

# Application of a Wigner Ville Distribution Based Method in Moving Target Detection

Ibrahim M. Ali, Fathy M. Ahmed

**Abstract** - In the present work, a sinusoidal detection method based on Wigner-Ville distribution (WVD) proposed in [1] is applied in the Moving Target Detection (MTD) for realizing a bank of Doppler filters instead of the direct Fast Fourier Transform (FFT) or WVD in a typical ground based radar. The proposed MTD scheme does not suffer from cross terms produced due to the bilinear nature of WVD. It enhances the target detection capabilities by providing higher detection probabilities and additional gain of 9 and 11 dB in the improvement factor, in the presence of ground and weather clutter, compared to WVD and direct FFT schemes respectively. Performance of the proposed MTD scheme and the other mentioned schemes is evaluated through computer simulation by generating receiver operating characteristics (ROCs) via Monte Carlo trials.

**Keywords**— Fast Fourier Transform (FFT), Moving Target Detection (MTD), Wigner Ville distribution (WVD).

## I. INTRODUCTION

The main task of radar signal processing is the detection of targets, with unknown frequency, in existence of unwanted echoes called clutter. The clutter may be classified as the following: (i) Ground clutter, reflects from fixed or slowly moving objects such as trees, vegetations and manmade structure; (ii) weather clutter, reflects from Storm clouds, precipitations, and movement flocks of birds. Ground clutter has very narrow spectrum, at or near zero Doppler, while weather clutter is more widely spread in frequency with the center frequency being shifted away from the zero Doppler. Conventionally, this task has been accomplished by using a bank of Doppler filters; which is designed with length ideally equal to the number of the returned pulses during each illumination time; each filter is tuned to certain Doppler shift, across the entire range of unambiguous frequencies. The MTD radar signal processor is designed to improve the target detection in variety forms of clutter and providing low false-alarm rate. A scheme of such a processor, which utilizes the direct FFT of the total received data sequence to realize the Doppler filter bank, is called MTD-I [2, 3] (path AA' shown in Fig. 1). MTD-I produces high sidelobes level which increases the probability of false alarm at the output of the Doppler filter bank. Attempts have been made to improve the performance of MTD-I [2, 4]. One of them is known as MTD-WVD (path BB' shown in Fig. 1). It depends on applying WVD in realizing Doppler filter bank instead of the direct FFT

MTD-WVD presents low sidelobes level without restoring to additional weighting like MTD-I. Also, it improves the detection performance, and enhances the improvement factor [4].

Unfortunately, MTD-WVD suffers from cross terms, in case of multiple targets, which are produced due to the bilinear nature of WVD that results in increasing the probability of false alarm.

In this paper, a proposed MTD scheme, designated as MTD-SWVD (path CC' in Fig. 1), is introduced. The proposed MTD-SWVD overcomes the problem of cross terms appeared in MTD-WVD, reduces the noise variance, and improves the improvement factor. The proposed MTD-SWVD realizes the Doppler filter bank by a method presented in [1], based on WVD and called SWVD, for detecting noisy sinusoidal signals in frequency domain.

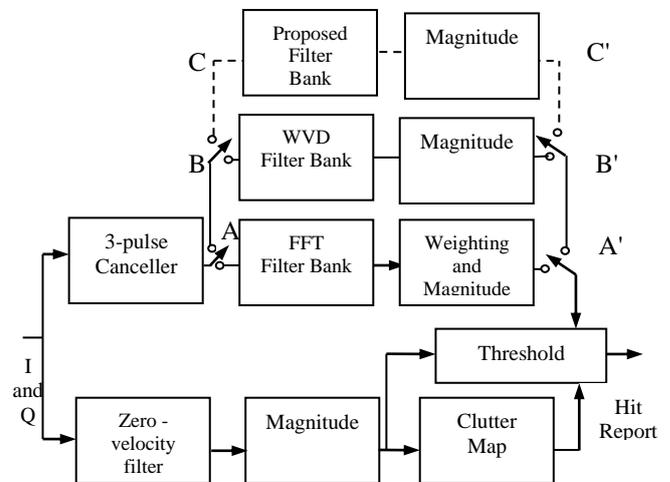


Fig. 1 Block diagram of MTD-I (path AA'), MTD-WVD (path BB'), and the proposed MTD-SWVD (path CC')

This paper is organized as follows: after the introduction, section 2 discusses MTD overview and MTD-I realization. Section 3 demonstrates WVD theory and MTD-WVD. Section 4 details the proposed MTD-SWVD. Section 5 presents computer simulation. Finally section 6 concludes this work and presents future work.

## II. MTD OVERVIEW AND MTD-I REALIZATION

### A MTD Overview

The MTD radar signal processor, which takes place in high-performance coherent-radar systems such as Airport Surveillance Radar (ASR), employs coherent, linear Doppler filtering, adaptive thresholding and a fine ground clutter map as shown in Fig. 1.

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It is designed to improve the performance of radar systems in the presence of various forms of clutter, achieving good probability of detection ( $P_d$ ) and false-alarm rate ( $P_{fa}$ ) [3].

The radar coverage area is divided into a number of Range-Azimuth (RA) cells. The whole range is divided into 'P' range cells and the azimuth into 'M' cells, as shown in Fig. 2 [5].

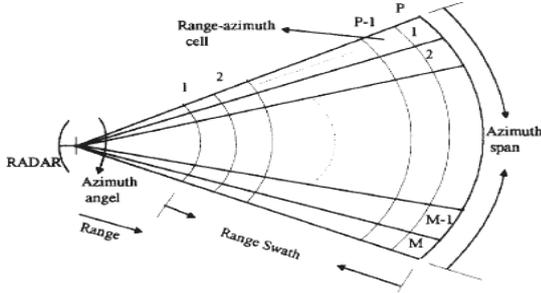


Fig. 2 Radar coverage area.

Since the antenna has to spend certain time in each of these 'M' azimuth rays, a fixed number of pulses are always reflected from each of these 'P' rang cells. The number of reflected pulses, M, in each range azimuth cell can be calculated by [6]:

$$M = \frac{f_r \theta_b}{\Omega} \quad (1)$$

Where,

$f_r$  is the pulse repetition frequency.

$\theta_b$  is the azimuth beam width .

$\Omega$  is the antenna scanning rate.

This bunch of pulses from each RA cell is called the Coherent Processing Interval (CPI). These returned pulses must be quickly processed, to decide whether the returned signal is caused by presence of target or clutter and noise alone. This is achieved by implementing pulse Doppler processing. The elimination of clutter is dependent on the Doppler-filter bank sidelobes, i.e. the lower sidelobes is better for target discrimination [3].

### B MTD-I Realization

The main part of the MTD signal processor is a bank of Doppler filters designed for better discrimination of targets and clutter elimination. In MTD-I the realization of the bank of Doppler filters depends on performing FFT for the total received data sequence, which in this paper is assumed to be 8 pulses out of CPI=10 pulses. The FFT is relied on the discrete Fourier transform which is given by:

$$Y(m) = \sum_{n=0}^{N-1} y(n) \exp(-j2\pi mn/N) \quad (2)$$

Where,

$m = 0, 1, 2, \dots, N-1$ .

$y(n)$  is the input data sequence.

$N$  is the length of this sequence which equal to 8.

The three-pulse canceller, followed by the Doppler filter bank, as shown in Fig. 1 eliminates stationary clutter and

generates  $N$  overlapping filters covering the total Doppler frequency interval.

Direct FFT application results in high sidelobes level of about -13 dB. This high sidelobes level increases the false alarm probability at the output of the Doppler filter bank.

A solution for reducing the effect of high sidelobes and improving clutter rejection is the application of window function; it can be used to decrease the sidelobes level to about -15 to -25 dB or more according to the type of window function [4]. This solution results in widening the main spectral lobes and reduces the Doppler resolution [2].

## III. WVD THEORY AND MTD-WVD

### A WVD Theory

Time-varying signals whose spectral characteristics vary with time, such as radar signals, are analyzed perfectly with the Wigner-Ville distribution (WVD) which is a time-frequency signal representation. The discrete-time version of WVD of the analytic discrete data  $s(n)$  is defined as [7]:

$$WVD(n, 2f) = \sum_{m=-\infty}^{\infty} s\{(n + m/2)\Delta t\} s^*\{(n - m/2)\Delta t\} \times \exp(-j2\pi f m \Delta t) \quad (3)$$

Where  $\Delta t$  is the sampling interval and  $n, m$  are integers. The term  $s\{(n + m/2)\Delta t\} s^*\{(n - m/2)\Delta t\}$  is called the kernel sequence.

The windowed WVD, which is referred as pseudo-Wigner-Ville distribution (PWVD), is used to analyze an infinite duration signal. The returned data of ground-based radar is a discrete short sequence, so there is no need to any window and PWVD can be represented with redefined kernel and implemented by FFT operation as follows[4]:

$$PWVD(n, m) = \sum_{k=0}^{N-1} x(n, k) \exp(-j2\pi km/N) \quad (4)$$

Where  $N$  is the (CPI-2) length,  $PWVD(n, m)$  is the energy distribution of complex data at time index  $n$  and frequency index  $m$ . For  $N$  even, Let  $Q(n, k) = s(n, k)$ , then:

$$= \begin{cases} Q(n, k) \cdot Q^*(n, -k) & k = 0, \dots, \frac{N}{2} - 1 \\ Q(n, k - N) \cdot Q^*(n, -k + N) & k = \frac{N}{2}, \dots, N - 1 \end{cases} \quad (5)$$

For  $N$  is odd, a zero is added, i.e.  $Q(n, -N/2) = 0$ .

### B MTD-WVD

Here, in the application of WVD in MTD for realizing the Doppler filter bank, the frequency domain properties of WVD are considered, for which the time index,  $n$ , is frozen in  $PWVD(n, m)$ . Moreover, the value of  $n$  is set such that all the input data samples are considered in the WVD process. Also, signal interpolated by a factor of 2 is carried out before kernel calculation as shown in Fig. 3 [4].



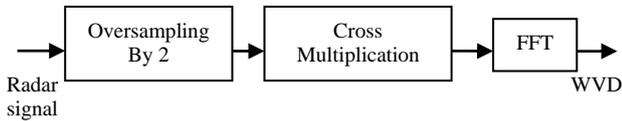


Fig. 3 WVD computation.

For a number of complex data samples with length  $N=8$  at the WVD block. The interpolation process increases the data length to 15 and so a value of  $n=8$  will include all the data samples in the WVD process, i.e. computation of WVD will be at the center of data sequence.

It has been reported that effective classifying of the radar return has been achieved due to using the WVD in frequency domain by this way [8].

The WVD offers better sidelobes level and does not require an additional weighting operation, as is needed for FFT-based MTD. It is considered a good alternative of FFT-based MTD. The main problems here are the cross term effect and the mathematical complexity.

#### IV. THE PROPOSED MTD-SWVD

A method proposed in [1] for detection and frequency estimation of sinusoidal signals from a finite number of noisy discrete-time measurement. This method based on WVD and called SWVD. Here this method is applied in the MTD system for realizing the Doppler filter bank, referred as MTD-SWVD (path CC' in Fig. 1). For CPI radar data of length equal  $M$  pulses which MTD processes coherently.  $N= M-2$  complex data samples, out of three pulse canceller, are fed to the WVD-SWVD block.

In SWVD method, kernel calculation differs from that of WVD calculated in [4] such that; cross-products are calculated for all time index  $n=0$  to  $N-1$ , then the summation is applied over time on complex cross-products to generate summed kernel, instead of calculating the kernel only at time  $n=0$ . The frequency spectrum of the signal under test is obtained by applying FFT on the summed kernel.

Mathematical description of SWVD is as follows [1]:

$$SWVD(m) = \frac{1}{N} \left| \sum_{k=0}^{N-1} SR(k) \exp(-j2\pi km/N) \right| \quad (6)$$

Where

$$m = 0, \dots, N-1 \text{ \& } n = 0, \dots, N-1$$

$$SR(k) = \begin{cases} \sum_{n=k}^{N-k-1} x(n+k)x^*(n-k), & k = 0, \dots, N/2 - 1 \\ 0, & k = N/2 \\ SR^*(N-k-1), & k = N/2 + 1, \dots, N-1 \end{cases}, N \text{ is even} \quad (7)$$

Where,  $m$  is normalized frequency index and  $n$  is discrete time index.  $SR(k)$  is the summed cross products of discrete signal  $x(n)$  calculated in equation 5. This method eliminates the problem of cross-terms of WVD generated due to its bilinear nature. It also reduces the noise variance and boosts the level of energy of signal spectrum. This method also does not need window operation for sidelobe reduction. Moreover, this method gave very good results at off-bin frequency [1]. For the previous reasons, SWVD is selected

to be used in the present work to realize the bank of Doppler filters in the MTD scheme.

#### V. COMPUTER SIMULATIONS

A composite radar video signal is generated for the purpose of simulation of MTD-I, MTD-WVD and the proposed MTD-SWVD schemes based on [9]. Radar environment consists of log-normally distributed ground clutter generated with zero Doppler shift, Rayleigh distributed weather clutter with nonzero Doppler shift, receiver thermal noise, and target returns. Power spectra due to various sources of radar clutter can be approximated by Gaussian spectra at the Doppler frequency,  $f_d$ , as follows [10]:

$$S(f) = S(0) \exp(-(f - f_d)^2 / 2\sigma_f^2) \quad (8)$$

Where  $\sigma_f^2$  is the spectral width.

Clutter spectral width,  $\sigma_f^2$ , Clutter to Noise Ratio (CNR) and Doppler frequency are chosen independently.

Here a CPI of length  $M=10$  pulses is used for the three MTD schemes simulation; the pulses at the Doppler filter bank block is  $N=8$ , because the three-pulse canceller gives  $M - 2$  samples for  $M$  samples at its inputs.

##### A Cross-Term Effect

The method proposed in [1] eliminates the cross terms that appears due to the bilinear nature of WVD. To prove that, a composite radar signal of two target returns at the same range cell with different normalized Doppler frequencies (one at  $f_d = 2/8$  and the other at  $f_d = 4/8$ ) is generated. The three discussed MTD schemes are applied and range Doppler information are plotted. Result shown in Fig. 4 depicts that for the proposed MTD-SWVD, the expected cross-term at  $f_d = 3/8$  is completely eliminated.

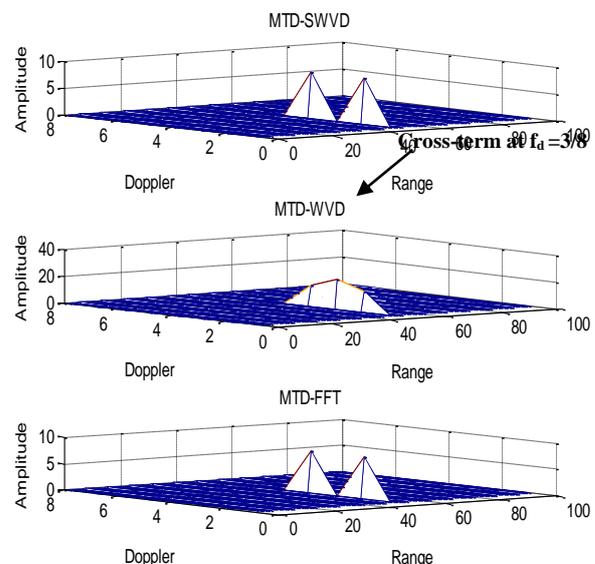


Fig. 4 Range Doppler information for MTD-I (bottom), MTD-WVD (middle), and the proposed MTD-SWVD (top).

B Detection Performance

The performance of the three systems, MTD-I, MTD-WVD and the proposed MTD-SWVD has been compared through the ROC curves, which measure the probability of detection at different probabilities of false alarm. Also, comparison was carried out with respect to the improvement factor (IF) of the three schemes.

The improvement factor for MTD is defined as the ratio of the Signal-to-Clutter + Noise-Ratio (SCNR) at the filters bank output to the SCNR at the filters bank input, averaged uniformly across all the target Doppler frequencies [6].

1 ROC curves

The setup procedure for measuring the detection performance is the same for the considered three MTD schemes. Target signal is added to the noise with a desired Signal-to-Noise-Ratio (SNR). The total normalized Doppler frequency is divided into 8 intervals. The number of range cells examined for each filter is chosen to be 100. Also the target is introduced with a known Doppler frequency in one of the range cells with a uniform probability of 1/100.

As a first step in simulation, the probability of false alarm,  $P_{fa}$ , after the CFAR block is computed for different threshold gains with only thermal noise exist. The results obtained are shown in Fig. 5 corresponding to MTD-I, MTD-WVD, and the proposed MTD-SWVD.

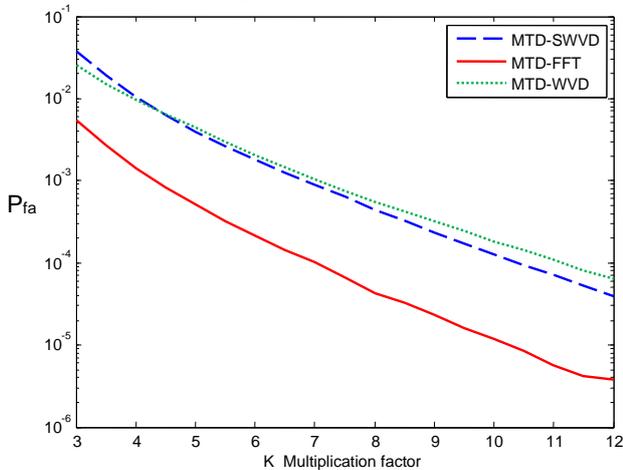


Fig. 5  $P_{fa}$  of MTD-I, MTD-WVD and the proposed MTD-SWVD against threshold gains.

For the case of target with normalized Doppler shift of  $f_d = 0.5$  (in-bin), the ROCs, for the MTD-I, MTD-WVD, and the proposed MTD-SWVD is shown in Fig. 6 at probabilities of false alarm,  $P_{fa}$  of  $10^{-4}$ ,  $10^{-5}$ , and  $10^{-6}$ . These results validate the superiority of the proposed MTD scheme over the two other approaches. For MTD-WVD, the improvement comes from the reduction of the sidelobe level due to the increase of the main lobe level. For the proposed MTD-SWVD, the improvement comes from both main level increasing and sidelobes mean and variance decreasing. Also, Fig. 6 shows that as  $P_{fa}$  gets lower, The ROC of MTD-WVD gets better than that of MTD-I.

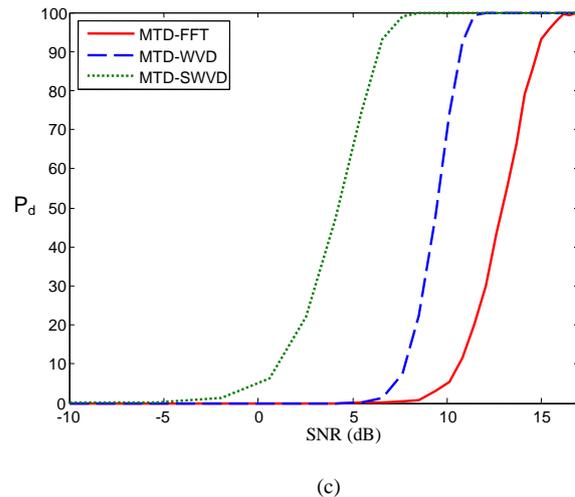
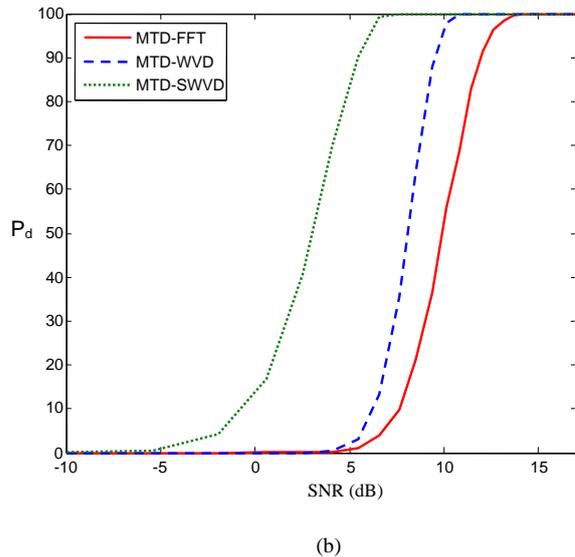
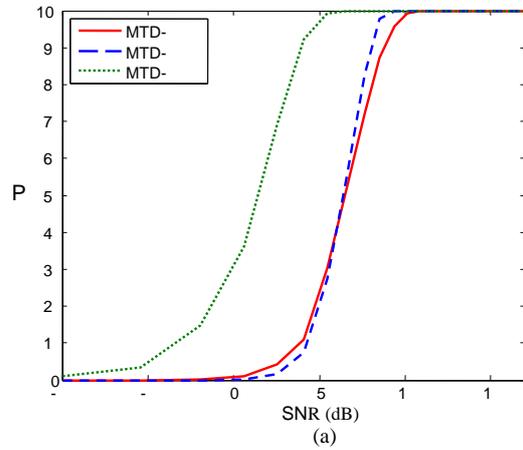


Fig. 6 ROC of MTD-I, MTD-WVD, and the proposed MTD-SWVD for in-bin Doppler frequency; (a)  $P_{fa} = 10^{-4}$ , (b)  $P_{fa} = 10^{-5}$ , and (c)  $P_{fa} = 10^{-6}$

Fig. 7 shows the ROC of MTD-I, MTD-WVD, and the proposed MTD-SWVD for target Doppler shift  $f_d = 0.5625$  (off-bin) at  $P_{fa}$  of  $10^{-4}$ ,  $10^{-5}$ , and  $10^{-6}$ . These figures confirm the superiority of the new approach over the two other approaches.



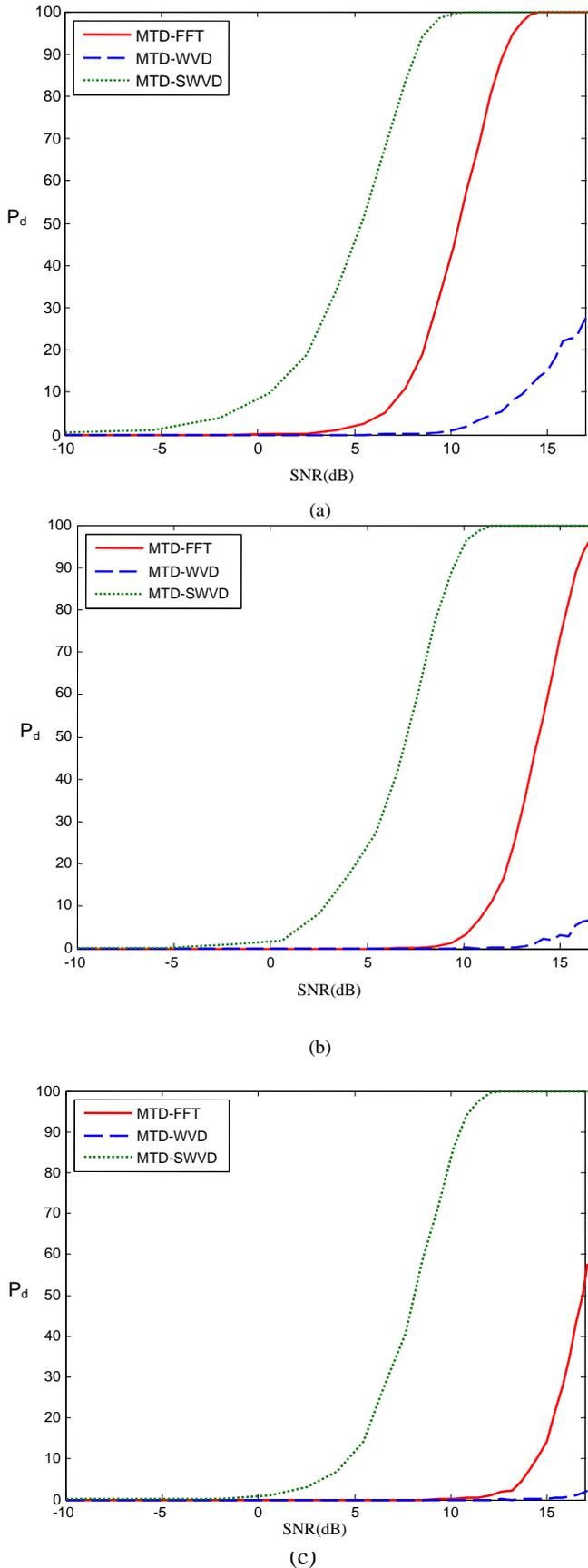


Fig. 7 ROC of MTD-I, MTD-WVD, and the proposed MTD-SWVD for off-bin Doppler frequency; (a)  $P_{fa} = 10^{-4}$ , (b)  $P_{fa} = 10^{-5}$ , and (c)  $P_{fa} = 10^{-6}$

### 2 Improvement factor

The improvement factor setup is also valid for the three MTD systems; MTD-I, MTD-WVD, and the proposed

MTD-SWVD. Initially clutter alone is fed to the MTD input and the output is collected for power computation just before the CFAR block. Later, target alone is introduced and a similar process is carried out. Target frequencies are selected such that it appears at the filter taps with a uniform probability of 1/8.

Fig. 8 shows the improvement factor (in dB) against normalized target Doppler shifts for the three MTD schemes in case of presence of the simulated ground and weather clutter (normalized  $f_d$  for weather clutter is 0.25). The CNR is 4 dB. The effect of weather clutter reduces the improvement factor for all schemes for target Doppler shifts lying in the clutter region. The obtained improvement factor for the proposed MTD-SWVD is about 23 dB, while it is about 14 dB for MTD-WVD, and 12 dB for MTD-I. Hence, a gain of 9 dB over MTD-WVD and 11 dB over MTD-I are achieved.

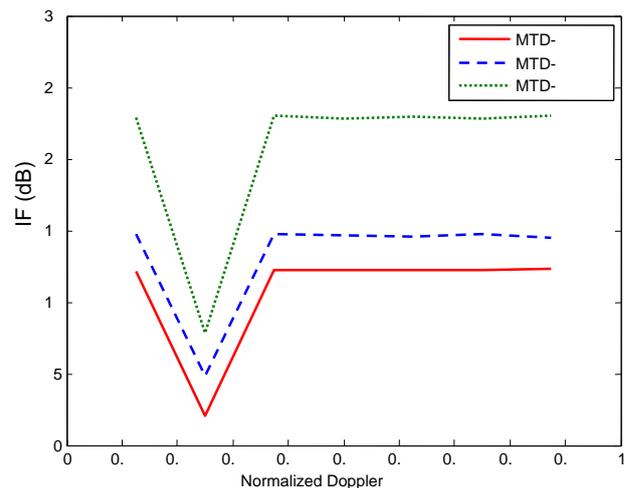


Fig. 8 Improvement factor of MTD-I, MTD-WVD, and the proposed MTD-SWVD against normalized Doppler shifts

## VI. CONCLUSION

A novel scheme, designated as MTD-SWVD, based on SWVD is proposed for realizing MTD. The cross-terms appeared in MTD-WVD were eliminated through the use of SWVD in the proposed MTD scheme. The proposed MTD-SWVD was compared with MTD-I and MTD-WVD and showed a superior performance from the viewpoint of detection probability and improvement factor in the presence of ground and weather clutter. It achieved an improvement factor gain of 9 dB over MTD-WVD, and 11 dB over MTD-I. This is achieved because the sidelobes obtained are very small in level and variance by applying SWVD compared to MTD-WVD or MTD-I. However, the new scheme is associated with the problem of mathematical complexity. Investigation and further analysis for the computational issue of the proposed scheme through the use of high speed processing devices and techniques for real time implementation may be taken into consideration.

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