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Modeling of Fractional order Buck-Boost Converter

Reena Chandran, Vrinda Prasad, Prima Vincent

Abstract—In this paper fractional order mathematical modeling and state space average modeling of Buck-Boost Converter is established in continuous conduction mode. This modeling is based on fractional calculus and an idea about the fractional calculus is describing here. To observe the performance of Buck-Boost converter in the fractional order modeling, simulation is done using MATLAB/SIMULINK.

Index Terms- Buck-Boost converter, Fractional order, mathematical model, Sate space average modeling

I. INTRODUCTION

The idea of fractional calculus has been known since development of the regular calculus. It is accepted today as a new tool that extends the descriptive power of the conventional calculus, supporting mathematical models [1-3]. In previous studies, the researchers in most cases used the integral-order model of the DC–DC converter. However, the capacitor and the inductor are all fractional in nature [7]. In fact, the dielectric materials exhibit a fractional behavior, yielding electrical impedances of the form $1/(j\omega CF)^{\alpha}$ with $\alpha \subset R+$. Inductor is also fractional in nature. Many real dynamical circuits were better characterized by using non-integer order models based on fractional calculus [4-6].The modeling of the fractional-order DC–DC converter has become the subject which has important significance in theory and practice. In this paper, we will analyze the modeling and some dynamical properties of the fractional-order Buck–Boost converter in continuous conduction mode.

II. FRACTIONAL CALCULUS AND THE MATHEMATICAL MODELING OF THE FRACTIONAL ORDER BUCK –BOOST CONVERTER

The Riemann–Liouville definition which is well known for the fractional differential operator is given as [8]

$$\frac{d^{\alpha} f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad (1)$$

Where $n - 1 < \alpha < n$, n is an integer number, and $\Gamma(\bullet)$ is the gamma function.

Another alternative definition of the Riemann–Liouville function is

$$\frac{d^{\alpha} f(t)}{dt^{\alpha}} = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \qquad (2)$$

Then we create the fractional-order mathematical model of the Buck–Boost converter by using the concept of fractional calculus.

For a general input voltage $V(t)$, the current is

$$i(t) = C_f \frac{d^{\alpha} u(t)}{dt^{\alpha}}, 0 < \alpha < 1 \qquad (3)$$

C_f - Capacitance of the capacitor. It is related to the kind of dielectric. Another constant α (order) is related to the loss of the capacitor.

For a general current in the inductor, the voltage is

$$u(t) = L_f \frac{d^{\beta} i(t)}{dt^{\beta}}, 0 < \beta < 1 \qquad (4)$$

Where L_f the inductance of the inductor and β (order) is the constant related to the “proximity effect”.

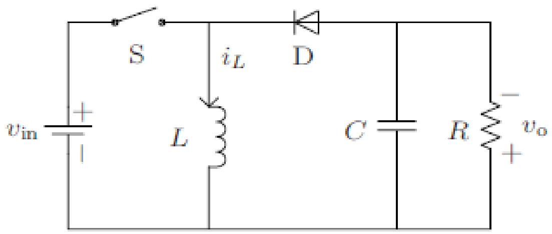

Fig.1 Buck-Boost converter

The Buck–Boost converter, sometimes called a step-up/down power stage, is an inverting power stage topology. The current-mode Buck–Boost converter is shown in Fig.1 [8]

In the CCM, there are two switching modes.

1) When switch S is on, the inductor current passes through the switch, and the diode is reverse-biased with the inductor current i_L rising.

2) When the switch S is turned off, the inductor maintains current flowing in the same direction so that the diode is forward-biased.

III. STATE SPACE AVERAGING

By canonical form,

$\dot{X}$=AX+BU, Y=CX+DU

$$X = \begin{bmatrix} i_L(t) \\ v_o(t) \end{bmatrix}, U = [v_{in}(t)], Y = i_{in}(t)$$

Switch ON mode:-

$$v_{in}(t) = L\frac{d^{\alpha} i_L(t)}{dt^{\alpha}} \Rightarrow \frac{d^{\alpha} i_L(t)}{dt^{\alpha}} = \frac{v_{in}(t)}{L} \qquad (5)$$

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$$C\frac{d^{\beta}v_o(t)}{dt^{\beta}}=\frac{-v_o(t)}{R}\Rightarrow\frac{d^{\beta}v_o(t)}{dt^{\beta}}=\frac{-v_o(t)}{RC} \tag{6}$$

$$i_{in}(t)=i_L(t) \tag{7}$$

$$\begin{bmatrix}\frac{d^{\alpha}{}_{i_L}(t)}{dt^{\alpha}}\\ \frac{d^{\beta}v_o(t)}{dt^{\beta}}\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & \frac{-1}{RC}\end{bmatrix}\begin{bmatrix}i_L(t)\\ v_o(t)\end{bmatrix}+\begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix}[v_{in}(t)] \tag{8}$$

$$A_1=\begin{bmatrix}0 & 0\\ 0 & \frac{-1}{RC}\end{bmatrix}\qquad B_1=\begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix}\quad C_1=\begin{bmatrix}1 & 0\end{bmatrix}$$

$$E_1=[0]$$

Switch OFF mode:-

$$L\frac{d^{\alpha}{}_{i_L}(t)}{dt^{\alpha}}=v_o(t)\Rightarrow\frac{1}{L}v_o(t) \tag{9}$$

$$C\frac{d^{\beta}v_o(t)}{dt^{\beta}}=-i(t)-\frac{v_o(t)}{R}\Rightarrow\frac{d^{\beta}v_o(t)}{dt^{\beta}}=\frac{-1}{C}i(t)-\frac{v_o(t)}{RC} \tag{10}$$

$$\begin{bmatrix}\frac{d^{\alpha}{}_{i_L}(t)}{dt^{\alpha}}\\ \frac{d^{\beta}v_o(t)}{dt^{\beta}}\end{bmatrix}=\begin{bmatrix}0 & \frac{1}{L}\\ \frac{-1}{C} & \frac{-1}{RC}\end{bmatrix}\begin{bmatrix}i_L(t)\\ v_o(t)\end{bmatrix}+\begin{bmatrix}0\\ 0\end{bmatrix}[v_{in}(t)] \tag{11}$$

$$i_{in}(t)=0$$

$$A_2=\begin{bmatrix}0 & \frac{1}{L}\\ \frac{-1}{C} & \frac{-1}{RC}\end{bmatrix}\quad B_2=\begin{bmatrix}0\\ 0\end{bmatrix}\quad C_2=\begin{bmatrix}0 & 0\end{bmatrix}$$

$$E_2=[0]$$

In comparison with the integer-order model, the fractional-order model of the Buck–Boost converter in the CCM described by is relevant to α and β.

IV. STATE SPACE AVERAGING MODEL OF FRACTIONAL-ORDER BUCK BOOST CONVERTER

According to the operational features of the Buck–Boost converter, the circuit variables in the Buck–Boost converter, such as inductance current $i_L$and the output voltage V_O have all the high frequency switching harmonics. These harmonics can be removed by averaging a circuit variable over one switching period.

$$\langle x(t)\rangle_T=\frac{1}{T}\int_t^{t+T}x(\tau)d\tau \tag{12}$$

x is an arbitrary circuit variable of the Buck–Boost converter.

$$\frac{d^{\alpha}\langle x(t)\rangle_T}{dt^{\alpha}}=\frac{d\left(\frac{1}{T}\int_t^{t+T}x(\tau)d\tau\right)}{dt^{\alpha}} \tag{13}$$

$$=\frac{1}{T}\int_t^{t+\tau}\frac{d^{\alpha}x(\tau)}{dt^{\alpha}}d\tau$$

$$=\left\langle\frac{d^{\alpha}x(t)}{dt^{\alpha}}\right\rangle$$

α -is the order and $0<\alpha<1$.

The average values of the circuit variables $\langle i_L(t)\rangle,\langle v_o(t)\rangle,\langle v_{in}(t)\rangle$ and $d(t)$ can be described in the following forms

$$\langle i_L(t)\rangle=I_L+\hat{i}_L(t),\quad\langle v_{in}(t)\rangle=V_{in}+\hat{v}_{in}(t)$$

$$\langle v_o(t)\rangle=V_o+\hat{v}_o(t)\quad,\quad d(t)=D+\hat{d}(t)$$

where I_L, Vo, D, and Vin are the DC components of $i_L(t)$, $v_o(t)$, $d(t)$ and $d(t)$ respectively, $\hat{i}_L(t)$, $\hat{v}_{in}(t),\hat{v}_o(t),\hat{d}(t)$ are the AC components respectively.

$$A=DA_1+D'A_2$$

$$A=D\begin{bmatrix}0 & 0\\ 0 & \frac{-1}{CR}\end{bmatrix}+D'\begin{bmatrix}0 & \frac{1}{L}\\ \frac{-1}{C} & \frac{-1}{CR}\end{bmatrix}$$

$$A=\begin{bmatrix}0 & \frac{D'}{L}\\ -\frac{D'}{C} & -\frac{1}{CR}\end{bmatrix}$$

$$B=DB_1+D'B_2$$

$$B=D\begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix}+D'\begin{bmatrix}0\\ 0\end{bmatrix}\qquad B=\begin{bmatrix}\frac{D}{L}\\ 0\end{bmatrix}$$

$$\begin{bmatrix}I_L\\ V_O\end{bmatrix}=-A^{-1}BU$$

$$=\frac{-1}{\frac{D'^2}{LC}}\begin{bmatrix}\frac{-1}{CR} & \frac{-D'}{L}\\ \frac{D'}{C} & 0\end{bmatrix}\begin{bmatrix}\frac{D}{L}\\ 0\end{bmatrix}[v_{in}]$$

$$=\begin{bmatrix}\frac{v_{in}D}{D'^2R}\\ \frac{-Dv_{in}}{D'}\end{bmatrix}$$

$$\dot{X}(t)=A\hat{X}(t)+B\hat{U}(t)+\left[(A_1-A_2)X+(B_1-B_2)U\right]\hat{d}(t)$$

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$$\begin{bmatrix} \dfrac{d^{\alpha}{}_{i_L}(t)}{dt^{\alpha}} \\ \dfrac{d^{\beta}v_o(t)}{dt^{\beta}} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{D'}{L} \\ \dfrac{-D'}{C} & \dfrac{-1}{CR} \end{bmatrix} \begin{bmatrix} \hat{i}_L(t) \\ \hat{v}_o(t) \end{bmatrix} + \begin{bmatrix} \dfrac{D}{L} \\ 0 \end{bmatrix} \begin{bmatrix} \hat{v}_{in}(t) \end{bmatrix} + \left[\begin{bmatrix} 0 & \dfrac{-1}{L} \\ \dfrac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_O \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} [V_{in}] \right] \hat{d}(t) \tag{14}$$

$$\frac{d^{\alpha}{}_{i_L}(t)}{dt^{\alpha}} = \frac{D'}{L}\hat{v}_o(t) + \frac{D}{L}\hat{v}_{in}(t) - \frac{V_O}{L}\hat{d}(t) + \frac{V_{in}}{L}\hat{d}(t) \tag{15}$$

$$\frac{d^{\beta}v_o(t)}{dt^{\beta}} = \frac{-D'}{C}\hat{i}_L(t) - \frac{1}{CR}\hat{v}_o(t) + \frac{I_L}{C}\hat{d}(t) \tag{16}$$

$$\hat{Y}(t) = C\hat{X}(t) + E\hat{U}(t)\left[(C_1 - C_2)X + (E_1 - E_2)U\right]\hat{d}(t)$$

$$C = DC_1 + D'C_2$$

$$= D[D \quad 0] + D'[0 \quad 0]$$

$$= [D \quad 0]$$

$$\hat{Y}(t) = [D \quad 0]\begin{bmatrix} \hat{i}_L(t) \\ \hat{v}_O(t) \end{bmatrix} + [1 \quad 0]\begin{bmatrix} I_L \\ V_O \end{bmatrix}\hat{d}(t)$$

$$\hat{i}_{in}(t) = D\hat{i}_L(t) + I\hat{d}(t) \tag{17}$$

Using the definition of fractional-order derivative and considering the input voltage V_{in} to be a constant, we can obtain the inductor current ripple Δi_L in (0, *DT*) as

$$\Delta i_L = \frac{V_{in}[DT]^{\alpha}}{L\alpha\Gamma(\alpha)} \tag{18}$$

$\Gamma(\bullet)$ is the gamma function. The inductor current ripple Δi_L is related not only to the inductance, the input voltage V_{in}, the duty ratio *D*, and the switching period *T*, but also to the order of inductor *L*. When α is increased Δi_L is reduced. Especially when $\alpha = 1$, we can obtain the integral-order model.

$$i_{L\max} = I_L + \frac{1}{2}\Delta i_L = \frac{V_{in}D}{(1-D)^2 R} + \frac{V_{in}(DT)^{\alpha}}{2L\alpha\Gamma(\alpha)} \tag{19}$$

The continuous-conduction mode of operation occurs when the current through the inductor is continuous. That means the inductor current is always greater than zero, i.e.,

$$I_L > \frac{1}{2}\Delta i_L$$

Then we consider the variation of output voltage *vo* using the Adomian decomposition method, we can solve the following fractional differential equation.

$$D^{\alpha}x(t) = Ax(t) + f(t)$$

$$0 < \alpha < 1, 0 < t < T$$

$$D^{\beta}v_o(t) = -\frac{1}{CR}v_o(t) \qquad A = -\frac{1}{CR} \text{ And } f(t) = 0$$

$$v_o(t) = E_{\beta,1}(At^{\beta})V_{os}$$

$E_{\beta}(\bullet)$ is the Mittag–Leffler function [5] and *Vos* is the initial value of the output voltage. When $t = 0$, the switch is on, so

$$v_o(0) = E_{\beta,1}(0)V_{os} = V_{os}$$

When t=DT, the output voltage is

$$v_o(DT) = E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]V_{os}$$

Then, we can obtain the output voltage ripple Δ*vo* as

$$\Delta v_o = v_o(0) - v_o(DT)$$

$$= V_{os} - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]V_{os} = \left\{1 - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}V_{os}$$

The approximate expression of *Vos* is $V_{os} = V_O + \frac{\Delta v_o}{2}$

$$\Delta v_o = 2V_O\frac{\left\{1 - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}}{\left\{1 + E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}} = -\frac{2V_{in}D}{(1-D)}\frac{\left\{1 - E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}}{\left\{1 + E_{\beta,1}\left[-\frac{(DT)^{\beta}}{CR}\right]\right\}} \tag{20}$$

The output voltage ripple is related not only to capacitor *C*, load resistance *R*, input constant voltage V_{in}, duty ratio *D*, and switching period *T*, but also to *β*. When *β* increases, Δ*vo* is reduced.

From (15) & (16), by using Laplace transform based on the FC

$$s^{\beta}\hat{v}_o(s) = -\frac{(1-D)\hat{i}_L(s)}{C} + \frac{\hat{d}(s)I_L}{C} - \frac{\hat{v}_o(s)}{CR}$$

$$s^{\alpha}\hat{i}_L(s) = \frac{V_{in}\hat{d}(s) + D\hat{v}_{in}(s) + (1-D)\hat{v}_o(s) - V_O\hat{d}(s)}{L} \tag{21}$$

The transfer function of $\hat{v}_o(t)$ to $\hat{v}_{in}(t)$ is

$$G_{v_o v_{in}}(s) = \frac{\hat{v}_o(s)}{\hat{v}_{in}(s)}\bigg|_{\hat{d}(s)=0}$$, From equations (21)

$$\left(Ls^{\beta} + \frac{1}{CR}\right)\hat{v}_o(s) = -\frac{(1-D)}{C}\hat{i}_L(s)$$

$$\hat{v}_o(s) = -\frac{(1-D)\hat{i}_L(s)}{\left(LCs^{\beta} + \frac{1}{R}\right)}$$

$$Ls^{\alpha}\hat{i}_L(s) = D\hat{v}_{in}(s) + (1-D)\hat{v}_o(s)$$

$$\hat{v}_{in}(s) = \frac{\left(LCs^{\alpha+\beta} + \frac{L}{R}s^{\alpha}\right) + (1-D)^2}{\left(LCs^{\beta} + \frac{1}{R}\right)D}\hat{i}_L(s)$$

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$$\frac{\hat{v}_o(s)}{\hat{v}_{in}(s)} = -\frac{D(1-D)}{LCs^{\alpha+\beta}+\frac{L}{R}s^{\alpha}+(1-D)^2} \tag{22}$$

The transfer function of $\hat{v}_o(t)$ to $\hat{d}(t)$ is

$$G_{v_o d}(s) = \frac{\hat{v}_o(s)}{\hat{d}(s)}\bigg|_{\hat{v}_{in}(s)=0}$$

From (21)

$$\hat{v}_o(s)\left(s^{\beta}+\frac{1}{CR}\right) = -\frac{(1-D)}{C}\hat{i}_L(s)+\frac{\hat{d}(s)}{C}I_L$$

$$\hat{i}_L(s) = -\hat{v}_o(s)\left(Cs^{\beta}+\frac{1}{R}\right)\frac{1}{(1-D)}+\frac{I_L}{(1-D)}\hat{d}(s)$$

$$\frac{\hat{v}_o(s)}{\hat{d}(s)} = -\frac{\left[(V_{in}-V_o)-\frac{Ls^{\alpha}}{(1-D)}I_L\right]}{\left(LCs^{\alpha+\beta}+\frac{L}{R}s^{\alpha}+(1-D)^2\right)}$$

Using the below equations we can solve the above mentioned equation

$$V_O = \frac{-D}{1-D}V_{in}$$

$$I_L = \frac{-V_o}{(1-D)R}$$

$$\frac{\hat{v}_o(s)}{\hat{d}(s)} = -V_{in}\left[1-\frac{LD}{(1-D)^2R}s^{\alpha}\right] \tag{23}$$

The transfer function of $\hat{i}_L(t)$ to $\hat{v}_{in}(t)$ is

$$G_{i_L v_{in}}(s) = \frac{\hat{i}_L(s)}{\hat{v}_{in}(s)}\bigg|_{\hat{d}(s)=0}$$

$$Ls^{\alpha}\frac{\hat{i}_L(s)}{\hat{v}_{in}(s)} = D+(1-D)\frac{\hat{v}_o(s)}{\hat{v}_{in}(s)}$$

$$= D-\frac{D(1-D)^2}{\left(LCs^{\alpha+\beta}+\frac{L}{R}s^{\alpha}+(1-D)^2\right)}$$

$$\frac{\hat{i}_L(s)}{\hat{v}_{in}(s)} = \frac{D\left(Cs^{\beta}+\frac{1}{R}\right)}{\left(LCs^{\alpha+\beta}+\frac{L}{R}s^{\alpha}+(1-D)^2\right)} \tag{24}$$

V. SIMULATION STUDY OF THE CURRENT CONTROLLED FRACTIONAL-ORDER BUCK-BOOST CONVERTER

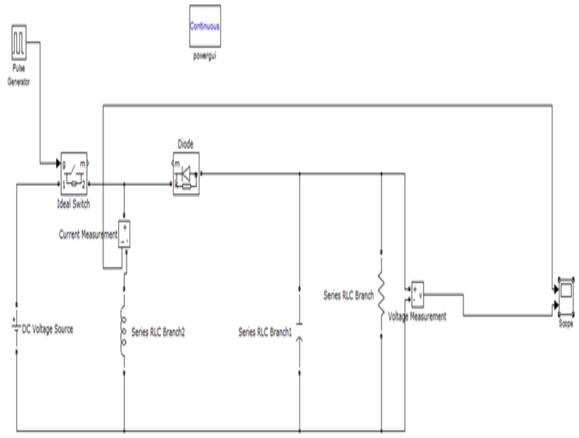

Fig.2 Simulation circuit of Buck-Boost converter

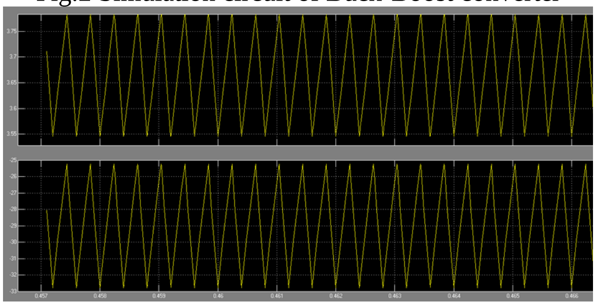

Fig.3 When α=1, β=1 i_L/A and v_o/V

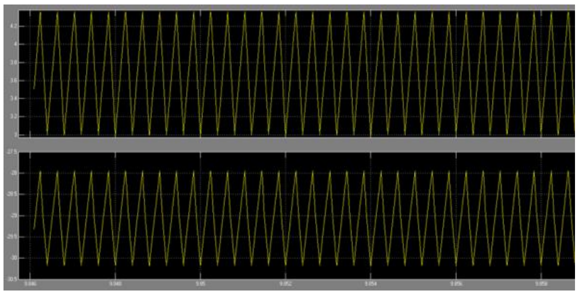

Fig.4 When α=0.8 and β= 0.95 i_L/A and v_o/V

In the simulation use the following parameters:
f = 2.5 kHz, V_{in} = 20 V, d = 0:6, R = 20 Ω,
L = 0:02 H, and C = 47 µF.
When α=1 and β=1 according to equations

I_L =3.75A, Δi_L =0.24A, V_o = -30V, Δv_o =-7.62V

$i_{L\max}$ =3.87 A

$v_{o\min}$ = -33.81V

The practical simulation results are

$i_{L\max}$ =3.85A $v_{o\min}$ = -32.8V

When α=0.8 and β=0.95 according to equations

I_L =3.75A, Δi_L =1.36A, V_o = -30V, Δv_o = -13.8V

$i_{L\max}$ =4.43 A

$v_{o\min}$ = -36.9V

The practical simulation results are

$i_{L\max}$ =4.4A $v_{o\min}$ = -30.3V

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VI. CONCLUSION

The mathematical model and the state space averaging model of the fractional-order Buck–Boost converter is created based on Fractional calculus. The inductor current ripple, the peak inductor current, the output voltage ripple, the initial value of the output voltage and the transfer functions (*Gvovin* (*s*), *Gvod*(*s*), and *GiLvin* (*s*)) are all related to fractional-order α or β .When α increases, the inductor current ripple and the peak inductor current decrease. When other parameters are fixed, the output voltage ripple and the initial value of the output voltage decrease with β increasing. SIMULATION results also verify the equations.

REFERENCES

[1] Nigmatullin R R, Arbuzov A A, Salehli F, Giz A, BayrakI and Catalgil G L 2007 Physica B 388 418

[2] Westerlund S 1994 IEEE Transactions on Dielectrics and Electrical Insulation 1 826

[3] Wu C J, Zhang Y B and Yang N N 2011 Chin. Phys. B 20 060505

[4] Westerlund S 2002 Dead Matter Has Memory! (Kalmar, Sweden: Causal Consulting) Chap. 7

[5] Podlubny I 1999 Fractional Differential Equations (San Diego: Academic Press) Chap. 2

[6] Nakagava M and Sorimachi K 1992 IEICE Trans. Fundam. E75-A 1814

[7] Jonscher A K 1983 Dielectric Relaxation in Solids (London: Chelsea Dielectric Press) Chap. 3

[8] Yang Ning –Ning, Liu Chong-Xin and Wu Chao-Jun ," Modeling and dynamics analysis of the fractional-order Buck-Boost converter in continuous conduction mode", Chin.Phys.B Vol.21,2012

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