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Optimal Homotopy Asymptotic Method to MHD Flow of a Viscoelastic Fluid Aver a Stretching Sheet

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Abstract—In this study, the problem of steady two-dimensional magnetohydrodynamic (MHD) flow of a viscoelastic fluid near a stretching sheet in has been investigated using a simulation method called Optimal Homotopy Asymptotic Method (OHAM). The concept of OHAM is briefly introduced, and then employed it to derive solutions of governing nonlinear equation. The obtained results from this method are compared with those from the numerical solution to verify the accuracy of the proposed method. It is found that the OHAM can achieve suitable results in predicting the solution of such problems. Moreover, the effect of important parameters on temperature distribution, mass transfer as well as stream function along the stretching sheet are presented and discussed in detail.

Index MHD flow; viscoelastic fluid; stretching sheet; Optimal Homotopy Asymptotic Method.

I. INTRODUCTION

Over the last decade, the study of boundary layer flow and heat transfer over a stretching surface has achieved a lot of success because of its wide applications (e.g. [1,2]). After the pioneering work by Sakiadis [3], a large amount of literature is available on boundary layer flow both of Newtonian and non-Newtonian fluids over linear and nonlinear continuous moving surface ([4-8]). Nevertheless, all the above-mentioned researches did not consider the situations, where hydro-magnetic effects arose.

Analytical solutions have been done for boundary layer flow over a stretching sheet. Yazdi et al. [9] investigated friction and heat transfer in the slip flow boundary layer at constant heat flux boundary conditions. It has been found that suction makes a significant effect on the velocity adjacent to the wall in the presence of slip. Rana and Bhargava [10] studied the problem of steady laminar boundary fluid flow, which results from the non-linear stretching of a flat surface in a nanofluid. Javed et al. [11] studied heat transfer of a viscous fluid over a non-linear shrinking sheet in the presence of magnetic field, where they have obtained dual solutions for the exact and numerical solutions in the shrinking sheet problem. Hydrodynamic nano-boundary layer flow over permeable stretching surface by employing Homotopy Analysis Method (HAM) and Boundary Value Problem solver (BVP) was studied by Van Gorder et al. [12]. Kechil and Hashim [13] studied the boundary-layer equation of flow over a nonlinearly stretching sheet in a magnetic field with chemical reaction. Most scientific problems demonstrate themselves in the mathematical relations that are modeled principally by ordinary or partial differential equations but there are few phenomena in different fields of science occurring linearly.

Not always exact solution does exist to solve these equations generally because they are innately of nonlinearity; therefore, special techniques should be applied to solve them. In most cases, the solution can be obtained either by numerical techniques [14–15] or by method of perturbation [16–17]. In the case of numerical methods, stability and convergency should be considered due to avoid inappropriate results. On the other hand, in perturbation method, the small parameter should be exerted on the equation. Thus, finding the small parameter is deficiency of this method. For this reason, some different techniques have recently introduced to eliminate the small parameter including Homotopy Perturbation Method [18–20], Differential Transformation Method [21–23], Homotopy Analysis Method [24–28] and so forth.

One of the semi-exact methods which does not require the small parameter is OHAM which is powerful method for solving nonlinear problems. In this research, we employ this powerful technique to solve the governing equations which arise to describe MHD flow MHD flow of a viscoelastic fluid over a stretching sheet filled with nanofluids

II. GOVERNING EQUATIONS

Let us consider the steady-state 2D incompressible electrically conducting viscoelastic Maxwell fluid flow over a stretching sheet in presence of nanoparticles (see Fig. 1). It is assumed that a uniform magnetic field of strength B_0 is applied in the negative y-direction normal to the sheet. The induced magnetic field due to the motion of the electrically conducting fluid and the pressure gradient are neglected. The equations governing transport of heat, momentum and mass can be written as (Abel et al., [29]),

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\upsilon\frac{\partial^2 v}{\partial y^2} -k_0\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right) +\frac{\sigma}{\rho}\left(E_0B_0-B_0{}^2u\right), \qquad (2)$$

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$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{(uB_0-E_0)^2\sigma}{\rho c_p}$$
$$+\tau\left\{\begin{matrix} D_B\left(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right) \\ +\left(\frac{D_T}{T_\infty}\right)\left[\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right]\end{matrix}\right\},$$

(3)

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial x^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2},$$

(4)

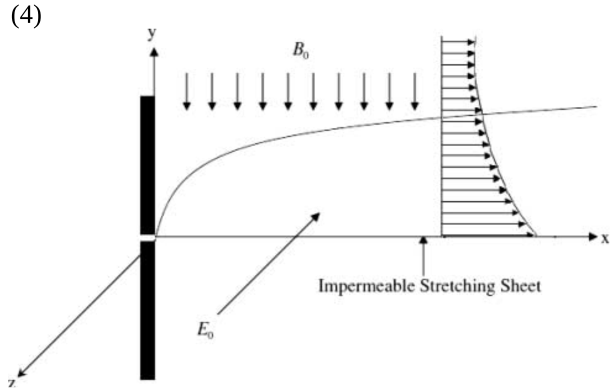

Fig. 1. Geometry of problem under investigation.

where (u,v) are the velocity components in (x,y) directions, ρf is the fluid density of base fluid, υ is the kinematic viscosity, T is the temperature, C is the nanoparticle volume fraction, α is the thermal diffusivity of the base fluid, $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio of nanoparticle heat capacity and the base fluid heat capacity, DB is the Brownian diffusion coefficient and DT is the thermophoretic diffusion coefficient.

The boundary conditions for this problem will be:

$$y=0,\ T=T_w,\ u=U_w(x),\ v=0,\ C=C_w, \qquad (5)$$
$$y\to\infty,\ T=T_\infty,\ u=0,\ C=C_\infty,$$

(6)

We introduce the following dimensionless quantities:

$$\eta=\left(\frac{b}{\upsilon}\right)^{1/2}y,\ u=bxf'(\eta),\ v=-\sqrt{b\upsilon}\,f(\eta),$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

(7)

Here *f(η)* is the dimensionless stream function, *θ(η)* is dimensionless temperature and *ϕ(η)* is dimensionless concentration.

After transformation we have:

$$f'''+ff''-f'^2-k_1^*(2f'f''-ff'''-f''^2)$$
$$+Ha^2\left(E_1-f'\right)=0, \qquad (8)$$

$$\theta''+\Pr\left(f\theta'-2\theta f''\right)+\Pr Ec\left(f''^2-Ha^2\left(f'^2+E_1{}^2-2E_1f'\right)\right)+$$
$$N_b\theta'\phi'+N_t\theta'^2=0,$$

(9)

$$\phi''+Lef\phi'+\frac{N_t}{N_b}\phi^2=0,$$

(10)

The associated boundary conditions are,

$$f(0)=0,\ \ f'(0)=1,\ \ \theta(0)=1,\ \ \phi(0)=1, \qquad (11)$$
$$f'(\infty)=0,\ \ \theta(\infty)=0,\ \ \phi(\infty)=0, \qquad (12)$$

where $k_1^*=\frac{k_0 b}{\gamma}$ is the dimensionless viscoelastic parameter, $Ha=\sqrt{\frac{\sigma}{\rho b}}B_0$ is Hartmann number, $E_1=\frac{E_0}{B_0 bx}$ is the local electric parameter, and the prime stands for differentiation with respect to η. The four parameters are defined by:

$$\Pr=\frac{\upsilon}{\alpha},Le=\frac{\upsilon}{D_B},N_b=\frac{(\rho c)_p D_B(\phi_w-\phi_\infty)}{(\rho c)_f\upsilon},$$
$$N_t=\frac{(\rho c)_p D_T(T_w-T_\infty)}{(\rho c)_f T_\infty\upsilon} \qquad (13)$$

III. BASIC IDEA OF OPTIMAL HOMOTOPY ASYMPTOTIC METHOD

We apply the OHAM to the following differential equation:

$$L(u(\tau))+g(\tau)+N(u(\tau))=0,\quad B(u)=0 \qquad (14)$$

Where L is a linear operator, τ denotes independent variable, u(τ) is an unknown function, g(τ) is a known function, N(u(τ)) is a nonlinear operator and B is a boundary operator. Through OHAM one first constructs a family of equation:

$$(1-p)[L(\phi(\tau,p))+g(\tau)]=H(p)$$
$$[L(\phi(\tau,p))+g(\tau)+N(\phi(\tau,p))],\ \ B(\phi(\tau,p))=0 \qquad (15)$$

Where p∈[0,1] is an embedding parameter, H(p) is a nonzero auxiliary function for p≠0 and H(0)=0, ϕ(τ,p) is an unknown function, respectively. Clearly, when p=0 and p=1, it holds that:

$$\phi(\tau,0)=u_0(\tau),\quad \phi(\tau,1)=u(\tau) \qquad (16)$$

Hence, when p increases from 0 to 1, the solution ϕ(τ,p) varies from u0(τ) to the solution u(τ), where u0(τ) is obtained from Eq. (15) for p= 0:

$$L(u_0(\tau))+g(\tau)=0,\quad B(u_0)=0 \qquad (17)$$

We choose the auxiliary function H(p) in the form

$$H(p)=pC_1+p^2C_2+\ldots \qquad (18)$$

Where C1, C2, … are constants which can be determined later. Expanding ϕ(τ,p) in a series with respect to p, one has

$$\phi(\tau,p,C_i)=u_0(\tau)+\sum_{k\ge1}u_k(\tau,C_i)p^k,i=1,2,.. \qquad (19)$$

Substituting Eq. (19) into Eq. (15), collecting the same powers of p, and equating each coefficient of p to zero, we obtain set of differential equation with boundary conditions. Solving differential equations by boundary conditions, u0(τ), u1(τ,C1), and u2(τ,C2),… are obtained. Generally, the solution of Eq. (14) can be determined approximately in the form:

$$\tilde{u}^{(m)}=u_0(\tau)+\sum_{k=1}^{m}u_k(\tau,C_i) \qquad (20)$$

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Considering that the last coefficient Cm can be function of τ. Substituting Eq. (20) into Eq. (14), there results the following residual:

$$R(\tau, C_i) = L(\tilde{u}^{(m)}(\tau, C_i)) + g(\tau) + N(\tilde{u}^{(m)}((\tau, C_i)) \quad (21)$$

If R(τ,Ci)=0 then $\tilde{u}^{(m)}(\tau, C_i)$ would be the exact solution. As a whole, such a case will not arise for nonlinear problems, but we can minimize the functional:

$$J(C_1, C_2, ..., C_n) = \int_a^b R^2(\tau, C_1, C_2, ..., C_m)\, d\tau \quad (22)$$

Where a and b are two values, depending on the given problem. The unknown constants Ci (i=1, 2, …, m) can be identified from the conditions

$$\frac{\partial J}{\partial C_1} = \frac{\partial J}{\partial C_2} = ... = 0 \quad (23)$$

With these constants, the approximate solution (of order m) is well obtained.

IV. EMPLOYING OPTIMAL HOMOTOPY ASYMPTOTIC METHOD

In this section, we will apply OHAM to solve Eqs. (8–10), subjected to boundary conditions Eqs. (11–12). According to OHAM, we can construct a Homotopy of Eqs. (8–10),

$$(1-p)\left(f''' - Ha^2 f'\right) - H_1(p)\begin{pmatrix} f''' + ff'' - f'^2 - k_1^*(2ff'' - ff''' - f''^2) + \\ Ha^2(E_1 - f') \end{pmatrix} = 0, \quad (24)$$

$$(1-p)(\theta'') - H_2(p)\begin{pmatrix} \theta'' + \Pr\left(f\theta' - 2\theta f''\right) + \Pr Ec \\ \begin{pmatrix} f''^2 - Ha^2 \\ \left(f'^2 + E_1^2 - 2E_1 f'\right) \end{pmatrix} + \\ N_b\theta'\phi' + N_t\theta'^2 \end{pmatrix} = 0, \quad (25)$$

$$(1-p)(\phi'') - H_3(p)\left(\phi'' + Lef\phi' + \frac{N_t}{N_b}\phi^2\right) = 0, \quad (26)$$

where primes denote differentiation with respect to η. We consider $f, \theta, \phi, H_1(p), H_2(p)$ and $H_3(p)$ as follows:

$$\begin{aligned} f &= f_0 + pf_1 + p^2 f_2, \\ \theta &= \theta_0 + p\theta_1 + p^2\theta_2, \\ \phi &= \phi_0 + p\phi_1 + p^2\phi_2, \\ H_1(p) &= pc_{11} + p^2 c_{12}, \\ H_2(p) &= pc_{21} + p^2 c_{22}, \\ H_3(p) &= pc_{31} + p^2 c_{32}, \end{aligned} \quad (27)$$

Substituting Eq. (27) into Eqs. (24–26) and some simplification and rearrangement based on powers of p-terms, we can get the results. The final expression for $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ would be:

$$\begin{aligned} f(\eta) &= f_0(\eta) + f_1(\eta) + f_2(\eta) + \cdots, \\ \theta(\eta) &= \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) + \cdots, \\ \phi(\eta) &= \phi_0(\eta) + \phi_1(\eta) + \phi_2(\eta) + \cdots, \end{aligned} \quad (28)$$

Now, we should construct following expressions. Note that R1, R2 and R3 are the residual of Eqs. (8-10):

$$\begin{aligned} J_1(c_{11}, c_{12}) &= \int_0^\infty R_1^2(\eta, c_{11}, c_{12})\, d\eta, \\ J_2(c_{21}, c_{22}) &= \int_0^\infty R_2^2(\eta, c_{21}, c_{22})\, d\eta, \\ J_3(c_{31}, c_{32}) &= \int_0^\infty R_3^2(\eta, c_{31}, c_{32})\, d\eta, \end{aligned} \quad (29)$$

We can find the constants c11, c12, c21, c22, c31, c32 by solving the following equation simultaneously.

$$\begin{aligned} \frac{\partial J_1(c_{11}, c_{12})}{\partial c_{11}} &= \frac{\partial J_1(c_{11}, c_{12})}{\partial c_{12}} = 0, \\ \frac{\partial J_2(c_{21}, c_{22})}{\partial c_{21}} &= \frac{\partial J_2(c_{21}, c_{22})}{\partial c_{22}} = 0, \\ \frac{\partial J_3(c_{31}, c_{32})}{\partial c_{31}} &= \frac{\partial J_1(c_{31}, c_{32})}{\partial c_{32}} = 0, \end{aligned} \quad (30)$$

V. RESULTS AND DISCUSSION

The system of Eqs. (8–10), along with boundary conditions of Eqs. (11,12), has been solved by using the optimal homotopy asymptotic method. The results are presented graphically through Figs. 2–10. The significance of parameter involved on the MHD flow and heat & mass transfer has been discussed. To check the validity of our results, we have made a comparison between the present analytical solution and numerical solution through the fourth-order Runge–Kutta method that is depicted in ab. 1. It can be realized that our analytical approximations are in good agreement with the numerical ones.

The non-dimensional temperature distribution $\theta(\eta)$ for various parameters including the Prandtl number *Pr*, thermophoresis parameters N_t, and viscoelastic parameter k_1^* are plotted in Figs. 2–4.

Figure 2 shows the effect of Prandtl number on temperature distributions along the streching sheet. As seen, there is a reduction in temperature as prandtl number goes up. Moreover, the higher Pr numbers result in an increase in the heat transfer rate at the sheet.

Table 1. Comparison between the present analytical solution and numerical solution for *θ(η)*.

η	OHAM Solution	Numerical Solution	Relative Error
0	1	1	0
0.5	0.584463	0.583593	0.00087
1	0.343715	0.342955	0.00076
1.5	0.202815	0.202195	0.00062
2	0.119858	0.119338	0.00052
2.5	0.070856	0.070396	0.00046
3	0.041869	0.041539	0.00033
3.5	0.024716	0.024456	0.00026
4	0.01457	0.0143	0.00027
4.5	0.008575	0.008335	0.00024
5	0.005037	0.004857	0.00018
5.5	0.002952	0.002812	0.00014
6	0.001726	0.001616	0.00011
6.5	0.001006	0.000936	0.00007
7	0.000585	0.000555	0.00003

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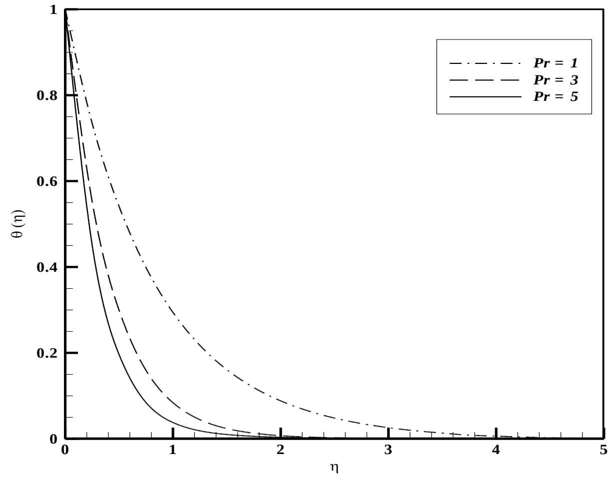

Fig 2. Temperature profiles as function of η for different values of Prandtl number when $k_1^* = E_1 = Ha = 0.2.$

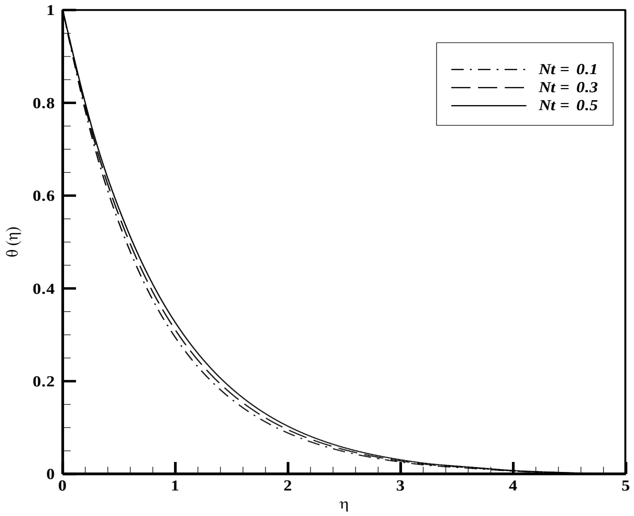

Fig 3. Temperature profiles as function of η **for different values of** N_t **when** $k_1^* = E_1 = Ha = 0.2.$

The variation of non-dimensional temperature with the thermophoresis parameterof nano-particles has been shown in fig. 3 when the other parameters are kept at $k_1^* = E_1 = Ha = 0.2$. It can be found that increasing the value of N_t results in an increase in temperature as well as thickening thermal boundary layer thickness.

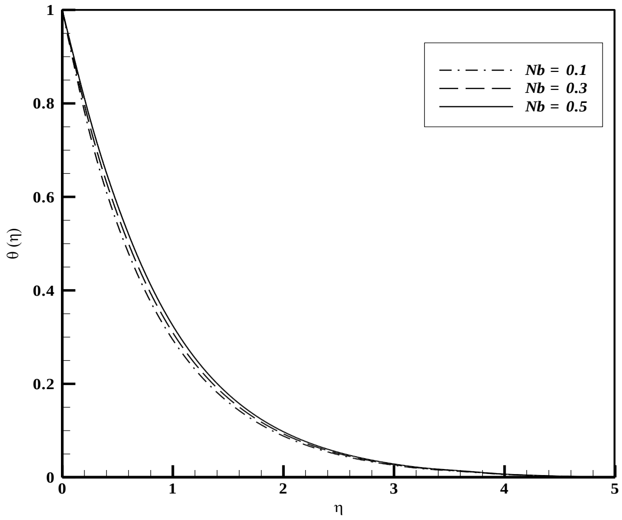

Fig 4. Temperature profiles as function of η **for different values of** N_b **when** $k_1^* = E_1 = Ha = 0.2.$

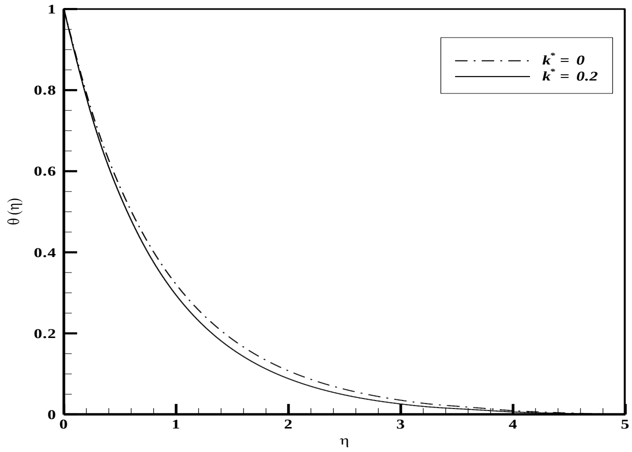

Fig 5. Temperature profiles as function of η **for different values of** k^* **when** $E_1 = Ha = 0.2.$

Figure 4 indicates the effects of brownian motion parameter on non-dimensional temperature. Likewise thermophoresis parameter, brownian motion parameter increases temperature of the sheet surface. As the Brownian motion intensifies, it impacts a larger extent of the fluid, causing the thermal boundary layer to thicken, which in turn decreases the reduced Nusselt number. On the other hand, the reduced Nusselt number decreases as the thermophoresis diffusion penetrates deeper into the fluid and causes the thermal boundary layer to thicken.

The effect of viscoelastic fluid on non-dimensional temperature profiles has been shown in the fig. 5. It is obvious that using a viscoelastic fluid can lead to decrease in the temperature of the stretching sheet.

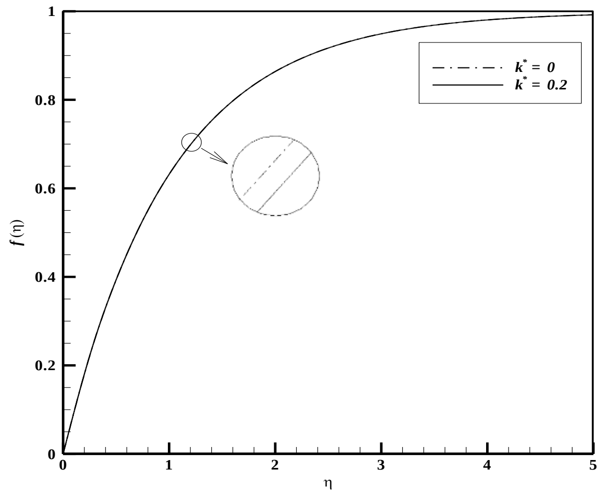

Fig 6. Stream function profiles for different values of k^* **when** $E_1 = Ha = 0.2.$

Figure 6 illustrates the variation of stream function with different amounts of viscoelastic parameter. It can be realized that viscoelastic tend to adopt the stream function inward to the plane and consequently decrease the momentum thickness. On the other hand, we examined the effect of Hartman number on stream function profile as shown in the fig. 7. It is found that Harman number affects stream function in a similar manner as the viscoelastic parameter does.

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In fact, it can be realized that the Hartmann number decreases the boundary layer velocity throughout the boundary layer but significantly near the stretching sheet.

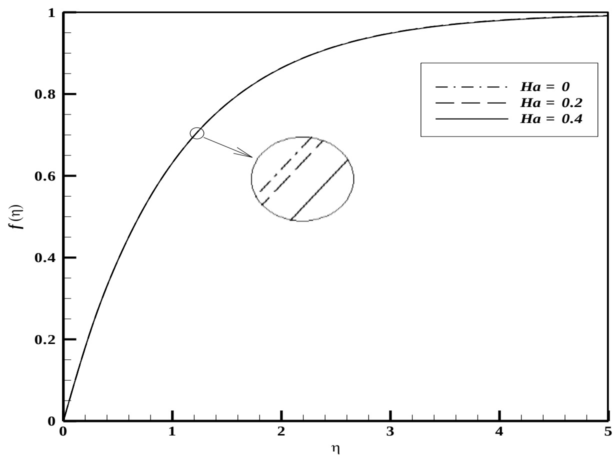

Fig 7. Stream function profiles for different values of *Ha* when $k_1^* = E_1 = Ha = 0.2$.

The influence of Lewis number, Brownian motion parameter and thermophoresis parameters on concentration profiles has been represented in figs. 8-10, respectively. It can be interpreted that thermophoresis parameter deteriorates the rate of mass transfer, whereas by increasing the Brownian motion as well as Lewis number enhances it.

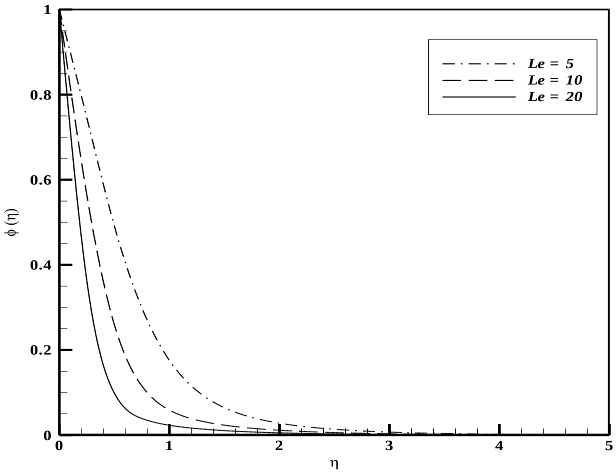

Fig 8. Concentration profiles for different values of *Le* when $k_1^* = E_1 = Ha = 0.2$.

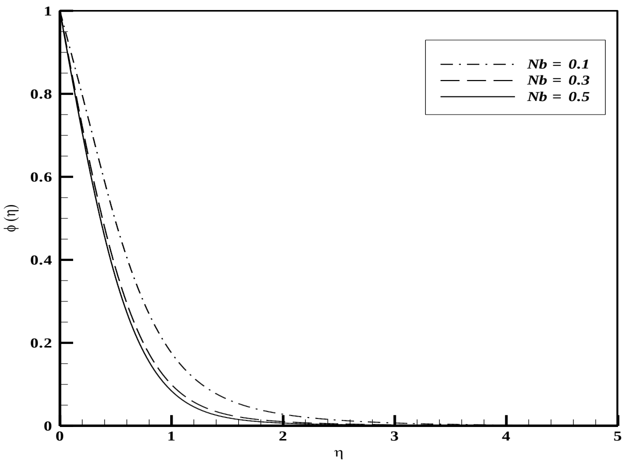

Fig 9. Concentration profiles for different values of N_b **when** $k_1^* = E_1 = Ha = 0.2$.

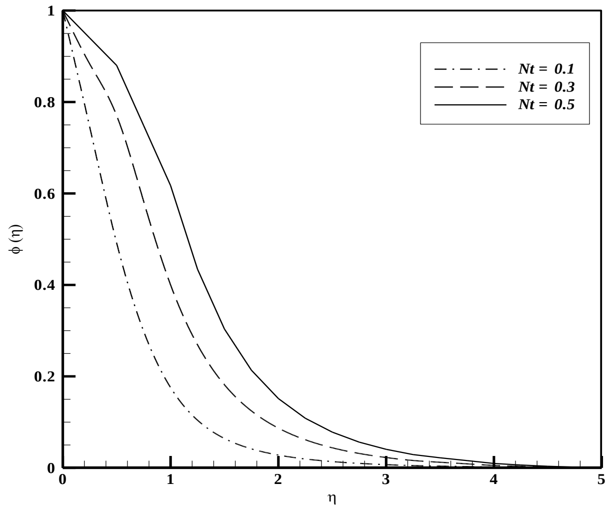

Fig 10. Concentration profiles for different values of N_t **when** $k_1^* = E_1 = Ha = 0.2$.

VI. CONCLUSIONS

The flow field and heat transfer as well as mass transfer due to streching sheet have been investigated for an MHD flow of a viscoelastic fluid immersed in nano-partices. The governing partial differential equations were converted into ordinary differential equations by using a suitable similarity trans- formation, which were analytically solved by employing the OHAM technique. Comparisons with the numerical solution were performed and the results were found to be in good agreement. Trusting these validities, effects of some pertinent parameters were discussed. It can be found the rate of heat transfer at the sheet surface increases due to an increase in the Prandtl number. Conversely, there is a decrease in local Nusselt number as thermophoresis parameter and Brownian motion parameter of the nanoparticles increase. Moreover, the rate of mass transfer intensifies with the Brownian motion as well as Lewis number. Ultimately, it could be concluded that OHAM has a great reliability in solving the set of coupled nonlinear differential equations arising in heat transfer applications.

Nomenclature

B_0 magnetic field
C concentration
C_f local skin-friction coefficient
c_p heat capacity
D_B Brownian diffusion coefficient
D_T thermophoretic diffusion coefficient,
E Eckert number
E_1 local electromagnetic parameter
E_0 electric field
f stream function
Ha Hartmann number
k thermal conductivity
Le Lewis number
k_0 elastic parameter
k_1^* viscoelastic parameter
N_b Brownian motion parameters
N_t thermophoresis parameters
Pr Prandtl number
T temperature

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u,v dimensional velocity components along x and y axes
x,y dimensional Cartesian coordinates

Greek symbols

ν Kinematic viscosity
ρ density
θ dimensionless temperature
η similarity variable
σ electrical conductivity
ϕ dimensionless concentration
α thermal diffusivity
ε velocity ratio parameter
ψ stream function
σ electric conductivity
γ chemical reaction parameter

Subscripts

f fluid
p nanoparticles
w condition at wall
∞ condition at infinity

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