limited life cycle, which could cause severe damages to the operators and products. To solve this problem, an adaptive PD controller incorporated with a nonlinear compensation term is developed. This controller is designed based on conventional PD control scheme combined with adaptive control algorithm. Theoretical proof for the closed-loop dynamic system is given via Lyapunov theorem and La Salle's theorem. To demonstrate the validity of the controller, a number of computer simulations as well as experiments are also performed.*

***Index Terms**—Adaptive control, Underactuated mechanical system, PD control.*

## I. INTRODUCTION

Robot manipulators have been ubiquitously equipped in many industrial plants for manufacturing automation and most of them were fully actuated so as to satisfy the control requirements. However, actuators failure might occur due to unfrequent maintenance or limited life cycle, which could cause severe damages to the operators and products. In general, the parts with actuator failures would be underactuated, and thus, unexpected motion of the overall system might result. Hence, to keep a safe operation, it is inevitable to embed the manipulators with fail-safe controllers. In this paper, the underactuation problem for a class of mechanical systems is considered and appropriate nonlinear adaptive PD control is developed based on the assumption that all the system parameters are unknown. There have been abundant literatures discussing the fully actuated robots in the past years. Nevertheless, only a few regard the underactuated cases. In [1], a robust controller is designed for a multi-link manipulator in the presence of heavy loads and external disturbances. In [2], the reduction and control of underactuated mechanical systems with first order nonholonomic constraints and kinetic symmetry is addressed. The authors [3] develop a simple sufficient condition for testing STLC in underactuated mechanical systems with n degrees of freedom and n - 1 control inputs, directly based on the terms of the system inertia matrix. In the paper [4], a six-step motion strategy of the pendulum driven cart-pole system is presented to form the desired velocity and acceleration profiles of the pendulum joint. A closed loop control approach by using the partial feedback linearization technique is also devised. In [5], a control method is proposed based on Lyapunov approach.

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* Correspondence Author (s)
**Yang. Jung Hua***, Department of Vehicle Engineering, National Pingtung University of Science and Technology, Pingtung, Tawan.
**Yu. Shih Shien**, Department of Vehicle Engineering, National Pingtung University of Science and Technology, Pingtung, Tawan.



Asymptotical stability is ensured by solving some linear and partial differential equations. An adaptive control [6] with recurrent neural networks is designed for double-joint manipulators. The control is composed of the feedback feedforward part, in which the feedback part is used to stabilize the overall system while the feedforward part being used as a neural approximator for the inverse dynamics of the plant. In [7]-[8], a sliding mode controller is design for class of underactuated mechanical systems. The sliding surfaces consists second-layer structures. The first layer is constructed from stablility while the second layer is made of the first layer. The stability of the whole closed loop system is guaranteed via the Lyapunov theory. The paper [9] exploits the dynamic surface control technique to achieve exponential stability of underactuated mechanical systems. by using singular perturbation method. In [10], a design procedure by using backstepping approach is presented for underactuated mechanical systems. By using non-regular static state feedback, the original system is transformed into a class of nonlinear system with triangular structure. Furthermore, a hybrid switching control technique which alternatively switches the proportional and differential control is devised in [11]. The authors in [12] proposed a genetic algorithm based stable control approach for a class of underactuated mechanical systems. The real-valued GA is utilized to tune the control parameters in order to enhance the system performance. In [13], the nonlinear $H_\infty$ control approach is synthesized for the tracking control of a 2 DOF underactuated mechanical system. In the proposed scheme, only position measurement is needed for feedback. In [14], the authors developed a two-layer sliding surface control for a class of underactuated systems with mismatched uncertainty. The authors in [15] proposed a new concept called kinematic controllability for the stabilization of an underactuated mechanical system. This concept adopts a two-stage philosophy for the controller design. The paper is organized as follows: In Section 2, we develop an adaptive PD controller for the acrobot system, and verified by computer simulations. In section 3, the experiments are also performed to verify the effectiveness of the designed controller. The experimental result will be presented in following subsections. And finally some conclusions are given in section 4.

## II. ADAPTIVE PD CONTROLLER DESIGN

For convenience, the dynamic equations of a general underactuated mechanical system can be expressed as

$$R(q)\ddot{q} + B(q,\dot{q})\dot{q} = \bar{u} \tag{1}$$


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where

$$q = [\alpha_1 \quad \alpha_2]^T$$
$$\bar{u} = \begin{bmatrix} u \\ 0 \end{bmatrix}$$

where $q$ represents the generalized coordinate vector, $u$ is the applied torque vector, $R(q) \in R^{n \times n}$ is the inertia matrix which is symmetric and positive definite, $\alpha_1 \in R^m$ is defined as the actuated joint, $\alpha_2 \in R^{n-m}$ denotes the unactuated joint, and $m$ is the number of actuated joint. The centripetal and Coriolis terms are collected in the vector $B(q,\dot{q})\dot{q}$. Equivalently, (1) can be rewritten in following partitioned form

$$\begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} \ddot{\alpha}_1 \\ \ddot{\alpha}_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \dot{\alpha}_1 \\ \dot{\alpha}_2 \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} \tag{2}$$

Define $a_1^{\ d}$ and $a_2^{\ d}$ are the desired position of $a_1$ and $a_2$, respectively, and $e_1 = a_1 - a_1^{\ d}$, $e_2 = a_2 - a_2^{\ d}$, then the following holds

$$R\ddot{e} + B\dot{e} = \begin{bmatrix} u \\ 0 \end{bmatrix} \tag{3}$$

Now let $V_P = \begin{bmatrix} v_{11} & 0 \\ 0 & v_{12} \end{bmatrix}, V_D = \begin{bmatrix} v_{21} & 0 \\ 0 & v_{22} \end{bmatrix}$, then the equation (3) can be rewritten in following equation.

$$R\ddot{e} + B\dot{e} + V_D\dot{e} + V_P e = \begin{bmatrix} u + v_{11}e_1 + v_{21}\dot{e}_1 \\ v_{12}e_2 + v_{22}\dot{e}_2 \end{bmatrix} \tag{4}$$

Let the control input and adaptive gain be designed as

$$u = -v_{11}e_1 - v_{21}\dot{e}_1 - (e_2 + \dot{e}_2)\xi \tag{5}$$

$$\dot{\xi} = \sqrt[2n+1]{\xi}\left[\dot{e}_1(e_2 + \dot{e}_2)\xi - \dot{e}_2 v_{12} e_2 - \dot{e}_2 v_{22} \dot{e}_2\right] \tag{6}$$

In the following, it is shown that, by applying the adaptive PD controller designed above, the asymptotical stabilization of overall closed-loop system can be achieved. A complete statement of the results will be given in theorem 1.

**Theorem 1:** Consider the robot system (2) with imprecise system parameters. By applying the control laws (5) and adaptive gain (6), the objective of asymptotical stabilization can be achieved, i.e., all signal inside the close-loop system are bounded and $e_1, e_2 \to 0$ asymptotically.

***Proof*：**

To prove the theorem, let the Lyapunov function candidate be defined as:

$$L = \frac{1}{2}\dot{e}^T R\dot{e} + \frac{1}{2}e^T V_p e + \frac{2n+1}{2n}\xi^{\frac{2n}{2n+1}}$$

By taking the time derivative of $V$ we get:

$$\dot{L} = \dot{e}R\ddot{e}^T + \frac{1}{2}\dot{e}\dot{R}\dot{e} + \dot{e}^T V_p e + \dot{\xi}\xi^{\frac{-1}{2n+1}}$$

$$= \frac{1}{2}\dot{e}^T\dot{R}\dot{e} + \dot{e}^T(-B\dot{e} - V_p e - V_D\dot{e} + \begin{bmatrix} u + v_{11}e_1 + v_{21}\dot{e}_1 \\ v_{12}e_2 + v_{22}\dot{e}_2 \end{bmatrix}) + \dot{e}^T V_p e + \dot{\xi}\xi^{\frac{-1}{2n+1}}$$

$$= \dot{e}^T(-V_D\dot{e} + \begin{bmatrix} u + v_{11}e_1 + v_{21}\dot{e}_1 \\ v_{12}e_2 + v_{22}\dot{e}_2 \end{bmatrix}) + \dot{\xi}\xi^{\frac{-1}{2n+1}} \tag{7}$$

It is noted that the fact $\dot{R} - 2B$ is skew symmetrical has been applied. If we plug in the designed controller (5) and the adaptive gain (6), it is obtained that

$$\dot{L} = -\dot{e}^T V_D\dot{e} + \dot{e}_1 a + \dot{e}_1 v_{11} e_1 + \dot{e}_1 v_{21}\dot{e}_1 + \dot{e}_2 v_{12} e_2 \dot{e}_2 v_{22}\dot{e}_2 + \dot{\xi}\xi^{\frac{-1}{2n+1}}$$

$$= -\dot{e}^T V_D\dot{e} + \dot{e}_1(\dot{e}_2 + e_2)\xi + \dot{e}_2 v_{12} e_2 + \dot{e}_2 v_{22}\dot{e}_2 + \dot{\xi}\xi^{\frac{-1}{2n+1}}$$

$$= -\dot{e}^T V_D\dot{e} \tag{8}$$

From (8), it is clear that $\dot{L} = 0 \Rightarrow \ddot{e}, \dot{e} \equiv 0 \Rightarrow e = 0$ and that $\{\dot{e} \mid \dot{e} = 0, \dot{L} = 0\}$ is the largest invariant set. Hence by LaSalle's invariance theorem, the error signal $e$ and $\dot{e}$ in the closed loop system will tend to zero asymptotically.

## III. COMPUTER SIMULATION

In this section, the adaptive PD control scheme developed previously will be applied to the acrobot system via computer simulation. The acrobot system is a two link planner manipulator in which the first link is driven by an actuator but the second link is unactuated, as shown in Figure 1 in which link 1 and link 2 are connected by revolute joints.

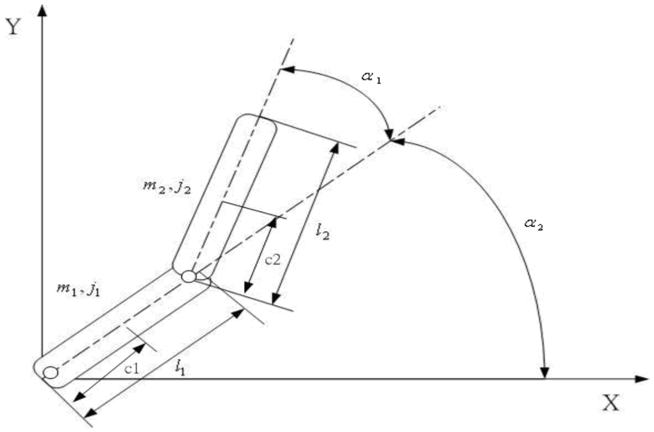


**Figure (1) Acrobot System**

The equation of motion for this system can represented in following form

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = \bar{u} \tag{9}$$

where

$$q = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, M(q) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}, \bar{u} = \begin{bmatrix} u \\ 0 \end{bmatrix}$$


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$$m_{11} = m_1 c_1^{\;2} + m_2 l_1^{\;2} + m_2 c_2^{\;2} + 2 m_2 l_1 c_2 \cos\alpha_2 + J_1 + J_2$$

$$m_{12} = m_2 c_2^{\;2} + m_2 l_1 c_2 \cos\alpha_2 + J_2$$

$$m_{21} = m_2 c_2^{\;2} + m_2 l_1 c_2 \cos\alpha_2 + J_2$$

$$m_{22} = m_2 c_2^{\;2} + J_2$$

$$c_{11} = -m_2 l_1 c_2 \dot{\alpha}_2 \sin \varepsilon_2$$

$$c_{12} = -m_2 l_1 c_2 \dot{\alpha}_1 \sin \theta_2 - m_2 l_1 c_2 \dot{\alpha}_2 \sin \alpha_2$$

$$c_{21} = m_2 l_1 c_2 \dot{\alpha}_1 \sin \alpha_2$$

$$c_{22} = 0$$

The system variables and parameters are defined as follows:

$m_1$ : mass of link 1

$m_2$ : mass of link 2

$l_1$ : length of link 1

$l_2$ : length of link 2

$c_1$ : distance to link 1 center of mass

$c_2$ : distance to link 2 center of mass

$J_1$ : inertia of link 1

$J_2$ : inertia of link 2

$\alpha_1$ : displacement of link 1

$\alpha_2$ : displacement of link 2

$u$ : applied torque

The desired position of link 1 is set to be $\alpha_1^{\;d} = 45$ degrees and link 2 is desired to maintain at original location, i.e. $\alpha_2^{\;d} = 0$ degree. To demonstrate the effectiveness of our proposed scheme, we first apply a conventional PD controller $u = -v_{11} e_1 - v_{21} \dot{e}_1$ without any compensation to the acrobot system. It is observed from figure 2 and 3 that link1 could converge to target quickly, but link 2 was unstable.

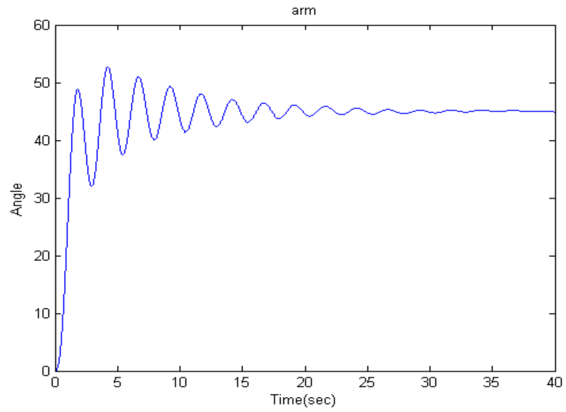


**Figure 2. Link 1 Response with Conventional PD Control**

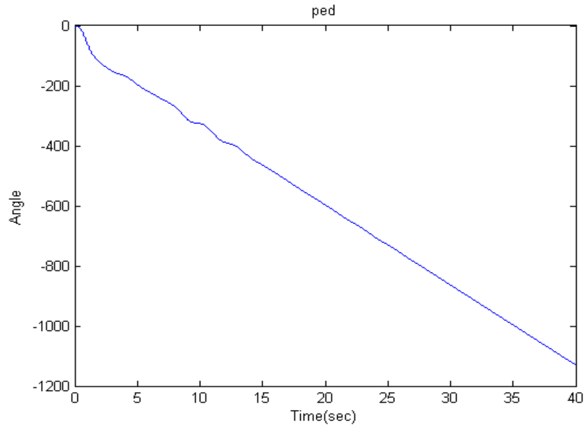


**Figure 3. Link 2 Response with Conventional PD Control**

To show the validity of the proposed adaptive PD control, the algorithms developed in (5) and (6) are then applied. Figure 4 and 5 depict the tracking and stabilization responses for $\alpha_1$ and $\alpha_2$. In Figure 6, it is observed that the adaptive gain $\xi$ converge to some constant when the error tends to zero. The control input magnitude is plotted in Figure 7. The control gains are chosen to be $v_{11} = 7.2$ , $v_{12} = 0.3$ , $v_{21} = 1900$ , $v_{22} = 170$ .

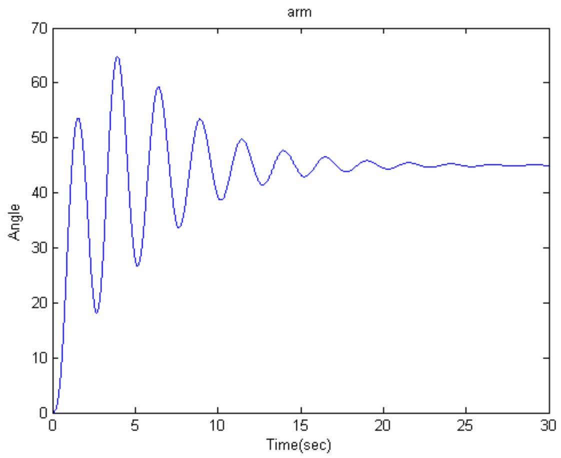


**Figure 4. The Link 1 Response with Adaptive PD Control**

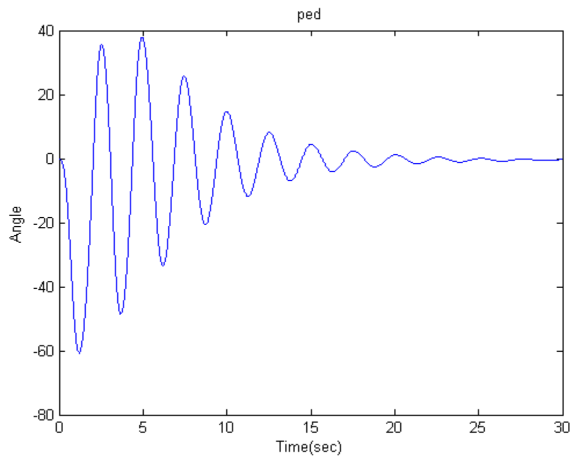


**Figure 5. The Link 2 Response with Adaptive PD Control**



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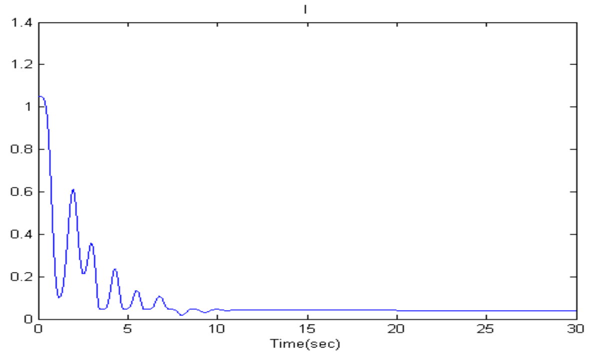


**Figure 6. Adaptive Gain $\xi$**

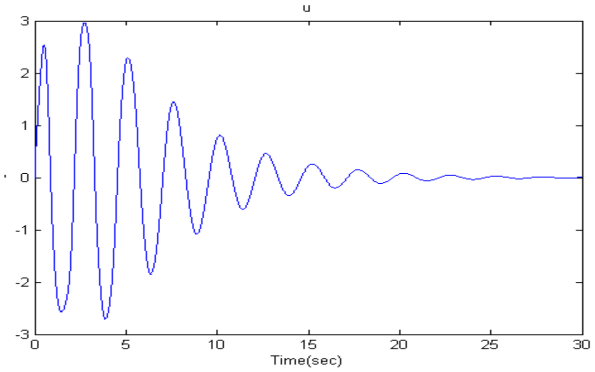


**Figure 7. Control Input U**

## IV. EXPERIMENTAL VERIFICATION

In this section, practical experiments are conducted to validate the application of the proposed algorithm for the underactuated mechanical system mentioned above. The mechanical system is set up with free rotation at the second joint. The first joint is embedded with a DC motor whose rotational angle is measured by an encoder. The second is only equipped with an encoder for measurement of angular position. Figure 8 shows the overview of the experimental apparatus.

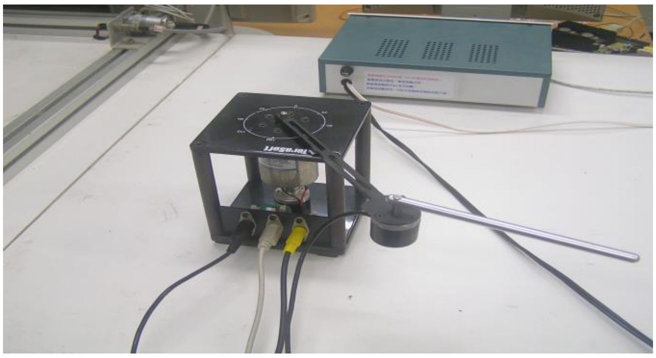

**Figure 8. Experimental Setup for the Underactuated Mechanical System**

The proposed adaptive PD controller is applied to the acrobot robot with the following controlled gains $v_{11}$=1, $v_{21}$=1.5, $v_{12}$=2, $v_{22}$=1. Figure 9 and 10 depict the position response of link 1 and link 2. The adaptive gain $k$ are shown in the Figure 11. Finally the control input is described in the Figure 12.

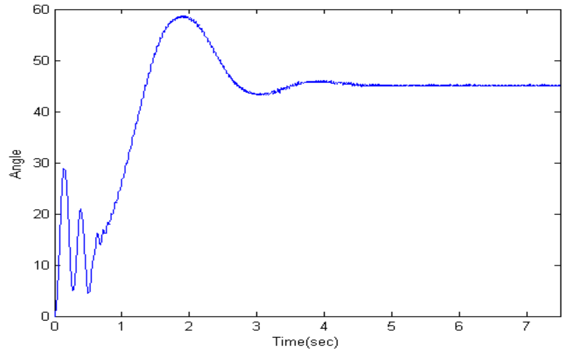


**Figure 9. Link 1 Response with Adaptive PD Controller**

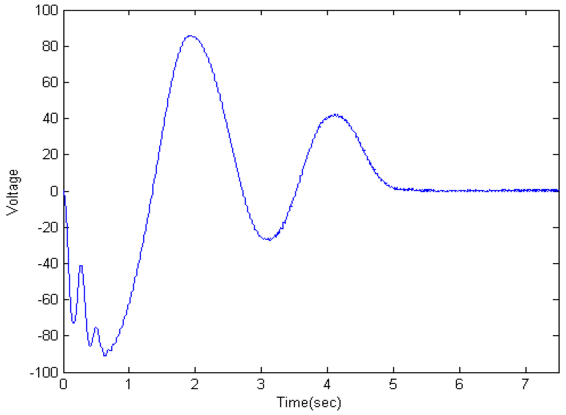


**Figure 10. Link 2 Response with Adaptive PD Controller**

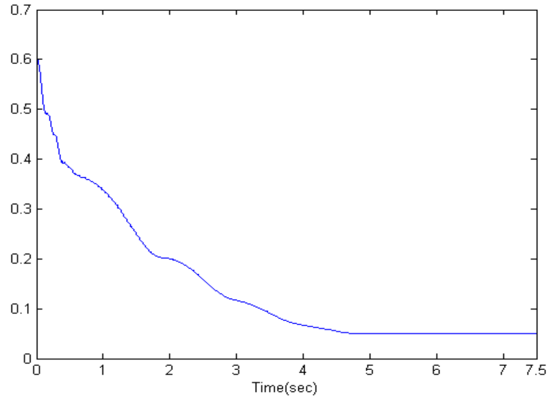


**Figure 11. Adaptive Gain $\xi$**

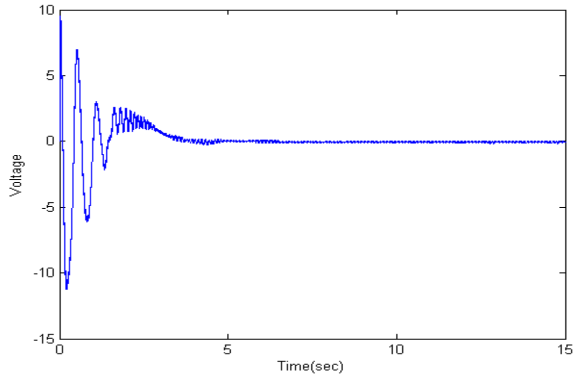


**Figure 12. Control Input**



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## V. CONCLUSION

In this paper, an adaptive PD control design has been presented for the underactuated mechanical manipulator system. By utilizing Lyapunov-based stability analysis asymptotical stabilization of the closed-loop system can be guaranteed. In addition to the evaluation from computer simulation, the control schemes are also practically implemented in the acrobot system to verify the performance of the proposed controller.

## REFERENCES


[1] A. Chemori and A. Loria (2004) Control of a Planar Underactuated Biped on a Complete Walking Cycle, IEEE Transaction on Automatic Control, Vol. 49, No. 5, pp. 838-843.

[2] F. Mnif (2003) On the Reduction and Control for a Class of Non-holonomic Underactuated Systems, Journal of ELECTRICAL ENGINEERING, Vol. 54, No. 1-2, pp. 22-29.

[3] A.D. Luca and S. Iannitti (2002) A Simple STLC Test for Mechanical Systems Under-actuated by One Control, Proc. of the IEEE International Conference on Robotics and Automation, pp. 1735-1740.

[4] H. Yu, Y. Liu, and T. Yang (2007) Tracking Control of A Peddulum-driven Cart-pole Underactuated System, IEEE International Conference on Systems, Man, and Cybernetics, Montreal, QC, Canada, 7-10, Oct.

[5] W. N. White and M. Foss (2006) A Direct Lyapunov Approach for a Class of Underactuated Mechanical Systems, Proc. of the American Control Conference Minneapolis, Minnesota, USA, June 14-16.

[66] C. K. Lin (2003) A Reinforcement Learning Adaptive Fuzzy Controller for Robots, Fuzzy set and systems, pp. 339-352.

[7] C. Y. Su and Y. Stepanenko (1999) Adaptive Variable Structure Set-point Control of Underactated Robot, IEEE Transactions on Automatic Control, Vol. 44, No. 11.

[8] M. Zhang and T. J. Tarn (2003) A Hybrid Switching Control Strategy for Nonlinear and Underactuated Mechanical Systems, IEEE Transaction on Automatic Control, VOL. 48, NO.10, pp. 1777-1782.

[9] Nadeem Qaiser, Naeem Iqbal, Amir Hussain, and Naeem Qaiser (2007), Exponential Stabilization of a Class of Underactuated Mechanical Systems using Dynamic Surface Cont, International Journal of Control, Automation, and Systems, vol. 5, no. 5, pp. 547-558, October.

[10] Z. Sun, S. S. Ge, and T. H. Lee (2001) Stabilization of Underactuated Mechanical Systems: A Non-regular Backstepping Approach, International Journal of Control, Vol. 74, No. 11, pp. 1045-1051.

[11] G. Hu, ,C. Makkar, and W. E. Dixon (2007) Energy-Based Nonlinear Control of Underactuated Euler-Lagrange Systems Subject to Impacts, IEEE Trans. Automatic Control, Vol. 52, No. 9, pp. 1742-1748.

[12] D. Liu, W. Guo, and J. Yi (2008) Dynamics and GA-Based Stable Control for a Class of Underactuated Mechanical Systems, International Journal of Control, Automation, and Systems, Vol. 6, No. 1, pp. 35-43.

[13] L. T. Aguilar (2009) Output Feedback Nonlinear $H_\infty$ Tracking Control of a Nonminimum Phase 2 DOF Underactuated Mechanical Systems, Journal of Robotics, Vol. 2009.

[14] Y. Hao, J. Yi, D. Zhao, and D. Qian. (2008), Robust Control Using Incremental Sliding Mode for Underactuated Systems with Mismatched Uncertainties, Proc. of American Control Conference Westin Seattle Hotel, Seattle, Washington, USA , June 11-13.

[15] J.A. Acosta and M. L'opez-Martınez (2005) Constructive Feedback Linearization of Underactuated Mechanical Systems with 2-DOF, Proc. of the 44th IEEE Conference on Decision and Control Seville, pp.12-15.



**Yang. Jung-Hua**, received the BS in Electrical Engineering from the Chung-Yuan Christian University, Chung-Li, Taiwan, in 1989, and the MS and PhD in Electrical Engineering from National Taiwan University, Taipei, Taiwan, in 1991 and 1995, respectively. Currently, he is a Professor in the Department of Vehicle Engineering, National Pingtung University of Science and Technology where he has been since 2002. His current research interests include non-linear adaptive control, sliding mode control, vehicle motion control and applications in mechatronic systems.

**Yu. Shieh Shien**, received the M..S. Degree from the Graduate Institute of Vehicle Engineering at National Pingtung University of Science and Technology in 2007. His research interests include control application, robot manipulator, and vehicle control.




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