Design & Analysis of a Spur Gear in different Geometric Conditions

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Abstract—This paper present the designing of the spur gear on different geometric conditions and finding the effect of these on tooth load like by changing the concentration of SIC in SIC based aluminium gear. Addition of SIC increases the strength of Spur Gear. Effect is also analyzed by changing the modules of the gear and by changing the tooth width. Tooth load is calculated with help of Lewis equation & dynamic tooth load is calculated with help of Buckingham equation. Static analysis of the gear is done to find the Von-mises stress on the tooth of the gear in while meshing.

Index Terms—Buckingham equation, Lewis equation, Module, PTC Creo

I. INTRODUCTION

The purpose of gear mechanism is to transmit torque & rotation between two shafts. Spur gear is cylindrical in form and has teeth, which are of involute form in most cases. There are several kinds of stresses present in loaded and rotating gear teeth. Each gear tooth may be considering as a cantilever beam, when it transmits the load, it subjected to bending. The bending stress is highest at the fillet and can caused breakage or fatigue failure of tooth in root region. Calculation of the tooth load is done by Lewis equation and dynamic tooth load by Buckingham equation. The four major failure modes in gear systems are tooth bending fatigue, contact fatigue, surface wear and scoring.

First Gear Profile is designed taking the standard module. Other parameters are find out with respect to that module. Then Failure Load is found out with respect to static and dynamic condition. A 3D model of the designed is made on PTC Creo 2.0 and further analysis of the gear is done in solidworks for induced Von-mises stresses.

II. MATERIAL SELECTION

Different materials used for gear manufacturing are:

- Steel
- Nylon
- Aluminium
- Bronze
- Cast iron
- Phenolic
- Bakelite
- Plastics

A. Aluminium

- Aluminium is a lightweight, reasonably cheap metal widely used for packaging and transport. It has only been widely available and used for the last 60 years.
- Raw aluminium has low strength and high ductility (ideal for foil). Strength is increased by alloying, e.g. with Si, Mg, Cu, Zn, and heat treatment. Some alloys are cast, others are used for wrought products.
- Aluminium is quite reactive, but protects itself very effectively with a thin oxide layer. The surface can be “anodised”, to resist corrosion and to give decorative effects.

B. Steel

- Steels are the most important engineering materials, and cover a wide range of alloys based on iron and carbon. The strength of iron-carbon alloys, particularly after heat treatment, has been exploited for thousands of years (since the “Iron Age”). Modern steels and ferrous alloys have mostly been developed since the Industrial Revolution.
- Mild steel contains 0.1-0.2% C. They are cheap, strong steels used for construction, transport and packaging.
- All steels have a high density and a high Young’s modulus. The strength of mild steel is improved by cold working. It is inherently very tough.
- Mild steel rusts easily, and must be protected by painting, galvanising or other coatings.

C. Cast Iron

- Cast irons were the forerunners to steels, being iron alloys of high carbon content (2-4%). The strength of iron-carbon alloys, particularly after heat treatment, has been exploited for thousands of years (since the “Iron Age”). Modern steels and ferrous alloys have mostly been developed since the Industrial Revolution.
- Cast irons are cheap, high carbon alloys of moderate strength and which can easily be cast to shape. Cast irons have a high density and a high Young’s modulus. They tend to have poor toughness, but their strength and toughness can be improved by alloying and heat treatment.
- Cast irons rust easily, and must be protected by painting or other coatings.

D. Silicon carbide

- Silicon carbide is a covalent ceramic. It is mainly used for its very high hardness (e.g. cutting tools), and for its electrical properties.
- Like all ceramics, silicon carbide is intrinsically hard and strong in compression, but has low toughness and tensile strength.
- Due to its high melting point, silicon carbide can only be processed in powder form.

E. Nylon

- A partially crystalline thermoplastic polymer
- Good strength (for a polymer)
F. SIC based Aluminium Gears

Silicon carbide also reacts with aluminium to yield Al4C3. This conversion limits the mechanical applications of SiC, because Al4C3 is more brittle than SiC.

$4 \text{Al} + 3\text{SiC} \rightarrow \text{Al}_4\text{C}_3 + 3\text{Si}$

In aluminium-matrix composites reinforced with silicon carbide, the chemical reactions between silicon carbide and molten aluminium generate a layer of aluminium carbide on the silicon carbide particles, which decreases the strength of the material, although it increases the wettability of the SiC particles. This tendency can be decreased by coating the silicon carbide particles with a suitable oxide or nitride, preoxidation of the particles to form a silica coating, or using a layer of sacrificial metal.

An aluminium-silicon carbide composite material can be made by mechanical alloying, by milling aluminium powder with graphite particles.

III. BASIC TERMS OF SPUR GEAR

1) MODULE: Module of a gear is defined as ratio of diameter to number of teeth. $m = \frac{D}{N}$
2) FACE WIDTH: The width along the contact surface between the gears is called the face width.
3) TOOTH THICKNESS: The thickness of the tooth along the pitch circle is called the tooth thickness.
4) ADDENDUM: The radial distance between the pitch circle and the top land of the gear is called the addendum.
5) DEDENDUM: The radial distance between the pitch circle and the bottomland of the gear is called the dedendum.
6) PRESSURE ANGLE: The angle between the line joining the centers of the two gears and the common tangent to the base circles.

IV. DESIGN PROCEDURE OF SPUR GEAR

First of all, the design tangential tooth load is obtained from the power transmitted and the pitch line velocity by using the following relation:

$W_T = \frac{P}{v} \times C_S$

Where,

$W_T$ = Permissible tangential tooth load in newtons,
$P$ = Power transmitted in watts,
$v$ = Pitch line velocity in m / s,
$D$ = Pitch circle diameter in metres
$N$ = Speed in r.p.m.,
$C_S$ = Service factor.

A. Apply the Lewis equation as follows

$W_T = \sigma_w b p_c y = \sigma_w b \pi m y$

i. The Lewis equation is applied only to the weaker of the two wheels (i.e. pinion or gear).
ii. When both the pinion and the gear are made of the same material, then pinion is the weaker.
iii. When the pinion and the gear are made of different materials, then the product of $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is the deciding factor. The Lewis equation is used to that wheel for which $(\sigma_w \times y)$ or $(\sigma_o \times y)$ is less.
iv. The product $(\sigma_w \times y)$ is called strength factor of the gear.
v. The face width (b) may be taken as 3 $p_c$ to 4 $p_c$ (or 9.5 m to 12.5 m) for cut teeth and 2 $p_c$ to 3 $p_c$ (or 6.5 m to 9.5 m) for cast teeth.

B. Calculate the dynamic load ($W_D$) on the tooth by using Buckingham equation

$W_D = W_T + \frac{W_f}{21 + \frac{b.C + W_T}{21}}$

C. Find the static tooth load (i.e. beam strength or the endurance strength of the tooth) by using the relation,

$W_S = \sigma_b b p_c y = \sigma_b b \pi m y$

For safety against breakage, $W_S$ should be greater than $W_D$. Finally, find the wear tooth load by using the relation

$W_W = D_p b.Q.K$

The wear load ($W_W$) should not be less than the dynamic load ($W_D$).

V. DESIGNING OF GEAR TOOTH PROFILE

Teeth (T) = 20
Pitch Circle Diameter = D
Pressure Angle(φ) = 200
Take module (m) = 2.5

We know, $m = \frac{D}{T}$

$2.5 = \frac{D}{20}$
$D = 50$ mm

So, Pitch Circle Diameter (D) = 50 mm
Circular pitch (C.p) = $P_c = \Pi D/T = \Pi 50/20 = 7.85$
Diametral pitch (D.p) = $T/D = 20/50 = 0.4$
Addendum = $m = 2.5$ mm
Dedendum = 1.25m = 3.125 mm
Tooth Thickness = 1.5708m = 3.927 mm
Fillet radius = 0.4m = 1mm
Working Depth = 2 m = 5 mm
Minimum Total Depth = 2.25m = 5.625 mm

Top diameter = Addendum+ Pitch Circle Diameter = 25+2.5 = 27.5 mm
Bottom Diameter = Pitch Circle Diameter-Dedendum = 25-3.125 = 21.875
Clearance Depth = Total Depth - (Addendum + Dedendum) = 0.625 mm

VII. DESIGNING OF PURE ALUMINIUM GEAR

A. Lewis equation:

\[ W_r = \sigma_w b \rho_c y \]
\[ \sigma_w = \sigma_o C_v \]
\[ W_r = \sigma_o C_v b \pi m y \]

Take \( \sigma_o = 12.6 \text{ kgf/mm}^2 = 126 \text{ N/mm}^2 \)
\[ C_v = \frac{3}{3 + v} \]
\[ Y = 0.154 - 0.912/T \text{ for 200 full depth involute teeth} \]

B. Dynamic tooth load:

\[ W_D = W_T + W_f \]
\[ W_f = \frac{21v(b.C + W_T)}{21v + \sqrt{b.C + W_T}} \]

Where, \( c = \frac{K.e}{E_p + E_g} \)

For Pure Aluminium:

For 20 teeths, \( Y = 0.154 - 0.912/T \)
\( Y = 0.154 - 0.912/20, \)
\( y = 0.1084 \)

Take \( \sigma_o = 12.6 \text{ kgf/mm}^2 = 126 \text{ N/mm}^2 \)
\[ C_v = \frac{3}{3 + 3} = 0.5 \text{ (as } v = 3 \text{ m/s)} \]

\[ W_T = \sigma_w b \pi m y \]
\[ W_T = 126 \times 0.5 \times 15 \times 2.5 \times 0.1084 \times \pi = 804.55 \text{ N} \]
So, tangential tooth load = 804.55 N

Dynamic tooth load:

\[ W_D = W_T + W_f \]
\[ W_f = \frac{21v(b.C + W_T)}{21v + \sqrt{b.C + W_T}} \]

Where, \( c = \frac{K.e}{E_p + E_g} \)

Take \( K = 0.111 \)
\( E = 0.025 \)
\( E_p = E_g = 6.75 \times 10^4 \)
\[ c = \frac{1}{E_p + E_g} = 0.111 \times 0.025 \times 6.75 \times 10^4 / 2 = 93066 \]
\[ W_f = \frac{21v(b.C + W_T)}{21v + \sqrt{b.C + W_T}} \]
\[ = 21 \times 3 \times (15 \times 93.66 + 804.55) / 21 \]
\[ + \sqrt{((15 \times 93.66) + 804.55)} = 1265 \text{ N} \]
\[ W_D = W_T + W_f = 804.55 + 1265 = 2069.96 \text{ N} \]
VIII. DESIGNING OF SIC BASED ALUMINIUM GEAR

A. For Aluminium & 10% SIC:
Take $\sigma_0 = 14.7$ kgf/mm$^2 = 147$ N/mm$^2$

$$C_v = \frac{3}{3 + v}$$
$$C_v = \frac{3}{3 + 3} = 0.5 \text{ (as } v = 3 \text{ m/s)}$$

$$W_T = \sigma_w b \pi m y$$
$$W_T = 147 \times 0.5 \times 15 \times 2.5 \times 0.1084 \times \pi = 1072.73 N$$

B. For Aluminium & 15% SIC:
Take $\sigma_0 = 17.3$ kgf/mm$^2 = 173$ N/mm$^2$

$$W_T = \sigma_w b \pi m y$$
$$W_T = 173 \times 0.5 \times 15 \times 2.5 \times 0.1084 \times \pi = 1340.91 N$$

IX. CALCULATION OF TOOTH LOAD BY CHANGING MODULES

A. If we take module(m) = 2
Then b = 6m = 12
Take $\sigma_0 = 12.6$ kgf/mm$^2 = 126$ N/mm$^2$

$$C_v = \frac{3}{3 + v}$$
$$C_v = \frac{3}{3 + 3} = 0.5 \text{ (as } v = 3 \text{ m/s)}$$

$$W_T = \sigma_w b \pi m y$$
$$W_T = 126 \times 0.5 \times 12 \times 2 \times 0.1084 \times \pi = 514.88 N$$
So, tangential tooth load = 514.88 N

B. If we take module(m) = 3
Then b = 6m = 18
Take $\sigma_0 = 12.6$ kgf/mm$^2 = 126$ N/mm$^2$

$$C_v = \frac{3}{3 + v}$$
$$C_v = \frac{3}{3 + 3} = 0.5 \text{ (as } v = 3 \text{ m/s)}$$

$$W_T = \sigma_w b \pi m y$$
$$W_T = 126 \times 0.5 \times 18 \times 3 \times 0.1084 \times \pi = 1158.88 N$$
So, tangential tooth load = 1158.88 N

C. If we take module(m) = 4
Then b = 6m = 24
Take $\sigma_0 = 12.6$ kgf/mm$^2 = 126$ N/mm$^2$

$$C_v = \frac{3}{3 + v}$$
$$C_v = \frac{3}{3 + 3} = 0.5 \text{ (as } v = 3 \text{ m/s)}$$

$$W_T = \sigma_w b \pi m y$$
$$W_T = 126 \times 0.5 \times 24 \times 4 \times 0.1084 \times \pi = 2059.52 N$$

X. CALCULATION OF TOOTH LOAD BY CHANGING TOOTH WIDTH

A. If we take width b = 8m = 20 mm
Take $\sigma_0 = 12.6$ kgf/mm$^2 = 126$ N/mm$^2$

$$C_v = \frac{3}{3 + v}$$
$$C_v = \frac{3}{3 + 3} = 0.5 \text{ (as } v = 3 \text{ m/s)}$$

$$W_T = \sigma_w b \pi m y$$
$$W_T = 126 \times 0.5 \times 20 \times 2.5 \times 0.1084 \times \pi = 1072.73 N$$
So, tangential tooth load = 1072.73 N

B. If we take width b = 10m = 25 mm
Take $\sigma_0 = 12.6$ kgf/mm$^2 = 126$ N/mm$^2$

$$C_v = \frac{3}{3 + v}$$
$$C_v = \frac{3}{3 + 3} = 0.5 \text{ (as } v = 3 \text{ m/s)}$$

$$W_T = \sigma_w b \pi m y$$
$$W_T = 126 \times 0.5 \times 25 \times 2.5 \times 0.1084 \times \pi = 1340.91 N$$

XI. ANALYSIS OF GEAR TOOTH USING SOLID WORKS

Fig. 7 Model exported in solidworks
A. Properties:
- Mass: 0.0702584 kg
- Volume: 2.60216e-005 m^3
- Density: 2700 kg/m^3
- Weight: 0.688533 N

B. Material properties:
Name: 1060 Alloy
Model type: Linear Elastic Isotropic
Default failure criterion: Max von Mises Stress
Yield strength: 2.75742e+007 N/m^2
Tensile strength: 6.89356e+007 N/m^2

C. Loads and Fixtures:
Fixed-1

D. Mesh Information:
Total Nodes: 22392
Total Elements: 14031
Element Size: 2.96436 mm
Mesh Quality: High
Jacobian points: 4 Points

E. Results:
- Stress Type - von Mises Stress
  - MIN. Value: -827.971 N/m^2
  - At Node: 17359
  - Max. Value: 6.03174e+007 N/m^2
  - Node: 21007

XII. CONCLUSION

Present day competitive business in global market has brought increasing awareness to optimize gear design. Current trends in engineering globalization require results to comply with various normalized standards to determine their common fundamentals and those approaches needed to identify “best practices” in industries. This can lead to various benefits including reduction in redundancies, cost containment related to adjustments between manufacturers for missing part interchangeability, and performance due to incompatibility of different standards. From the study of effect of various parameters (viz. bending stress, dynamic tooth load, beam strength) on the optimum design of helical gears for marine applications, the induced bending stresses are much lower than those of the results obtained theoretically. Also the bending stresses are much lower than the design stresses, thus the design is safe from the structural point of view. It is observed that the induced bending stresses...
are less than that of the theoretical calculations. Aluminum alloy reduces the weight up to 55-67% compared to other materials like steel. Weight reduction is a very important criterion, in order to minimize the unbalanced forces setup in the marine gear system, thereby improving the system performance. The helical gear parameters that constitute the design are found to be safe from strength and rigidity point of view. Hence Aluminum alloy may be the best possible material for marine gear in the high speed applications.

Optimum parameters for maximum dynamic tooth load: The effect of gear ratio, face width and normal module on optimum dynamic tooth load is carried out. If face width, speed and normal module except gear ratio are kept constant and gear ratio is increased, the corresponding dynamic tooth load remained constant. When gear ratio, speed, normal module except face width are kept constant and face width is increased, the corresponding dynamic tooth load remained constant. The face width, gear ratio and speed except normal module are kept constant and normal module is increased, the corresponding dynamic tooth load increases. The normal module 2.5mm, corresponding to maximum dynamic tooth load is taken as constant.

Optimum parameters for maximum beam strength: The effect of gear ratio, face width, normal module on optimum beam strength is carried out. If face width, speed and normal module except gear ratio are kept constant and gear ratio is increased, the corresponding beam strength remained constant. The gear ratio, speed, normal module except face width are kept constant and face width is increased, the corresponding beam strength found to increase. The face width 49cm, corresponding to maximum beam strength is taken as constant. The face width, gear ratio and speed except normal module are kept constant and normal module is increased, the corresponding beam strength found to increase. The normal module 2.5mm, corresponding to maximum beam strength is taken as constant.

Manganese has been known to be an alloying element of Al alloys that contributes to uniform deformation. Recently, it was found that as the manganese content increases over 0.5% in such aluminum alloys, both yield and ultimate tensile strength increase significantly without decreasing ductility. The added manganese forms a manganese dispersoid, this dispersoid has an incoherent structural relationship with respect to the matrix, FCC, in retarding the motion of dislocations that increase strength. Once the dislocation is blocked by the dispersoid, it tends to change the slip system by means of cross-slip. This cross-slip allows the deformation to maintain uniformly good ductility. Adding manganese to aluminum alloys not only enhances tensile strength but also significantly improves low-cycle fatigue resistance. Corrosion resistance is also measurable improved by the addition of manganese. After extrusion, the recrystallization is also retarded so that a very small grain size is maintained, contributing to an improvement in the mechanical properties. The addition of magnesium to aluminum increases strength through solid solution strengthening and improves their strain hardening ability. These alloys are the highest strength non-heat-treatable aluminum alloys and are, therefore, used extensively for structural applications. The alloys are produced mainly as sheet and plate and only occasionally as extrusions. The reason for this is that these alloys strain harden quickly and, are therefore difficult and expensive to extrude.

REFERENCES


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