

# A Secure Elliptic Curve Digital Signature Approach without Inversion

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*Abstract— The Elliptic Curve Digital Signature Algorithm (ECDSA) is the elliptic curve analogue of the Digital Signature Algorithm (DSA). Unlike the ordinary discrete logarithm problem elliptic curve discrete logarithm problem (ECDLP) has no sub-exponential time algorithm, due to this the strength per key bit is substantially greater when compare with conventional discrete logarithm systems. Elliptic curve based digital signatures are stronger and ideal for constrained environments like smart cards due to smaller bit size, thereby reducing processing overhead. Considering the security of data it is lacking regarding random number choosing or determination. This lacking leads to recovery of the private key in original Elliptic Curve Digital Signature scheme. This problem is overcome by our proposed digital signature scheme which is presented in this paper.*

*Index Terms— Digital Signature, ECDSA, Elliptic Curve Cryptography, Elliptic Curve Digital Signature Algorithm.*

## I. INTRODUCTION

A digital signature is a checksum which depends on the time period during which it was produced. It depends on all the bits of a transmitted message, and also on a secret key, but which can be checked without knowledge of the secret key. The Digital Signature Algorithm (DSA) was specified in a U.S. Government Federal Information Processing Standard (FIPS) called the Digital Signature Standard (DSS). Its security is based on the computational intractability of the discrete logarithm problem (DLP) in prime-order subgroups of  $Z^*p$ . At this time, there are three popular public-key algorithms which can provide digital signatures:

1. Elliptic Curve Digital Signature Algorithm (ECDSA) [1]
2. RSA scheme [2]
3. ElGamal signature scheme [3]

Among these ECDSA provides a faster alternative for public-key cryptography, much smaller key lengths are required to provide a desired level of security [1].

In 1985, the Elliptic Curve Discrete Logarithm Problem (ECDLP) was proposed independently as a new cryptographic scheme by koblitz [4] and Miller [5]. It is considered that the security of ECC is sufficiently proved. Since the ECDLP appears to be significantly harder than the DLP, the elliptic curve systems appears efficient than in conventional discrete logarithm systems.

The Elliptic Curve Digital Signature Algorithm (ECDSA) is the elliptic curve analogue of the DSA. ECDSA was first proposed by Vanstone in 1992[6] in response to NIST's (National Institute of Standards and Technology) request for public comments on their first proposal for DSS. It was accepted in 1999 as an ANSI standard, and was accepted in 2000 as IEEE PI363 [7] and NIST's FIPS 186-2 [8] standards. It was also accepted in 1998 as an ISO standard, and is under consideration for inclusion in some other ISO standards

## II. OVERVIEW OF ELLIPTIC CURVE ARITHMETIC

Elliptic curve cryptosystems (ECC) were invented by Neal Koblitz [4] and Victor Miller [5] in 1985. The mathematical basis for the security of elliptic curve cryptosystems is the computational intractability of the elliptic curve discrete logarithm problem (ECDLP) [9].

An elliptic curve  $E$  over  $F_p$  is defined by an equation of the form:

$$y^2 = x^3 + ax + b$$

where  $a, b \in F_p$ , and  $4a^3 + 27b^2 \neq 0 \pmod{p}$ , together with a special point  $O$ , called the point at infinity. The set  $E(F_p)$  consists of all points  $(x, y)$ ,  $x \in F_p$ ,  $y \in F_p$ , which satisfy the defining equation, together with  $O$ . We denote the curve by  $E(F_p)$ .

Figures 1 and 2 show point addition and point doubling operations over an elliptic curve.

The basic EC operations are point addition and point doubling. Elliptic curve cryptographic primitives require scalar point multiplication. Say, given a point  $P(x, y)$  on an EC, one needs to compute  $kP$ , where  $k$  is a positive integer. This is achieved by a series of doubling and addition operations of  $P$  (see e.g [10]).

Definition of elliptic curve over  $F_p$  as follows [11].

Let  $p$  be a prime in  $F_p$  and  $a, b \in F_p$  such that  $4a^3 + 27b^2 \neq 0 \pmod{p}$  in  $F_p$ , then an elliptic curve  $E(F_p)$  is defined as

$$E(F_p) := \{ p(x, y), x, y \in F_p \}$$

Such that  $y^2 = x^3 + ax + b \pmod{p}$  together with a point  $O$ . Below is the definition of addition of points  $P$  and  $Q$  on the elliptic curve  $E(F_p)$ . Let  $P(x_1, y_1)$  and  $Q(x_2, y_2)$  then

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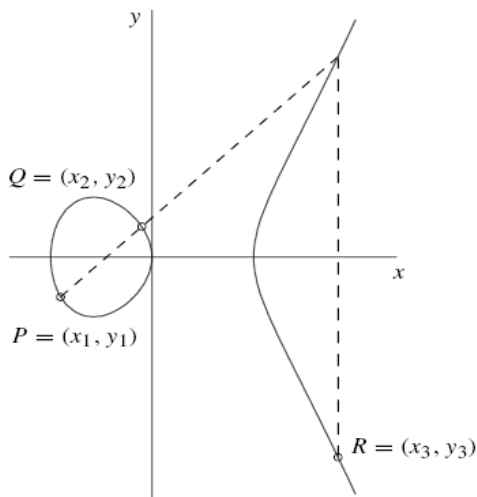


Fig.1. Addition:  $R=P+Q$

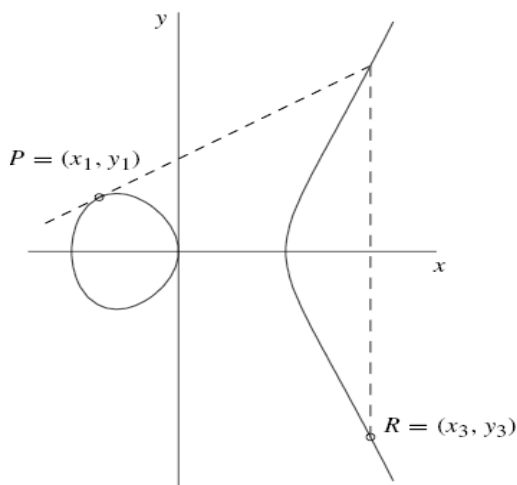


Fig.2. Doubling:  $R=P+P$

$$R = P+Q = \begin{cases} O & \text{If } x_1 = x_2 \text{ and } y_2 = -y_1 \\ Q = Q+P & \text{If } P = O \\ (x_3, y_3) & \text{otherwise} \end{cases}$$

Where

$$x_3 = \begin{cases} \lambda^2 - x_1 - x_2 & \text{If } P \neq \pm Q \text{ (Point Addition)} \\ \lambda^2 - 2x_1 & \text{If } P = Q \text{ (Point Doubling)} \end{cases}$$

$y_3 = \lambda(x_1 - x_3) - y_1$ , and

$$\lambda = \begin{cases} \frac{y_2 - y_1}{x_2 - x_1} & \text{If } P \neq \pm Q \text{ (Point Addition)} \\ \frac{3x_1^2 + a}{2y_1} & \text{If } P = Q \text{ (Point Doubling)} \end{cases}$$

The point  $p(x, -y)$  is said to be the negation of  $p(x, y)$ .

### III. PROPOSED DIGITAL SIGNATURE BASED ON ELLIPTIC CURVE

In original ECDSA the private can be recovered if two different messages are signed by using same random number.

To solve this problem, we proposed a new Digital Signature algorithm based on Elliptic Curve. Digital Signature scheme is developed without modular inversion operation in Signature generation and Verification algorithms. Proposed scheme signature contained three elements and the scheme consists of following three phases.

- Key pair generation
- Signature Generation
- Signature Verification

#### Notations:

To be convenient in description of our work the attributes are defined as

- d** : private key
- Q** : Public key
- m** : message
- H()** : a secure one-way hash function
- r, s<sub>1</sub>, s<sub>2</sub>** : Signature elements
- q**: field order
- FR**: field representation
- a, b**: coefficients
- G**: base point
- n**: Order of G
- h**: co-factor

Key pair  $d$  and  $Q$  generated by the Signer as follows

#### Key pair Generation Algorithm:

**INPUT:**  $D = (q, FR, a, b, G, n, h)$

**OUTPUT:**  $Q, d$

Select  $d \in [1, \dots, n-1]$

Compute  $Q = dG$

Return  $(Q, d)$

Message  $m$  can be signed by the signer as follows.

#### Signature Generation Algorithm:

**INPUT:**  $D = (q, FR, a, b, G, n, h), d, m$

**OUTPUT:** Signature  $(r, s_1, s_2)$

**Begin**

**repeat**

$k = \text{Random}[1, 2, \dots, n-1]$

$P = kG$

$c = X\text{-Co-ordinate}(P)$

$e = H(m) \bmod n$

$s_1 = ec \bmod n$

$s_2 = (dc + k) \bmod n$

$R = eP$

$r = X\text{-Co-ordinate}(R)$

**until**  $r \neq 0$  and  $s_1 \neq 0$  and  $s_2 \neq 0$

**return**  $(r, s_1, s_2)$

**End**

To verify the signature  $(r, s_1, s_2)$  on message  $m$ , receiver does the following:

#### Signature Verification Algorithm:

**INPUT:**  $D = (q, FR, a, b, G, n, h), Q, m, \text{Signature } (r, s_1, s_2)$

**OUTPUT:** Acceptance or rejection of the signature.

**Begin**

**if**  $r, s_1, s_2 \notin [1, \dots, n-1]$  **then**

Return (“Reject the signature”)

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end if
e = H(m)
t = es2
U1 = tG
U2 = s1Q
W = U1 - U2
v = X-Co-ordinate(W)
if v = r then
    Return ("Accept the signature")
else
    Return ("Reject the signature")
end if
End
    
```

#### IV. PERFORMANCE COMPARISON

We implemented original ECDSA and our proposed scheme and compared their performance over Elliptic Curve P-160 and presented the results below.

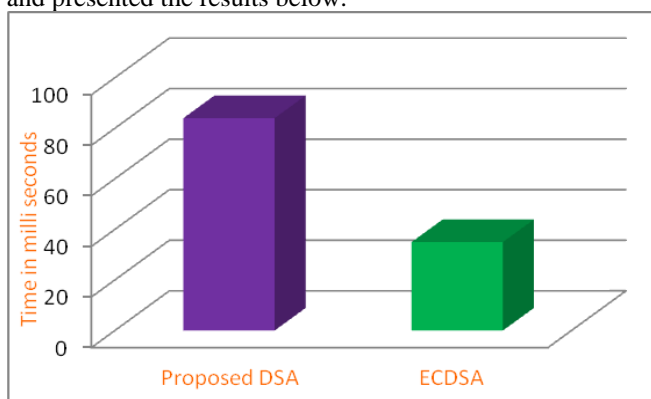


Fig.3. Performance Comparison of Signature Generation over EC P-160

From figure-3, proposed Digital Signature scheme performed poorly because security is inversely proportional to performance of the system.

From Figure-4, proposed scheme signature verification algorithm performed better than the existing verification scheme. This is enviable because to the application oriented point of view message is authorized by the individual only once, but verification may be required many times.

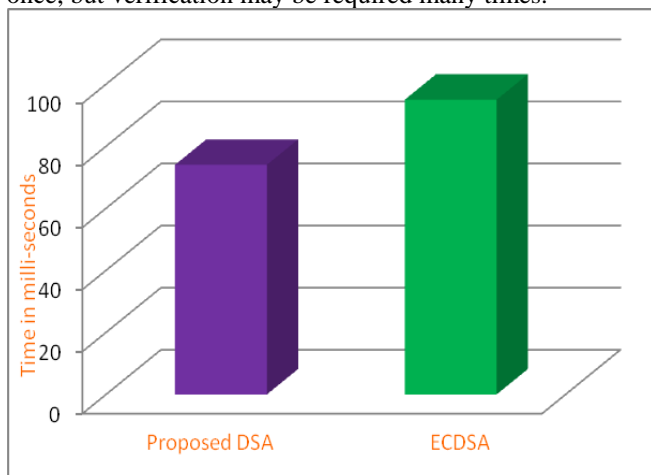


Fig.4. Performance Comparison of Signature Verification over EC P-160.

#### V. PROOF OF SIGNATURE VERIFICATION

We begin with  $W=U_1-U_2$

By substituting  $U_1$  with  $tG$  and  $U_2$  with  $s_1Q$

$$W = tG - s_1Q$$

By substituting  $t$  with  $es_2$  and  $Q$  with  $dG$

$$W = es_2G - s_1dG$$

By substituting  $s_1$  with  $ec$  and  $s_2$  with  $(dc + k)$

$$\begin{aligned}
 W &= e(dc + k)_G - ecdG \\
 &= edcG + ekG - ecdG \\
 &= ekG \\
 &= eP \\
 &= R
 \end{aligned}$$

$v = X\text{-Co-ordinate}(W)$  and  $r = X\text{-Co-ordinate}(R)$

Therefore  $v = r$ .

#### VI. CONCLUSION

Proposed Digital Signature scheme did not use any modular inversion operation. Modular inversion operation is more time consuming operation[12] and our scheme secure even the same random number chosen by the signer in generating more than one signature because it depends on difficulty of ECDLP. Thus, for applications requiring Signature verification more frequently than Signature Generation, proposed scheme may be better choice.

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