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Iterative Techniques for Load Flow Study: A Comparative Study for Nigerian 330kv Grid System as a Case Study

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Abstract: The purpose of any load flow analysis is to compute precise steady-state voltages magnitudes and angles of all buses in the network, the real and reactive power flows into every line and transformer, under the assumption of known generation and load. Model of power system generates non-linear algebraic equations and to solve these equations, three solution algorithms based on power equations of the methods were adopted. They were applied to two test power systems: IEEE 5-bus, IEEE 30-bus systems and a real power system, Nigerian 330kV 28-bus. The result of the application shows that number of iterations increases proportionally with the number of buses for Gauss-Seidel technique while that of Newton –Raphson method remained almost practically constant even with varying number of buses. The Fast Decoupled method presented a slight increase in number of iterations with increasing number of buses but with faster convergence, when compared with Newton –Raphson methods. The Newton-Raphson method provided the best characteristics of quadratic convergence in minimum number of iterations and this method is best suited for Nigerian system.

Keyword: Load Flow, Gauss-Seidel, Newton Raphson, Fast Decoupled, Power Flow, Iteration.

I. INTRODUCTION

The power flow problem is solved to determine the steady-state complex voltages at all buses of the network, from which the active and reactive power flows in every transmission line and transformer are calculated [2]. The set of equations representing the power system are nonlinear [3]. Owing to the nonlinear nature of the power flow equations, the numerical solution is reached by iteration [5]. Literature is well documented on the application of Gauss Seidel and Newton Raphson iterative methods to the Nigerian electrical power system while there is little known publication on Fast decoupled method to the Nigerian electrical power system. This work considers a comparative analysis of the three methods when it is applied to the Nigerian power system.

Test cases were carried out on IEEE 5-bus, IEEE 30-bus and Nigerian 28 –bus 330KV electrical power system network. In the investigation, a power flow analysis was carried out on the tested electrical power system network using the three methods of iterative techniques.

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The steps involved in carrying out the study o are as follows: Source for data of the Nigeria's 330kV transmission power system network, apply the power flow programs for Gauss-Seidel, Newton-Raphson and Fast decouple algorithms and performance evaluation of the three methods when applied to the Nigerian 330kV transmission network system.

II. LOAD FLOW SOLUTION TECNIQUES

The load flow analysis, in power system parlance, is the steady state solution of the power system network. The power flow problem involves determining voltages and line flows, in a large sparse electrical network, for a given load and generation schedule [10]. In this analysis, the power system network is modeled as an electric network and solved for the steady state power, voltages at various buses and hence the power at the slack bus and power flows through inter connecting power channels [7]. Four quantities which are associated with each bus are voltage magnitude $|v|$, phase angle δ, real power P, and reactive power Q. The system buses are generally classified into three types namely: Slack bus, Load buses and Generator buses where two variables are specified and others two to be determined [8].

A. Gauss-Seidel Method

The Gauss-Seidel method is an iterative algorithm for solving a set of non-linear algebraic equations. It was one of the methods used in load flow studies. Here a solution vector is assumed and one of the equations is used to obtain the revised value of a particular variable. The solution vector is immediately updated in respect of this variable. The process is then repeated for all the variables thereby completing one iteration. The iterative process is then repeated till the solution vector converges within prescribed accuracy. The convergence is quite sensitive to the starting values assumed [6]. The advantages of the method are the simplicity of the technique, small computer memory requirement, less computational time per iteration. However the disadvantages are slow rate of convergence, large numbers of iterations, increase of numbers of iteration directly with the increase in the number of buses and effect of convergence due to choice of slack bus. In view of these disadvantages, Gauss-Seidel method is used only for system having small number of buses [4].

B. Newton-Raphson Method

The origin of the formulation of the power flow problems and the solution based on Newton-Raphson's technique dates back to the late 1960s [(3]. It is an iterative method which approximates a set of non-linear simultaneous equations to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to the first approximation [12]. Many advantages are attributed to the Newton-Raphson(NR) approach. Its convergence characteristics are relatively more powerful compared to other alternative processes and the reliability of Newton-Raphson approach is comparatively good, since it can solve cases that lead to divergence with other popular processes.Failures do occur on some ill-conditional problems.[7].

C. Fast-Decoupled Method

It was demonstrated in the late 1970s that the storage and computing requirements of the Newton-Raphson method could be reduced very significantly by introducing a series of well-sustained simplifying assumptions. These assumptions are based on physical properties exhibited by electrical power system, in particular in high-voltage transmission system. The resulting formulation is no longer a Newton-Raphson method but a derived formulation described as Fast-decouple Newton- Raphson. The power mismatch equations of both methods are identical but their Jacobians are quite different; the Jacobians elements of the Newton –Raphson are voltage –dependent whereas those of the Fast decouple Newton- Raphson method are voltage-independent (i.e. constant parameters). Moreover, the number of Jacobian entries used in the Fast-decouple Newton- Raphson method is only half of those used in the Newton-Raphson method but has strong convergence characteristics. However, an asset of the Fast decouple Newton- Raphson method is the fact that one of its iterations takes only a fraction of the time required by Newton-Raphson's method of iterations [1]. The decoupled equation requires considerable less time to solve compared to the time required for the solution by the Newton- Raphson Method.

D. Power Flow Equation

Consider a typical bus of a power system network as shown in Figure 1. Transmission lines are represented by their equivalent π models where impedances have been converted to per – unit admittances on a common MVA base.

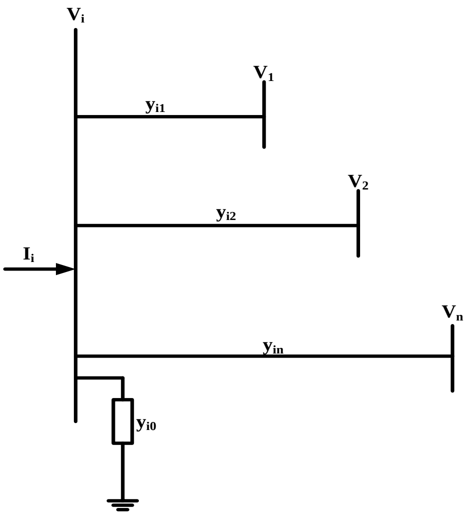

Figure 1: Typical bus of the power system [8]]

Application of Kirchhoff's current Law (KCL) to this bus results in

$$I_i = y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots\ y_{in}(V_i - V_n)$$

$$= (y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n \quad \ldots\ldots\ (1)$$

Or

$$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i \quad \ldots\ldots\ (2)$$

The apparent power at bus í is $S_i = P_i + jQ_i = V_i I_i^*$

$\ldots\ldots\ (3)$

Or

$$I_i = \frac{P_i - jQ_i}{V_i^*} \qquad \ldots\ldots\ (4)$$

Using equation (3) in equation (2) gives

$$I_i = \frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i \quad \ldots\ldots\ (5)$$

From equation (5), the mathematical formulation of the power flow problem results in a system of algebraic non-linear equations which must be solved by iterative techniques.

E. Gauss – Seidel power flow solution

In the power flow study, it is necessary to solve the set of non – linear equations represented by equation (5). In the Gauss – Seidel method equation (5) is solved for V_i and the iterative sequence becomes,

$$V^{(k+1)} = \frac{\dfrac{P^{sch} - jQ_i^{sch}}{V_i^{*(k)}} + \sum y_{ij}V_j^{(k)}}{\sum y_{ij}} \quad j \neq i \qquad \ldots\ldots\ldots\ (6)$$

In writing the KCL, current entering bus i was assumed positive. Thus, for buses where real and reactive powers are injected into the bus, such as generator buses, P^{sch} and Q^{sch} have positive values. For load buses where real and reactive powers are flowing away from the bus, P^{sch} and Q^{sch} have negative values. If equation (5) is solved for P_i and Q_i, we have

$$P_i^{(k+1)} = \mathrm{Real}\left[V_i^{*(k)}\left\{V_i^{(k)}\sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_i^{(k)}\right\}\right] \quad j \neq i \,.. \qquad (7)$$

$$Q_i^{(k+1)} = \mathrm{Imaginary}\left[V_i^{*(k)}\left\{V_i^{(k)}\sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_i^{(k)}\right\}\right] \quad j \neq i \quad \ldots(8)$$

The bus admittance matrix Y_{bus} is an important network description of the interconnected power system and the power flow equation is usually expressed in terms of the elements of the bus admittance matrix, Y_{bus}. Since the off-diagonal elements of the bus admittance matrix Y_{bus} are $Y_{ij} = -y_{ij}$ and the diagonal elements are $Y_{ii} = \sum yij$ then (6) above becomes,

$$V_i^{(k+1} = \frac{\frac{Pi^{sch} - jQi^{sch}}{V_i^{*(k)}} - \sum_{j=i}^{n} y_{ij} V_j^{(k)}}{Y_{ii}} \quad \ldots(9a)$$

$$Y_{ij} = G_{ij} - B_{ij} \quad \ldots (9b)$$

And,

$$P_i^{(k+!)} = \mathrm{Real}\left(V_i^{*(k)}\left\{V_i^{(k)}Y_{ii} + \sum_{i=1, j\neq i}^{n} y_{ij}V_j^{(k)}\right\}\right) \; j \neq i \quad \ldots.. \; (10)$$

$$Q_i^{(k+1)} = \mathrm{Im}aginary\left(V_i^{*(k)}\left\{V_i^{(k)}Yii + \sum_{i=1, j\neq i}^{n} y_{ij}V_j^{(k)}\right\}\right) \; j \neq i \, (11)$$

Y_{ii} includes the admittance to ground of line charging susceptance and any other fixed admittance to ground.

F. Newton –Raphson method

The load flow problem can also be solved by using Newton Raphson method. For the typical bus of the power system shown in Figure (1) the current entering bus i written in terms of the bus admittance matrix as.

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \quad \ldots\ldots. \quad (12)$$

In the above equation, j includes bus j. Expressing this equation in polar form, we have:

$$I_i = \sum_{j=1}^{n} |Y_{ij}||V_i| < (\theta_{ij} + \delta_j) \quad \ldots\ldots\ldots \; (13)$$

The complex power at bus i is

$$P_i - jQ_i = V_i^* I_i \quad \ldots\ldots. \; (14)$$

Substituting from (13) for I_i in (14)

$$P_i - jQ_i = |V_i| < -\delta_i \sum_{j=1}^{n} |Y_{ij}||V_j| < \delta_{ij} + \delta_j \quad \ldots.. \; (15)$$

Separating the real and imaginary parts

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad \ldots\ldots\ldots(16a)$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_i - \delta_i + \delta_j) \ldots\ldots\ldots(16b)$$

Equation (16a) and (16b) constitute a set of non – linear algebraic equations in terms of the independent variables, voltage magnitude in per – unit and phase angle in radians. We have two equations for each load bus given by (16a) and (16b), and one equation for each voltage controlled bus, given by (16a). Expanding (16a) and (16b) in Taylor's series about the initial estimate and neglecting all higher order terms results in a set of linear equations. These equations after linearization can be written in matrix form as.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \quad \ldots\ldots.. \; (17)$$

Where element J_1, J_2, J_3, J_4 are elements of Jacobian matrix. In obtaining the power flow solution by Newton-Raphson method, consider equation (17) $\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$

For voltage controlled buses, the voltage magnitudes are known. Therefore, if m buses of the system are voltage controlled, m equations involving ΔQ and ΔV and the corresponding column of the Jacobian matrix are eliminated. Accordingly, there are n-1 power constraints and n-1-m reactive power constraints, and the Jacobian matrix is of order $(2n-2-m)\times(2n-2-m)$. J_1 is of the order $(n-1)\times(n-1)$,J_2 is of the order $(n-1)\times(n-1-m)$, J_3 is of the order $(n-1-m)\times(n-1)$, and J_4 is of the order $(n-1-m)\times(n-1-m)$, where n is load bus and m is generator bus.

The diagonal and off diagonal element of J_1 are:

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{j\neq 1} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \quad \ldots\ldots \quad (18)$$

$$\frac{\partial P_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \; j \neq i \ldots. \; (19)$$

The diagonal and off diagonal elements of J_2 are:

$$\frac{\partial P_i}{\partial |Vi|} = 2|V_i||Y_{ij}|\cos\theta ii + \sum_{j\neq 1} |V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \ldots\ldots \; (20)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \; j \neq i \quad \ldots \quad (21)$$

The diagonal and off diagonal elements of J_3 are:

$$\frac{\partial Q_i}{\partial \delta_j} = \sum_{j\neq 1} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \quad \ldots. \quad (22)$$

$$\frac{\partial Q_i}{\partial \delta_j} = -|V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \; j \neq 1 \ldots \; (23)$$

The diagonal and off diagonal element of J_4 are:

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Yii| \sin\theta_{ij} + \sum_{j\neq 1} |Vj||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \; (24)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \; j \neq 1 \quad \ldots\ldots. \; (25)$$

The terms $\Delta P_i^{(k)}$ and $\Delta Q_i^{(k)}$ are the differences between the scheduled and calculated values, known as the power residuals given by:

$$\Delta P_i^{(k)} = P^{sch} - P_i^{(k)} \quad \ldots\ldots\ldots \ (26)$$

$$\Delta Q_i^{(k)} = Q^{sch} - Q_i^{(k)} \quad \ldots\ldots\ \ (27)$$

The new estimates for bus voltages are:

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)} \quad \ldots\ldots\ldots\ldots \ (28)$$

$$\left|V_i^{(k+1)}\right| = \left|V_i^k\right| + \Delta\left|V_i^k\right| \quad \ldots\ldots\ldots \ (29)$$

G. Fast decouple Newton- Raphson power flow solution

This is an extension of Newton – Raphson method formulated in polar coordinates with certain approximation which results in fast algorithm for load flow solution. Because power system transmission lines have a very high X/R ratio thus it is reasonably assumed that real power changes (ΔP) are less sensitive to changes in voltage magnitude and are mainly sensitive to changes in phase angle (Δδ). Similarly, the reactive power is less sensitive to changes in phase angle Δδ but mainly sensitive to changes in voltage magnitude. With these assumptions, equation (17) reduces to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \quad \ldots\ldots\ \ (30)$$

or $$\Delta P = J_1 \Delta\delta = \frac{|\partial P|}{\partial\delta} \Delta\delta \quad \ldots\ldots\ldots\ldots \ (31)$$

$$\Delta Q = J_4 \Delta|V| = \frac{|\partial Q|}{\partial|V|} \Delta|V| \quad \ldots\ldots\ldots\ldots \ (32)$$

Equations (32) and (33) show that the matrix equation is separated into two decoupled equations requiring considerably less time to solve compared to the time required for the solution of (18). Furthermore, considerable simplification can be made to eliminate the need for re-computing $J_1$and J_4 during each iteration. This procedure however, results in Fast Decoupled power flow equations. (Sadat 2010).

III. RESULTS AND DISCUSSION

Computing time requirement

From Table 1 it shows that the time per iteration in both Gauss-Seidel and Newton-Raphson methods increases almost directly as the number of buses of the system while the computational time per iteration of the Fast Decoupled is less than the Newton-Raphson method. Shown in Figures (2) is the bar chart showing the three algorithms with the computing time.

Table 1**:** Computing time (Seconds) requirement for the Algorithms

Test Systems	Gauss-Seidel	Newton- Raphson	Fast Decoupled
IEEE 5 Bus	0.0156002	0.0312002	0.0156001
IEEE 30 Bus	0.0312002	0.0468003	0.0124801
Nigerian 28 Bus	0.0780005	0.0936006	0.0780000

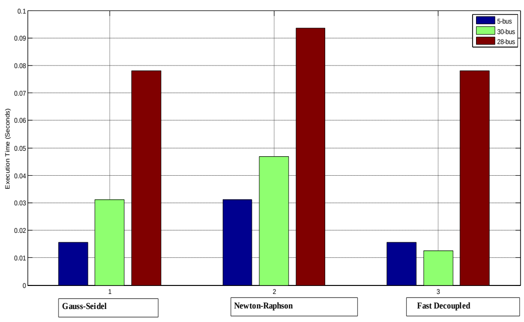

Figure 2: Comparison of the Execution Time obtained with the 3- Test Bus Systems.

As seen from Table 2 the Gauss-Seidel method needs a larger number of iterations to converge to a given power mismatch tolerance, compared to the other two methods. The Fast decouple method needs more iterations to converge than the Newton Raphson. But as indicated in the last discussion, the computing time requirement for Fast decouple method per iteration is much less than the Newton Raphson method. Shown in Figure (3) is the bar chart showing the three algorithms with the necessary number of iterations.

Iteration Requirements

Table .2: Iteration Number

Test Systems	Gauss-Seidel	Newton Raphson	Fast Decoupled
IEEE 5 bus	12	3	6
IEEE 30 bus	34	4	15
Nigerian 28 bus	69	4	5

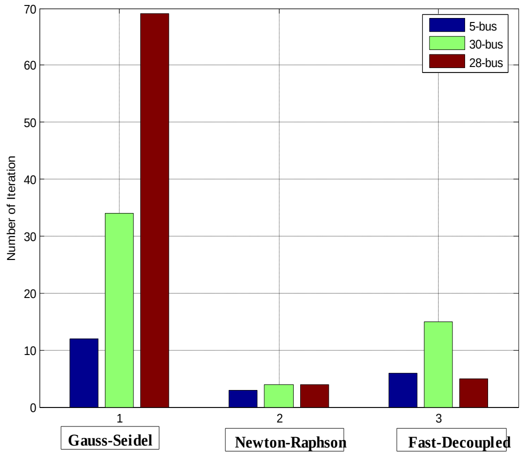

Figure 3: Comparison of the number of Iteration obtained with the 3- Test Bus Systems

Convergence Characteristics

The convergence characteristic of an algorithm is best described by plotting the maximum power mismatch at each iteration versus the number of iterations. Figure (3) through Figure (5) shows the maximum power mismatches of the three test systems as a function of the number of iterations for the three algorithms. From the figures the mismatches monotonously decrease for all cases under full load conditions and the Newton Raphson methods exhibit fastest convergence rate. These curves clearly show the unique characteristics of the three algorithms.

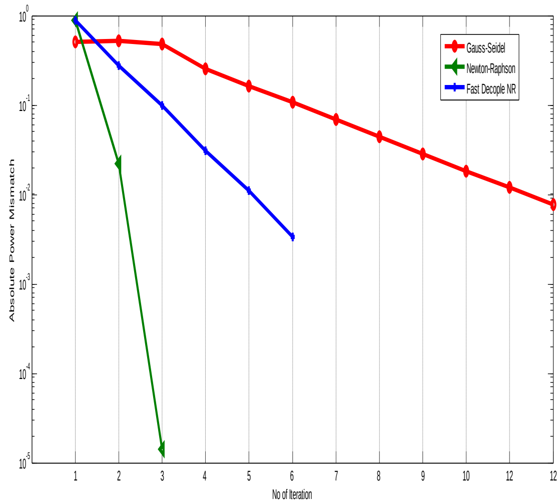

Figure 4. Absolute Power mismatches as function of number of Iteration for IEEE5-bus system

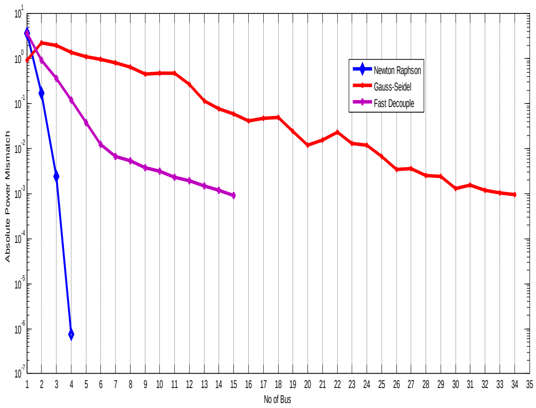

Figure 5: Absolute Power mismatches as function of number of Iteration for IEEE 30-bus system

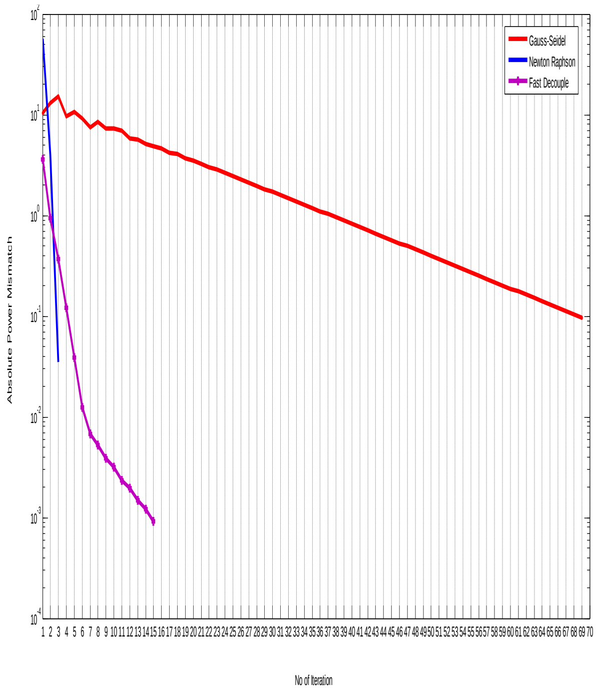

Figure 6: Absolute Power mismatches as a function of number of Iterations for 28-bus system

IV. CONCLUSION

The well known properties of these algorithms: speed, rate of convergence and the convergence characteristics were confirmed by the case study results. It indicates that Newton Raphson method is more reliable because it converges faster with quadratic convergence characteristics and least number of iterations when compared with the other two methods, In general the Newton Raphson algorithm takes the least number of iteration to converge despite its longer computing time. The number of iteration for the Gauss-Seidel increases directly as the number of the buses of the network, whereas the number of iterations for the Newton Raphson method remains practically constant, independent of the system size. However, since the convergence characteristics of the Fast decouple method is geometric compare to the quadratic convergence of the Newton Raphson, thus it has more number of iteration. Therefore because of high accuracies obtained in only a few iterations, the Newton Raphson method is important for use and more reliable than any of the methods.

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