

# Windowed Independent Update Algorithm for ODCT-I

Richa Sharma, Sachin Sharma, Nitin Pundir

**Abstract:** Data Compression is the process to eliminate redundancy between the neighboring pixel. To do this DCT is much more efficient. In this paper, an algorithm has been developed for DCT-V or ODCT-I using rectangular window. These updated DCT coefficient are independent of the Odd DST coefficient.

**Keywords:** DCT,DST,EDCT,ODCT

## I. Introduction

Discrete Cosine Transform (DCT) is widely used Fourier transform of real numbers introduced in 1974. DCT's are distinguished on the basis whether the denominator in the transform kernel is odd or even. The DCT's I, II, III, IV are called Even DCT's and are deeply studied and various Update Algorithms are provided for fast computation. DCT's V, VI, VII, VIII are called ODD DCT's. Odd transforms of ODCT's were discovered by A. Jain in 1979. Originally shift properties were discussed by Yip and Rao [2] of even DCT and its mate DST. Later on these results were extended to odd transforms by Le-Nan Wu [3]. To efficiently evaluate the ODCT of a long string of many data points, a suitable length of data N is taken. This set of N data points serve as Window [4][8]. When first data points are transformed this window is shifted one by one [2] or shift can vary from 1 to N-1 [1][4]. Previously, algorithms have been developed to shift the window [4][8] between 1 and N-1 [1] data points. In this paper we utilize that knowledge to provide the update algorithm of DCT-V/ODCT-I, as we interest in ODCT transform based applications in image/video coding. This fast algorithm can be utilized in transforms in HEVC (High Efficiency Video Coding) and future image and video coding standards [7].

### 1. Discrete cosine transform

#### 2.1 Odd Discrete Cosine Transform(ODCT)-I

The ODCT of a signal f(x) of length N is defined is-

$$C(n,k) = a P_k \sum_{x=0}^{N-1} P_x f(n-N+x) \cos \frac{2xk\pi}{M} \dots(1)$$

The ODST of a signal f(x) of length N is defined in [1][4] is

$$S(n,k) = a \sum_{x=1}^{N-1} f(n-N+x) \sin \frac{2xk\pi}{M} \dots\dots(2)$$

Where

$$x = 0,1,2 \dots N-1, k = 0,1,2 \dots N-1$$

$$M = 2N-1$$

$$a = \frac{2}{\sqrt{M}}$$

$$P_k = \begin{cases} 1 & \text{if } k \neq 0 \text{ or } N \\ \frac{1}{\sqrt{2}} & \text{if } k = 0 \text{ or } N \end{cases}$$

First we take some data input for which we are required to find out ODCT-I co-efficient. Generally we take 8 or 4 data points together for our convenience. Then we calculate it's ODCT coefficient using the formula.

$$C(n,k) = a k_k \sum_{x=0}^{N-1} k_x f(n-N+x) \cos \frac{2xk\pi}{M}$$

then we save these co-efficient and we take "r" number of more data point as input, and then with the help of old saved co-efficient and the new data point, we find out the ODCT-I of new data points. For this Operation we have derived formulae given below. We call it simultaneous update because while deriving we get one term of DST. So, for finding out co-efficient of new data points we have to track DCT as well as DST of the old data points. This problem has been short out using Independent Update

#### 2.2 ODCT Type-I Rectangular r-Point Update Derivation

The ODCT and ODST type-I update algorithms as developed by Kakad and Sherlock[1] for rectangular window are given by equations (1) and (2) respectively.

"r -Point" independent update algorithm is the algorithm which does not require to compute the ODST of the previous windowed data to calculate the ODCT of the coming windowed data, this algorithm is achieved by adding the final expressions of "(n+r)-point" equation and "(n-r) point" equation. In N+r point expression the DST term in it and in N-r point expression the DST term in it, so when we add these two expressions the DST terms in both get canceled out by each other and we get "r-point" independent update algorithm. Signal which is Non-windowed treated as windowed by the rectangular window W(x)=1 since multiplication by this window does not alter the signal data. Here r-point independent rectangular windowed independent update equation for ODCT-I is derived.

**Manuscript published on 30 June 2014.**

\* Correspondence Author (s)

**Richa Sharma** Assistant Professor at ABES Engineering College, Ghaziabad (U.P.), India.

**Sachin Sharma** Assistant Professor in ABES Engineering College, Ghaziabad (U.P.), India

**Nitin Pundir** B.Tech (Electronics and Communication Engineering) from Uttar Pradesh Technical University at ABES Engineering College, Ghaziabad (U.P.), India

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>



Let we have signal  $f(x) = 0, 1, 2, \dots, N-1$ , for which we have to calculate update ODCT co-efficient. If  $r$  new data points arrive (where  $0 < r < N-1$ ) then ODCT of that shifted signal is

$C(n+r, k)$ .

That effect of new data point we calculated using equation (1)

Using equation (1) and (2)

$$\begin{aligned}
 &= aP_k C(n, k) \cos\left(\frac{2\pi r k}{M}\right) + aP_k S(n, k) \sin\left(\frac{2\pi r k}{M}\right) \\
 &+ aP_k \sum_{y=0}^{r-1} P_{y+N-r} f(n+y) \cos\frac{2(y-r+N)\pi k}{M} \\
 &- aP_k \sum_{y=0}^{r-1} P_y \cos\frac{2(y-r)k\pi}{M} f(n-N+y) \\
 &- aP_k f(n-N+r) + \frac{aP_k}{\sqrt{2}} f(n-N+r) \\
 &= aP_k C(n, k) \cos\left(\frac{2\pi r k}{M}\right) + aP_k S(n, k) \sin\left(\frac{2\pi r k}{M}\right) \\
 &- aP_k \sum_{y=0}^{r-1} P_y \cos\frac{2(y-r)k\pi}{M} f(n-N+y) \\
 &+ \left(\frac{1}{\sqrt{2}} - 1\right) aP_k f(n-N+r) \\
 &+ aP_k \sum_{y=0}^{r-1} P_{y+N-r} f(n+y) \cos\left(\frac{2(y-r+N)\pi k}{M}\right) \\
 &= aP_k C(n, k) \cos\left(\frac{2\pi r k}{M}\right) + aP_k S(n, k) \sin\left(\frac{2\pi r k}{M}\right) \\
 &- aP_k \sum_{y=0}^{r-1} P_y \cos\left(\frac{2(y-r)k\pi}{M}\right) f(n-N+y) \\
 &+ \left(\frac{1}{\sqrt{2}} - 1\right) aP_k f(n-N+r) \\
 &+ aP_k \sum_{y=0}^{r-1} P_{y+N-r} f(n+y) \cos\left(\frac{2(y-r+N)\pi k}{M}\right) \\
 &= aP_k C(n, k) \cos\left(\frac{2\pi r k}{M}\right) + aP_k S(n, k) \sin\left(\frac{2\pi r k}{M}\right) \\
 &- aP_k \sum_{y=0}^{r-1} P_y \cos\left(\frac{2(y-r)k\pi}{M}\right) f(n-N+y) \\
 &+ \left(\frac{1}{\sqrt{2}} - 1\right) aP_k f(n-N+r) + aP_k \sum_{y=0}^{r-1} f(n+y) \cos\frac{2(y-r)\pi k}{M} \\
 &+ aP_k \sum_{y=0}^{r-1} f(n+y) \cos\frac{(2N-1)\pi k}{M}
 \end{aligned}$$

We know that  $2N-1=M$  so

$$\begin{aligned}
 &= aP_k C(n, k) \cos\left(\frac{2\pi r k}{M}\right) + aP_k S(n, k) \sin\left(\frac{2\pi r k}{M}\right) \\
 &- aP_k \sum_{y=0}^{r-1} P_y \cos\left(\frac{2(y-r)k\pi}{M}\right) f(n-N+y)
 \end{aligned}$$

$$\begin{aligned}
 &+ \left(\frac{1}{\sqrt{2}} - 1\right) aP_k f(n-N+r) + aP_k \sum_{y=0}^{r-1} f(n+y) \cos\left(\frac{2(y-r)\pi k}{M}\right) \\
 &+ aP_k \sum_{y=0}^{r-1} f(n+y) \cos\left(\frac{\pi k}{M}\right) \\
 &= aP_k C(n, k) \cos\left(\frac{2\pi r k}{M}\right) + aP_k S(n, k) \sin\left(\frac{2\pi r k}{M}\right) \\
 &- aP_k \sum_{y=0}^{r-1} P_y \cos\left(\frac{2(y-r)k\pi}{M}\right) f(n-N+y) + \left(\frac{1}{\sqrt{2}} - 1\right) aP_k f(n-N+r) \\
 &+ aP_k (-1)^k \sum_{y=0}^{r-1} f(n+y) \cos\left(\frac{(2y-2r+1)\pi k}{M}\right) \dots (3)
 \end{aligned}$$

Using the equation (1) the old points which exit from the window can be calculated as -

$$C(n-r, k) = aP_k \sum_{x=0}^{r-1} P_{y+r} f(n-N+x-r) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$C(n-r, k) = aP_k \sum_{y=-r}^{N-r-1} P_{y+r} f(n-N+y) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$= aP_k \sum_{y=-r}^{N-r-1} P_{y+r} f(n-N+y) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ aP_k \sum_{y=-r}^{-1} P_{y+r} f(n-N+y) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

Put  $P_{y+r} - P_y + P_y$  in first summation

$$= aP_k \sum_{y=-r}^{N-r-1} (P_{y+r} - P_y + P_y) f(n-N+y) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ aP_k \sum_{y=-r}^{-1} P_{y+r} f(n-N+y) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$= aP_k \sum_{y=-r}^{N-r-1} (P_{y+r} - P_y) f(n-N+y) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ aP_k \sum_{y=0}^{N-r-1} P_y f(n-N+y) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ aP_k \sum_{y=-r}^{-1} P_{y+r} f(n-N+y) \cos\left(\frac{2(y+r)k\pi}{M}\right) \dots (4)$$

By solving first summation -

$$= aP_k \sum_{y=-r}^{N-r-1} (P_{y+r} - P_y) f(n-N+y) \cos\left(\frac{2(y+r)\pi k}{M}\right)$$

At  $y=0$

$$P_r - P_0 = 1 - \frac{1}{\sqrt{2}}$$

At  $y=1$

$$P_{r+1} - P_1 = 1 - 1 = 0$$

At  $y=N-r-1$

$$P_{N-1} - P_{N-r-1} = 1 - 1 = 0$$



Now the term will be –

$$= a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \frac{2rk\pi}{M}$$

Solving the second summation –

$$\begin{aligned} &= a P_k \sum_{y=0}^{N-r-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &= a P_k \sum_{y=0}^{N-r-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &= a P_k \sum_{y=0}^{N-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &\quad - a P_k \sum_{y=N-r}^{N-1} P_y (n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &= a P_k \sum_{y=0}^{N-1} P_y f(n-N+y) \cos \left(\frac{2\pi yk}{M}\right) \cos \left(\frac{2\pi yk}{M}\right) \\ &\quad - a P_k \sum_{y=0}^{N-1} P_y f(n-N+y) \sin \left(\frac{2\pi yk}{M}\right) \sin \left(\frac{2\pi yk}{M}\right) \\ &\quad - a P_k \sum_{y=N-r}^{N-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \end{aligned}$$

Using equations (1) and (2) we can write as-

$$\begin{aligned} &= a P_k C(n,k) \cos \left(\frac{2\pi yk}{M}\right) - a P_k S(n,k) \sin \left(\frac{2\pi yk}{M}\right) \\ &\quad - a P_k \sum_{y=N-r}^{N-1} P_y f(n-N+y) \cos \frac{2(y+r)\pi k}{M} \end{aligned}$$

Now equation (4) will become –

$$\begin{aligned} &= a P_k C(n,k) \cos \left(\frac{2rk\pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2rk\pi}{M}\right) \\ &\quad + a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2rk\pi}{M}\right) \\ &\quad - a P_k \sum_{y=N-r}^{N-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &\quad + a P_k \sum_{y=-r}^{-1} P_{y+r} f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &= a P_k C(n,k) \cos \left(\frac{2rk\pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2rk\pi}{M}\right) \\ &\quad + a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2rk\pi}{M}\right) \\ &\quad - a P_k \sum_{y=N-r}^{N-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &\quad + a P_k \sum_{y=-r}^{-1} P_{y+r} f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \end{aligned}$$

Put  $y = -y$  in last summation –

$$= a P_k C(n,k) \cos \left(\frac{2rk\pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2rk\pi}{M}\right)$$

$$\begin{aligned} &+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2rk\pi}{M}\right) \\ &\quad - a P_k \sum_{y=N-r}^{N-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &\quad + a P_k \sum_{y=1}^r P_{y+r} f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \end{aligned}$$

Put  $x=y-N+r$  in the 4<sup>th</sup> term so the equation will become –

$$\begin{aligned} &= a P_k C(n,k) \cos \left(\frac{2rk\pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2rk\pi}{M}\right) \\ &\quad + a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2rk\pi}{M}\right) \\ &\quad - a P_k \sum_{x=0}^{r-1} P_{(x+N-r)} f(n-r+x) \cos \left(\frac{2(x+N)\pi k}{M}\right) \\ &\quad + a P_k \sum_{y=1}^r P_{y+r} f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &= a P_k C(n,k) \cos \left(\frac{2rk\pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2rk\pi}{M}\right) \\ &\quad + a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2rk\pi}{M}\right) \\ &\quad - a P_k \sum_{y=0}^{r-1} P_{(y+N-r)} f(n-r+y) \cos \left(\frac{2(y+N)\pi k}{M}\right) \\ &\quad + a P_k \sum_{y=1}^r P_{y+r} f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right) \\ &= a P_k C(n,k) \cos \left(\frac{2rk\pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2rk\pi}{M}\right) \\ &\quad + a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2rk\pi}{M}\right) \\ &\quad - a P_k \sum_{y=0}^{r-1} P_{(y+N-r)} f(n-r+y) \cos \left(\frac{2(y+N)\pi k}{M}\right) \\ &\quad + a P_k \sum_{y=0}^{r-1} P_{r-1} f(n-N+y) \cos \left(\frac{2(r-y)k\pi}{M}\right) \\ &\quad + a P_k \frac{1}{\sqrt{2}} f(n-N-r) - a P_k f(n-N) \cos \left(\frac{2rk\pi}{M}\right) \dots (5) \end{aligned}$$

Adding equation (3) and (5) –

$$\begin{aligned} &= 2C(n,k) \cos \left(\frac{2rk\pi}{M}\right) \\ &\quad - a P_k \sum_{y=0}^{r-1} P_y \cos \left(\frac{2(y-r)k\pi}{M}\right) f(n-N+y) \end{aligned}$$



$$\begin{aligned}
 &+ ak_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n - N + r) \\
 &+ a P_k (-1)^k \sum_{y=0}^{r-1} f(n+y) \cos\left(\frac{(2y-2r+1)k\pi}{M}\right) \\
 &+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n - N) \cos\left(\frac{2rk\pi}{M}\right) \\
 &- a P_k \sum_{y=0}^{r-1} P_{y+N-r} f(n+y-r) \cos\left(\frac{2(y+N)k\pi}{M}\right) \\
 &+ a P_k \sum_{y=0}^{r-1} P_{r-y} f(n-N-y) \cos\left(\frac{2(r-y)k\pi}{M}\right) \\
 &+ \frac{a P_k}{\sqrt{2}} f(n - N - r) - a P_k f(n - N) \cos\left(\frac{2rk\pi}{M}\right)
 \end{aligned}$$

Final expression for “n+r” point –

$$\begin{aligned}
 &= ak_k C(n, k) \cos\left(\frac{2\pi rk}{M}\right) + ak_k S(n, k) \sin\left(\frac{2\pi rk}{M}\right) \\
 &- ak_k \sum_{y=0}^{r-1} k_y \cos\frac{2(y-r)k\pi}{M} f(n - N + y) \\
 &+ \left(\frac{1}{\sqrt{2}} - 1\right) ak_k f(n - N + r) \\
 &+ ak_k \sum_{y=0}^{r-1} f(n+y) \cos\frac{2(y-r)\pi k}{M} \\
 &+ ak_k (-1)^k \sum_{y=0}^{r-1} f(n+y)
 \end{aligned}$$

Final expression for “n-r” point –

$$\begin{aligned}
 &= ak_k C(n, k) \cos\left(\frac{2r\pi k}{M}\right) - ak_k S(n, k) \sin\left(\frac{2r\pi k}{M}\right) \\
 &+ ak_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n - N) \cos\frac{2rk\pi}{M} \\
 &- ak_k \sum_{y=0}^{r-1} k_{(y+N-r)} f(n - r + y) \cos\frac{2(y+N)\pi k}{M} \\
 &+ ak_k \sum_{y=0}^{r-1} k_{r-1} f(n - N + y) \cos\frac{2(r-y)k\pi}{M} \\
 &+ ak_k * \frac{1}{\sqrt{2}} f(n - N - r) \\
 &- ak_k * 1 * f(n - N) \cos\frac{2rk\pi}{M}
 \end{aligned}$$

We got the expressions for “n+r” and “n-r” points now we add these two expressions to obtain the expression for r-point independent algorithm as –

$$\begin{aligned}
 &= 2C(n, k) \cos\left(\frac{2rk\pi}{M}\right) \\
 &- a P_k \sum_{y=0}^{r-1} P_y \cos\left(\frac{2(y-r)k\pi}{M}\right) f(n - N + y)
 \end{aligned}$$

$$\begin{aligned}
 &+ ak_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n - N + r) \\
 &+ a P_k (-1)^k \sum_{y=0}^{r-1} f(n+y) \cos\left(\frac{(2y-2r+1)k\pi}{M}\right) \\
 &+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n - N) \cos\left(\frac{2rk\pi}{M}\right) \\
 &- a P_k \sum_{y=0}^{r-1} P_{y+N-r} f(n+y-r) \cos\left(\frac{2(y+N)k\pi}{M}\right) \\
 &+ a P_k \sum_{y=0}^{r-1} P_{r-y} f(n-N-y) \cos\left(\frac{2(r-y)k\pi}{M}\right) \\
 &+ \frac{a P_k}{\sqrt{2}} f(n - N - r) - a P_k f(n - N) \cos\left(\frac{2rk\pi}{M}\right) \dots (6)
 \end{aligned}$$

### Conclusion

In this paper, computationally efficient algorithm has been proposed for developing the DCT shifting data sequence. The derived algorithm is independent of ODST coefficient. The data windowed by rectangular window. The size of shift can be any value from 0 to N-1 where N is the length of DCT. The proposed algorithm has been verified using Matlab and found it is efficient and fast than the finding coefficient by standard definition.

### References

1. B. G. Sherlock, Y. P. Kakad, A. Shukla, “Rapid update of odd DCT and DST for real-time signal processing, IEEE Trans. Proc. of SPIE Vol. 5809 471.
2. P. Yip and K. R. Rao, “On the Shift Property of DCT’s and DST’s”, IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-35, No.3, March 1987, pp. 404-406.
3. Le-Nan Wu, “Comments on ‘On the Shift Property of DCT’s and DST’s”, IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-38, No. 1, January 1990, pp. 186-188.
4. B.G. Sherlock and Y.P Kakad, “Windowed Discrete Cosine and Sine Transforms for Shifting Data”, submitted to Signal Processing, March 2000.
5. V. Britanak, P. C. Yip, K. R. Rao, "Discrete Cosine and Sine Transforms: General Properties, Fast Algorithms and Integer Approximations", Academic Press 2006.
6. Syed Ali Khayam, The Discrete Cosine Transform: Theory and Application, March 10th 2003, ECE 802 –602: Information Theory and Coding.
7. J.F.Blinn, “What is deal with DCT,” IEEE Computer Graphics and Application, Vol-13, March 1993, issue: 4, pp:78-83
8. H.Kwakernaak, R Sivan, Modern Signals and Systems, Prentice-Hall, Englewood Cliffs, NJ, 1991.



**Richa Sharma** received her B.E. degree from Rajiv Gandhi proudlyogiki vishwavidyalaya, Bhopal, Madhya Pradesh in 2010, M.tech (Electronics and communication) from Jaypee institute of information technology, Noida, India in 2012. she worked as Assistant Professor at ABES Engineering college, Ghaziabad, India. Her teaching and research are in signal processing and image processing. She is member of IAENG (International association of engineers) and (IACIT) International Association of Computer Science and Information Technology





**Sachin Sharma** received his B.tech degree from Kurukshetra university, India at M.M.E.C ,Mulana,M.tech from YMCA university of science and technology, Haryana, India. He is working as Assistant Professor in ABES Engineering college, Ghaziabad,India. He is member of IAENG (International association of engineers)



**Nitin Pundir** was born in Delhi, India on June 06,1993. He is pursuing his BTech (Electronics and Communication Engineering) from Uttar Pradesh Technical University at ABES Engineering College. He was an active member of SAE (Society Of Automotive Engineers) , Northern India Section. His area of work and interest is in HDL designing and FPGA implementation.