

# Modeling of 3 Link Biped Robot using DH Algorithm

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**Abstract:** This paper aims to develop a simple mathematical modeling of biped series of humanoid robots. Since the introduction of Wabot from Waseda University in 1973, many humanoid robots have been developed around the world that can walk with stability. Various biped robots have successfully shown their capabilities, bipedal walking methods are still one of the main technical challenges that robotics researchers are trying to solve. For modeling of biped robot or any multi body system different methods are used like Newton-Euler, Lagrange-Euler. By implementing use of Denavit Hartenberg convention for modeling kinematics for biped robots, we give an algorithm for derivation of Lagrange-Euler equations of motions for a general biped robots. We have considered a biped robot with three links having four degrees of freedom.

**Index Terms:** Humanoid, biped, D-H algorithm

## I. INTRODUCTION

A robot is introduced as an electro-mechanical machine that can walk or stir around, operate a mechanical limb, sense and change its environment, or exhibit intelligent behavior, especially behavior which mimics humans or other animals. Fact fully, the behavior was not of high intelligence, but nowadays robots have considered as machines that are stronger than humans and can do tasks faster and with higher accuracy. That's what makes the current field of robotics so incredibly fascinating. There are different kinds of robots in the field, from welding robotic arms in the automotive industry to tiny nanorobots. One specific group of robotic machines is classified as humanoid robots. In this work, humanoid robots are considered. Humanoid robotics is an rising technology that which would be an important part of our daily life in the coming decades. Researchers and engineers are developing humanoid and biped robots that show various human-like characteristics, since these robots should substitute humans in various tasks in fields. Humanoid robots that walk on two legs are called bipeds. The conventional formalism to derive the equations of motions such as Newton-Euler, Lagrange Euler [1].the complexity of such robot makes it very difficult to derive equations by hand. We will show that application of Denavit Hartenberg convention for modeling robot kinematics can facilitate

automatic derivation of Lagrange-Euler equations of motion[2][5]. In particular, we propose a mathematical formalism for modeling of a general humanoid robot. We are using mat lab for solving the Lagrange equation of motion .

## II. MODELING A HUMANOID ROBOT

### A. Robot description

We are considering a biped robot with 4 degree's of freedom, there is four joint one in thigh one in knee and tow joints on ankle and following are the concepts which we are using to model our robot the joint parameters are given below with the explanation of DH algorithm.

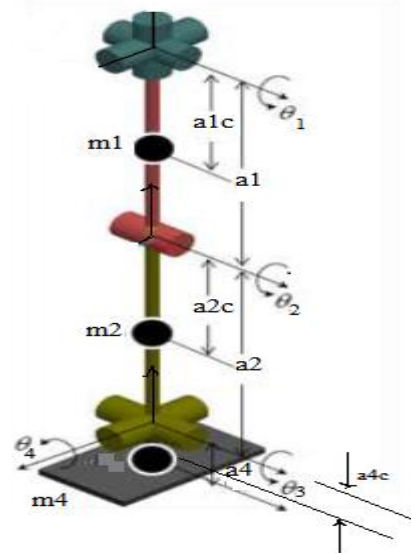


FIG 1 JOINT DEFINATION OF 3 LINK ROBOT

We have taken the value of mass and length of each link by some well known existing robots such as CHARLIE I [5]. The parameter we selected for 3 link biped robot are as follow:

Parameters	Specification
Mass of link 1 (m1)	1.7229 kg
Mass of link 2 (m2)	0.783 kg
Mass of link 3 (m4)	0.886 kg
Length of link 1(a1)	0.775 m
Length of link 2(a2)	0.425 m
Length of link 3(a4)	0.122 m
Position of centre of mass of link 1 (a1c)	0.627 m

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Position of centre of mass of link 2 (a2c)	0.265 m
Position of centre of mass of link 3 (a4c)	0.035 m

**B. Lagrange-Euler formulation of robot dynamics based on denavit hartenberg convention**

To derive Lagrange-Euler equations of motion in an automatic way, we can make use of Denavit Hartenberg convention for description of robot kinematics[2]. The Denavit Hartenberg convention is a commonly used method to assign coordinate frames to different link sin a robotic arm. This method ensures that the position and orientation of each frame can be described by only four parameters, which simplifies the kinematic analysis and allows us to derive the equations of motion using the kinetic and potential energy as in the Lagrange-Euler equations. The convention is normally used to model robotic arms with a fixed base and one tip at the end. Humanoid robots, however, do not have a fixed base and in general have two legs which can be seen as tips. This means that we, in some sense, have to modify the standard approach of deriving the equations of motion with the Denavit Hartenberg convention.

$$A_i^{i-1} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i) & a_i \cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & \sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For 3 link biped robot the joint angle parameters is calculated as in following table

Joint Angle	Joint Length	Link Angle	Link Length
$\theta_1$	0	0	$l_1$
$\theta_2$	0	0	$l_2$
$\theta_3$	0	-90	0
$\theta_4$	0	0	$l_3$

**C. Jacobian Matrices**

Jacobian matrices are the basic elements in building a dynamic model. Jacobian matrices are divided by actuation types. Each mass needs a Jacobian matrix, linear or revolution, based on whether the motions are rectilinear or rotary.

$$J_i = \begin{bmatrix} J_{vi} \\ J_{wi} \end{bmatrix} = \begin{bmatrix} z_{i-1} \times (O_n - O_{i-1}) \\ z_{i-1} \end{bmatrix}$$

Where  $J_{vi}$  represents the velocity jacobian matrix and  $J_{wi}$  represent the angular jacobian matrix.

**D. Equation of Motion**

The equations of motion of the robot can be derived using the well known Lagrange-Euler equations. This equation derives using the kinetic and potential energy of every link in the system:

$$\frac{d}{dt} \frac{\partial K}{\partial \dot{q}} - \frac{\partial K}{\partial q} + \frac{\partial P}{\partial q} = Q \tag{2}$$

Where  $K$  and  $P$  represent the kinetic and potential energy respectively and  $Q$  is a vector with non conservative forces like damping, friction and applied torques. The kinetic energy for a system .

$$K = \frac{1}{2} \dot{q}^T \left( \sum_{i=1}^n m_i J_{vi}^T J_{vi} + \sum_{wi} J_{wi}^T R_i I_i R_i^T J_{wi} \right) \dot{q} \tag{3}$$

We can simplified it as follow

$$D = \left( \sum_{i=1}^n m_i J_{vi}^T J_{vi} + \sum_{wi} J_{wi}^T R_i I_i R_i^T J_{wi} \right) \tag{4}$$

Where  $D$  is called the inertia matrix. The potential energy of a system or part of a system with links  $I$  in a gravity field  $g$  with its center of mass at position  $o_{ci}$  is:

$$P = \sum_{i=1}^n m_i g^T o_{ci} \tag{5}$$

The equations of motion can be calculated with Lagrange Euler equation, but we can further simplify the algorithm. If it's assumed that there are no external forces acting on the system but the applied torques, it can easily be seen that (2) can be rewritten as:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \Gamma_i \tag{6}$$

Where  $L = K - P$  and  $q_i$  and  $\Gamma_i$  are the joint variable and joint torque of link  $i$ . So we assume there is no damping or friction acting in the joint and  $Q$  only contains joint torques.

The Lagrange-Euler equations for each link  $i$  can be written as:

$$\sum_{j=1}^n d_{ij} \ddot{q}_j + \sum_{k=1}^n \sum_{j=1}^n \left( \frac{d_{ij}}{\partial q_k} - \frac{1}{2} \frac{\partial d_{kj}}{\partial q_i} \right) \dot{q}_k \dot{q}_j + \frac{\partial P}{\partial q_i} = \Gamma_i \tag{7}$$

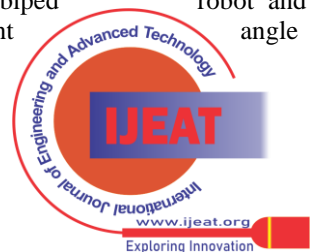
Which in matrix form can be written as :

$$D \ddot{q} + C \dot{q} + G = \Gamma \tag{8}$$

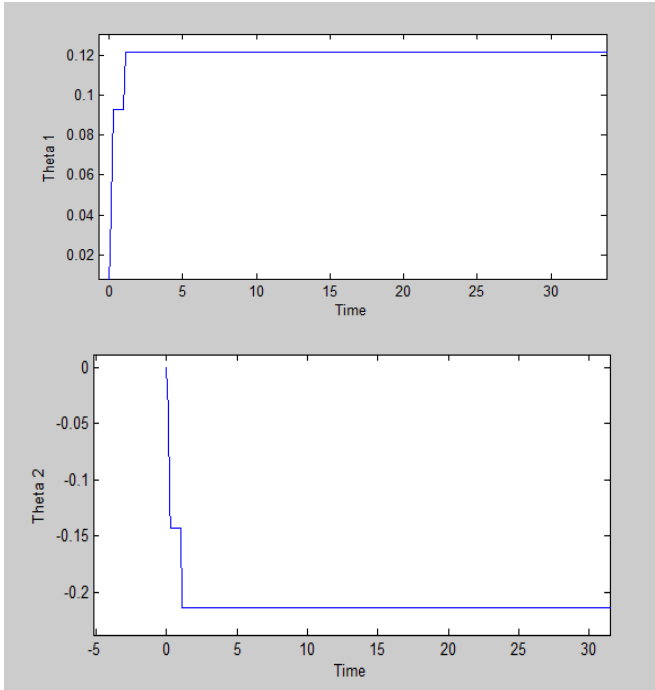
Where  $D$  is inertia matrix,  $C$  is coriolis and centrifugal matrix and  $G$  is gravity vector. The Denavit Hartenberg convention is perfectly suitable for modeling robotic arms and as we showed it can also be used to model a humanoid robot.

**III. SIMULATION AND RESULTS**

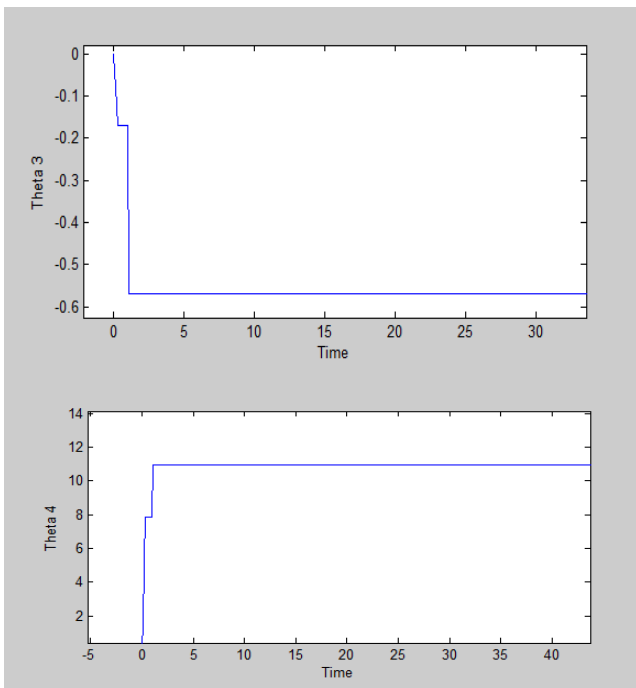
We are performing the solution on mat lab using function "ode45" to solve the differential equation which we have as equation of motion of 3 link biped robot and we got output as a plot of joint angle for each joint. We have considered step input to solve equations (i.e input is 4\*1



matrix with all element equals to 1) This is a time in sec versus angle in radian plot. Each joint angle plot is given as follow:



**FIG 2 PLOT OF THETA 1 AND THETA 2 (HIP AND KNEE ANGLE PLOT )**



**FIG 3 PLOT OF THETA 3 AND THEATA 4 (ANKEL ANGLE PLOT)**

We can conclude from the plot that each joint angles are varying in a bounded range so we can control each angle by using a proper controller and we can get the desire angle which we want. For future work we can apply different kinds of controller in each joint to control torque and similarly simulate the equation of motion using controller on the mat lab.

#### IV. CONCLUSION

We have shown an altogether a different frame work in which a system with degrees of freedom greater than two can be modeled and controlled using fundamentals of D-H algorithm signifying symbolisms. Here we notice that for a given control input we get an output which is bounded and it can be interpreted mechanically. But we have also observed that the system does not follow the reference value but has a potential for inclusion of a controller [9] to do so similar to the controller strategies explained in [9], we can have various other controllers (i.e. those which can be implemented for control of a standard double inverted pendulum) which could be designed. Due to the unavailability of strong computational hardware's, this forms the basis of our future work.

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