International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249 – 8958, Volume-3, Issue-5, June 2014

Windowed Independent Update Algorithm for ODCT-I

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***Abstract:** Data Compression is the process to eliminate redundancy between the neighboring pixel. To do this DCT is much more efficient. In this paper, an algorithm has been developed for DCT-V or ODCT-I using rectangular window. These updated DCT coefficient are independent of the Odd DST coefficient.*

***Keywords:** DCT,DST,EDCT,ODCT*

I. Introduction

Discrete Cosine Transform (DCT) is widely used Fourier transform of real numbers introduced in 1974. DCT's are distinguished on the basis whether the denominator in the transform kernel is odd or even. The DCT's I, II, III, IV are called Even DCT's and are deeply studied and various Update Algorithms are provided for fast computation. DCT's V, VI, VII, VIII are called ODD DCT's. Odd transforms of ODCT's were discovered by A. Jain in 1979. Originally shift properties were discussed by Yip and Rao [2] of even DCT and its mate DST. Later on these results were extended to odd transforms by Le-Nan Wu [3]. To efficiently evaluate the ODCT of a long string of many data points, a suitable length of data N is taken. This set of N data points serve as Window [4][8]. When first data points are transformed this window is shifted one by one [2] or shift can vary from 1 to N-1 [1][4] . Previously, algorithms have been developed to shift the window [4][8] between 1 and N-1 [1] data points. In this paper we utilize that knowledge to provide the update algorithm of DCT-V/ODCT-I, as we interest in ODCT transform based applications in image/video coding. This fast algorithm can be utilized in transforms in HEVC (High Efficiency Video Coding) and future image and video coding standards [7].

1. Discrete cosine transform

2.1 Odd Discrete Cosine Transform(ODCT)-I

The ODCT of a signal f(x) of length N is defined is-

$$C(n,k)=a\,P_k\sum_{x=0}^{N-1}P_x\,f(n-N+x)\cos\frac{2xk\pi}{M}\;....(1)$$

The ODST of a signal f(x) of length N is defined in [1][4] is

$$S(n,k)=a\sum_{x=1}^{N-1}f(n-N+x)\sin\frac{2xk\pi}{M}\;........(2)$$

Where

$$x=0,1,2\ldots N-1,\quad k=0,1,2\ldots N-1$$

$$M=2N-1$$

$$a=\frac{2}{\sqrt{M}}$$

$$P_k=\begin{cases}1 & \text{if } k\neq 0 \text{ or } N\\ \sqrt{\dfrac{1}{\sqrt{2}}} & \text{if } k=0 \text{ or } N\end{cases}$$

First we take some data input for which we are required to find out ODCT-I co-efficient. Generally we take 8 or 4 data points together for our convenience. Then we calculate it's ODCT coefficient using the formula.

$$C(n,k)=ak_k\sum_{x=0}^{N-1}k_x\,f(n-N+x)\cos\frac{2xk\pi}{M}$$

then we save these co-efficient and we take "r" number of more data point as input, and then with the help of old saved co-efficient and the new data point, we find out the ODCT-I of new data points. For this Operation we have derived formulae given below. We call it simultaneous update because while deriving we get one term of DST. So, for finding out co-efficient of new data points we have to track DCT as well as DST of the old data points. This problem has been short out using Independent Update

2.2 ODCT Type-I Rectangular r-Point Update Derivation

The ODCT and ODST type-I update algorithms as developed by Kakad and Sherlock[1] for rectangular window are given by equations (1) and (2) respectively.

"r –Point" independent update algorithm is the algorithm which does not require to compute the ODST of the previous windowed data to calculate the ODCT of the coming windowed data, this algorithm is achieved by adding the final expressions of "(n+r)-point" equation and "(n-r) point" equation. In N+r point expression the DST term in it and in N-r point expression the DST term in it, so when we add these two expressions the DST terms in both get canceled out by each other and we get "r-point" independent update algorithm. Signal which is Non-windowed treated as windowed by the rectangular window W(x)=1 since multiplication by this window does not alter the signal data. Here r-point independent rectangular windowed independent update equation for ODCT-I is derived.

Let we have signal f(x) = 0,1,2...N-1, for which we have to calculate update ODCT co-efficient. If r new data points arrive (where 0 < r < N-1) then ODCT of that shifted signal is

Manuscript Received on June 2014

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C(n+r,k).
That effect of new data point we calculated using equation (1)

Using equation (1) and (2)

$$= aP_k C(n,k)\cos\left(\frac{2\pi rk}{M}\right) + aP_k S(n,k)\sin\left(\frac{2\pi rk}{M}\right)$$

$$+ aP_k \sum_{y=0}^{r-1} P_{y+N-r} f(n+y)\cos\frac{2(y-r+N)\pi k}{M}$$

$$- aP_k \sum_{y=0}^{r-1} P_y \cos\frac{2(y-r)k\pi}{M} f(n-N+y)$$

$$- aP_k f(n-N+r) + \frac{aP_k}{\sqrt{2}} f(n-N+r)$$

$$= a P_k C(n,k)\cos\left(\frac{2\pi rk}{M}\right) + aP_k S(n,k)\sin\left(\frac{2\pi rk}{M}\right)$$

$$- aP_k \sum_{y=0}^{r-1} P y \cos\frac{2(y-r)k\pi}{M} f(n-N+y)$$

$$+ \left(\frac{1}{\sqrt{2}} - 1\right) a P_k f(n-N+r)$$

$$+ aP_k \sum_{y=0}^{r-1} P_{y+N-r} f(n+y)\cos\left(\frac{2(y-r+N)\pi k}{M}\right)$$

$$= a P_k C(n,k)\cos\left(\frac{2\pi rk}{M}\right) + a P_k S(n,k)\sin\left(\frac{2\pi rk}{M}\right)$$

$$- a P_k \sum_{y=0}^{r-1} k_y \cos\frac{2(y-r)k\pi}{M} f(n-N+y)$$

$$+ \left(\frac{1}{\sqrt{2}} - 1\right) a P_k f(n-N+r)$$

$$+ a P_k \sum_{y=0}^{r-1} P_{y+N-r} f(n+y)\cos\left(\frac{2(y-r+N)-1+1}{M}\right)\pi k$$

$$= a P_k C(n,k)\cos\left(\frac{2\pi rk}{M}\right) + a P_k S(n,k)\sin\left(\frac{2\pi rk}{M}\right)$$

$$- aP_k \sum_{y=0}^{r-1} Py\cos\left(\frac{2(y-r)k\pi}{M}\right) f(n-N+y)$$

$$+ \left(\frac{1}{\sqrt{2}} - 1\right) aP_k f(n-N+r) + aP_k \sum_{y=0}^{r-1} f(n+y)\cos\frac{2(y-r)\pi k}{M}$$

$$+ a P_k \sum_{y=0}^{r-1} f(n+y)\cos\frac{(2N-1)\pi k}{M}$$

We know that 2N-1=M so

$$= a P_k C(n,k)\cos\left(\frac{2\pi rk}{M}\right) + a P_k S(n,k)\sin\left(\frac{2\pi rk}{M}\right)$$

$$- a P_k \sum_{y=0}^{r-1} Py\cos\left(\frac{2(y-r)k\pi}{M}\right) f(n-N+y)$$

$$+ \left(\frac{1}{\sqrt{2}} - 1\right) aP_k f(n-N+r) + aP_k \sum_{y=0}^{r-1} f(n+y)\cos\left(\frac{2(y-r)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=0}^{r-1} f(n+y)\cos\left(\frac{\pi k}{M}\right)$$

$$= a P_k C(n,k)\cos\left(\frac{2\pi rk}{M}\right) + aP_k S(n,k)\sin\left(\frac{2\pi rk}{M}\right)$$

$$- aP_k \sum_{y=0}^{r-1} P_y \cos\left(\frac{2(y-r)k\pi}{M}\right) f(n-N+y) + \left(\frac{1}{\sqrt{2}} - 1\right) a P_k f(n-N+r)$$

$$+ a P_k (-1)^k \sum_{y=0}^{r-1} f(n+y)\cos\left(\frac{(2y-2r+1)\pi k}{M}\right) \quad \text{.....(3)}$$

Using the equation (1) the old points which exit from the window can be calculated as -

$$C(n-r,k) = aP_k \sum_{x=0}^{r-1} P_{y+r} f(n-N+x-r)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$C(n-r,k) = aP_k \sum_{y=-r}^{N-r-1} P_{y+r} f(n-N+y)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$= a P_k \sum_{y=-r}^{N-r-1} P_{y+r} f(n-N+y)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=-r}^{-1} P_{y+r} f(n-N+y)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

Put $P_{y+r} - P_y + P_y$ in first summation

$$= aP_k \sum_{y=-r}^{N-r-1} (P_{y+r} - P_y + P_y) f(n-N+y)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=-r}^{-1} P_{y+r} f(n-N+y)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$= aP_k \sum_{y=-r}^{N-r-1} (P_{y+r} - P_y) f(n-N+y)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=0}^{N-r-1} P_y f(n-N+y)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=-r}^{-1} P_{y+r} f(n-N+y)\cos\left(\frac{2(y+r)k\pi}{M}\right) \quad \text{......(4)}$$

By solving first summation –

$$= a P_k \sum_{y=-r}^{N-r-1} (P_{y+r} - P_y) f(n-N+y)\cos\left(\frac{2(y+r)\pi k}{M}\right)$$

At $y=0$

$$P_r - P_0 = 1 - 1/\sqrt{2}$$

At $y = 1$

$$P_{r+1} - P_1 = 1 - 1 = 0$$

At $y = N - r - 1$

$$P_{N-1} - P_{N-r-1} = 1 - 1 = 0$$

Now the term will be –

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$$= a P_k \left(1 - \frac{1}{\sqrt{2}}\right) \; f(n-N) \cos \frac{2 r \pi k}{M}$$

Solving the second summation –

$$= a P_k \sum_{y=0}^{N-r-1} P_y \; f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$= a P_k \sum_{y=0}^{N-r-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$= a P_k \sum_{y=0}^{N-1} P_y f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$- a P_k \sum_{y=N-r}^{N-1} P_y (n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$= a P_k \sum_{y=0}^{N-1} P_y f(n-N+y) \cos \left(\frac{2\pi y k}{M}\right) \cos \left(\frac{2\pi y k}{M}\right)$$

$$- a P_k \sum_{y=0}^{N-1} P_y f(n-N+y) \sin \left(\frac{2\pi y k}{M}\right) \; \sin \left(\frac{2\pi y k}{M}\right)$$

$$- a P_k \sum_{y=N-r}^{N-1} P_y \; f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

Using equations (1) and (2) we can write as-

$$= a P_k C(n,k) \mathrm{Cos} \left(\frac{2\pi y k}{M}\right) - a P_k S(n,k) \sin \left(\frac{2\pi y k}{M}\right)$$

$$- a P_k \sum_{y=N-r}^{N-1} P_y \; f(n-N+y) \cos \frac{2(y+r)\pi k}{M}$$

Now equation (4) will become –

$$= a P_k C(n,k) \; \cos \left(\frac{2 r k \pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2 r k \pi}{M}\right)$$

$$+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2 r k \pi}{M}\right)$$

$$- a P_k \sum_{y=N-r}^{N-1} P_y \; f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=-r}^{-1} P_{y+r} \; f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$= a P_k C(n,k) \cos \left(\frac{2 r k \pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2 r k \pi}{M}\right)$$

$$+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2 r k \pi}{M}\right)$$

$$- a P_k \sum_{y=N-r}^{N-1} P_y \; f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=-r}^{-1} P_{y+r} \, f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

Put y = -y in last summation –

$$= a P_k C(n,k) \cos \left(\frac{2 r k \pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2 r k \pi}{M}\right)$$

$$+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2 r k \pi}{M}\right)$$

$$- a P_k \sum_{y=N-r}^{N-1} P_y \, f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=1}^{r} P_{y+r} \, f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

Put x=y-N+r in the 4th term so the equation will become –

$$= a P_k C(n,k) \cos \left(\frac{2 r k \pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2 r k \pi}{M}\right)$$

$$+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2 r k \pi}{M}\right)$$

$$- a P_k \sum_{x=0}^{r-1} P_{(x+N-r)} f(n-r+x) \cos \left(\frac{2(x+N)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=1}^{r} P_{y+r} \, f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$= a P_k C(n,k) \cos \left(\frac{2 r k \pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2 r k \pi}{M}\right)$$

$$+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2 r k \pi}{M}\right)$$

$$- a P_k \sum_{y=0}^{r-1} P_{(y+N-r)} \, f(n-r+y) \cos \left(\frac{2(y+N)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=1}^{r} P_{y+r} \, f(n-N+y) \cos \left(\frac{2(y+r)\pi k}{M}\right)$$

$$= a P_k C(n,k) \cos \left(\frac{2 r k \pi}{M}\right) - a P_k S(n,k) \sin \left(\frac{2 r k \pi}{M}\right)$$

$$+ a P_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N) \cos \left(\frac{2 r k \pi}{M}\right)$$

$$- a P_k \sum_{y=0}^{r-1} P_{(y+N-r)} \, f(n-r+y) \cos \left(\frac{2(y+N)\pi k}{M}\right)$$

$$+ a P_k \sum_{y=0}^{r-1} P_{r-1} f(n-N+y) \cos \left(\frac{2(r-y)k\pi}{M}\right)$$

$$+ a P_k \frac{1}{\sqrt{2}} f(n-N-r) - a P_k f(n-N) \cos \left(\frac{2 r k \pi}{M}\right) \text{.....(5)}$$

Adding equation (3) and (5) –

$$= 2 C(n,k) \cos \left(\frac{2 r k \pi}{M}\right)$$

$$- a P_k \sum_{y=0}^{r-1} P_y \cos \left(\frac{2(y-r) k \pi}{M}\right) f(n-N+y)$$

$$+ a k_k \left(1 - \frac{1}{\sqrt{2}}\right) f(n-N+r)$$

$$+ a P_k (-1)^k \sum_{y=0}^{r-1} \; f(n+y) \cos \left((2y-2r+1) \frac{k\pi}{M}\right)$$

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$$+aP_k\left(1-\frac{1}{\sqrt{2}}\right)f(n-N)\cos\left(\frac{2rk\pi}{M}\right)$$

$$-aP_k\sum_{y=0}^{r-1}P_{y+N-r}f(n+y-r)\cos\left(\frac{2(y+N)k\pi}{M}\right)$$

$$+aP_k\sum_{y=0}^{r-1}P_{r-y}f(n-N-y)\cos\left(\frac{2(r-y)k\pi}{M}\right)$$

$$+\frac{aP_k}{\sqrt{2}}f(n-N-r)-aP_k\,f(n-N)\cos\left(\frac{2rk\pi}{M}\right)$$

Final expression for “n+r” point –

$$=ak_kC(n,k)\cos\left(\frac{2\pi rk}{M}\right)+ak_kS(n,k)\sin\left(\frac{2\pi rk}{M}\right)$$

$$-ak_k\sum_{y=0}^{r-1}k_y\cos\frac{2(y-r)k\pi}{M}f(n-N+y)$$

$$+\left(\frac{1}{\sqrt{2}}-1\right)ak_k\,f(n-N+r)$$

$$+ak_k\sum_{y=0}^{r-1}f(n+y)\cos\frac{2(y-r)\pi k}{M}$$

$$+ak_k(-1)^k\sum_{y=0}^{r-1}f(n+y)$$

Final expression for “n-r” point –

$$=ak_kC(n,k)Cos\left(\frac{2r\pi k}{M}\right)-ak_kS(n,k)\sin\left(\frac{2r\pi k}{M}\right)$$

$$+ak_k\left(1-\frac{1}{\sqrt{2}}\right)f(n-N)\cos\frac{2rk\pi}{M}$$

$$-ak_k\sum_{y=0}^{r-1}k_{(y+N-r)}f(n-r+y)\cos\frac{2(y+N)\pi k}{M}$$

$$+ak_k\sum_{y=0}^{r-1}k_{r-1}f(n-N+y)\cos\frac{2(r-y)k\pi}{M}$$

$$+ak_k*\frac{1}{\sqrt{2}}f(n-N-r)$$

$$-ak_k*1*f(n-N)\cos\frac{2rk\pi}{M}$$

We got the expressions for “n+r” and “n-r” points now we add these two expressions to obtain the expression for r-point independent algorithm as –

$$=2C(n,k)\cos\left(\frac{2rk\pi}{M}\right)$$

$$-aP_k\sum_{y=0}^{r-1}P_y\cos\left(\frac{2(y-r)k\pi}{M}\right)f(n-N+y)$$

$$+ak_k\left(1-\frac{1}{\sqrt{2}}\right)f(n-N+r)$$

$$+aP_k(-1)^k\sum_{y=0}^{r-1}f(n+y)\cos\left((2y-2r+1)\frac{k\pi}{M}\right)$$

$$+aP_k\left(1-\frac{1}{\sqrt{2}}\right)f(n-N)\cos\left(\frac{2rk\pi}{M}\right)$$

$$-aP_k\sum_{y=0}^{r-1}P_{y+N-r}f(n+y-r)\cos\left(\frac{2(y+N)k\pi}{M}\right)$$

$$+aP_k\sum_{y=0}^{r-1}P_{r-y}f(n-N-y)\cos\left(\frac{2(r-y)k\pi}{M}\right)$$

$$+\frac{aP_k}{\sqrt{2}}f(n-N-r)-aP_k\,f(n-N)\cos\left(\frac{2rk\pi}{M}\right).....(6)$$

Conclusion

In this paper, computationally efficient algorithm has been proposed for developing the DCT shifting data sequence. The derived algorithm is independent of ODST coefficient. The data windowed by rectangular window. The size of shift can be any value from 0 to N-1 where N is the length of DCT. The proposed algorithm has been verified using Matlab and found it is efficient and fast than the finding coefficient by standard definition.

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