

# Prediction of Behaviour of Steel Plate Subjected to Shear

Muhammad Aslam Bhutto, Abdul Aziz Ansari, Noor Ahmed Memon

**Abstract**— Steel plate-girders are generally subjected to high shear and low bending moment. The flanges primarily resist the applied moment, while the web primarily resists the shear. A thin plate in shear is a simple representation of the dominant loading case in a slender web panel of the plate-girder and is a combination of the principal tensile and compressive in-plane stresses. The elastic and plastic behaviour of a simply supported steel plate can be predicted using the existing design theories. This paper presents the details and results of finite element analyses (FEA) carried out for a thin square steel plate subjected to pure shear. The objective is to predict the linear elastic and nonlinear plastic behaviour of the plate using the FE analyses and compare the results of the analyses to the theoretical predictions for validation. The FEA results for the elastic critical load and the ultimate plastic load of the plate were in very good agreement with the theoretical predictions. The FE analyses also predicted correctly the elastic buckling and the plastic failure modes of the plate.

**Index Terms**— Finite element analyses, pure shear, shear buckling, steel plate, validation.

## I. INTRODUCTION

The steel plate-girders are generally subjected to high shear and low bending moment. The flanges primarily resist the applied moment, while the web primarily resists the shear. In the plate-girders, shear buckling of the thin web panels occurs when the applied shear approaches the critical shear stress of the panel. After buckling, the additional load is carried by a tensile membrane stress field and the flanges. The failure occurs when the web yields across the tensile stress field and plastic hinges develop in the top and bottom flanges [1].

Because the slender web of the plate-girder undergoes the shear buckling at an early stage of loading, the webs are strengthened with transverse or longitudinal stiffeners to increase their shear buckling strength [2-3].

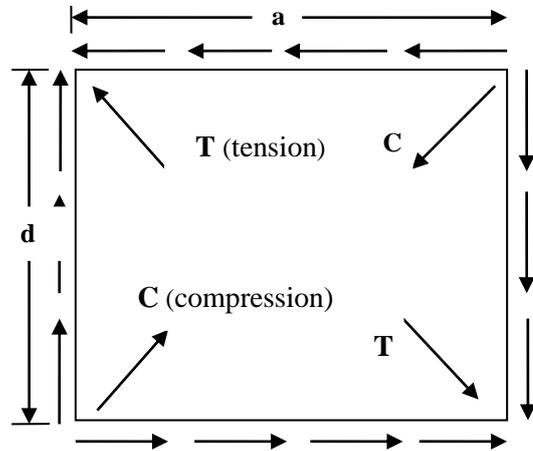


Figure 1 Principal tensile and compressive stresses in thin plate subjected to shear [4]

A thin plate in shear is a simple representation of the dominant loading case in a slender web panel of the plate-girder and can be regarded as a combination of the principal tensile and compressive in-plane stresses as shown in Fig 1. The compressive component eventually causes buckling while the tensile component tends to restrain buckling [4]. With the development of high-powered computers, together with state-of-art FE (finite element) software and user-friendly graphic interface, the FE analyses has become a popular choice to predict the behaviour of a structure or its part subjected to the different loading and boundary conditions [5].

## II. PRESENT STUDY

The present study consists of theoretical and finite element analyses (FEA) of a simply supported thin steel plate subjected to pure shear.

### A. Objective

Elastic and Elasto-plastic FE analyses of the simply supported thin steel plate were carried out to compare the results with the theoretical predictions for validation. The parameters compared are the elastic critical load, elastic maximum vertical displacement and ultimate plastic load of the plate.

### B. Model geometry, loading and boundary conditions

Fig 2 shows the dimensions, loading and boundary conditions of the plate. A uniformly distributed load (UDL), as shown in Fig 2, was applied along each edge of the plate in such a way that the plate was subjected to pure shear.

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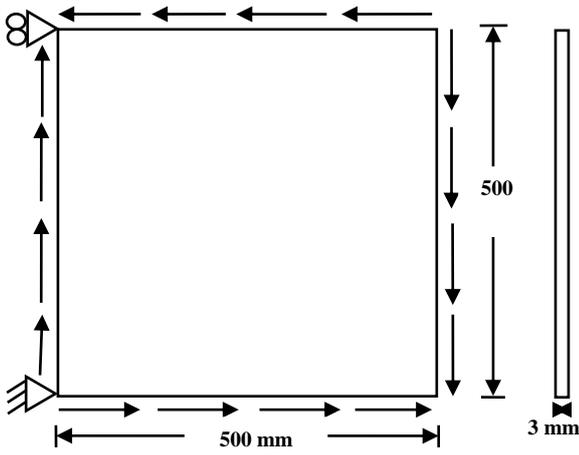


Figure 2 Square steel plate 500 x 500 x 3 mm

TABLE 1  
PROPERTIES OF STEEL

Property	Value
Modulus of elasticity (GPa)	205
Yield strength (MPa)	300
Shear modulus (GPa)	79
Poisson's Ratio	0.3

The self-weight of the plate was very small, less than 2%, compared to the applied load and was ignored in the theoretical predictions and FE analyses.

The steel plate was supported at both corners on the left edge as shown in Fig 2. The support at the bottom corner restrained it horizontally and vertically, but was free to rotate. The support at the top corner restrained the plate horizontally, but was free to move vertically and rotate. All four edges of the plate were restrained in the lateral direction.

### C. Material properties

The properties of the steel used in analyses of the plate are given in Table 1.

## III. THEORETICAL ANALYSES

### A. Theoretical predictions

The elastic shear buckling stress ( $\tau_{cr}$ ) of the simply supported steel plate is given by equation (1) [6]. The elastic critical load was obtained as the product of shear buckling stress and the area of cross section of the plate. The maximum elastic vertical displacement of the plate was determined at a shear load of 80 kN by using equation (2), while the ultimate plastic load ( $V_p$ ) of plate was calculated using equation (3) [7].

$$\tau_{cr} = \frac{\tau^2 KE}{12(1-\theta^2)} \frac{t^2}{a^2} \quad (1)$$

$$\delta_{s \max} = FL / AG \quad (2)$$

$$V_p = \sigma_y dt / \sqrt{3} \quad (3)$$

### B. Buckling mode

The simply supported thin plate subjected to pure shear tends to buckle out of plane along tensile diagonal and perpendicular to the compression diagonal [7]. The possible mode of an out of plane diagonal buckle to develop in the plate has accordingly been predicted and is shown in Fig 3.

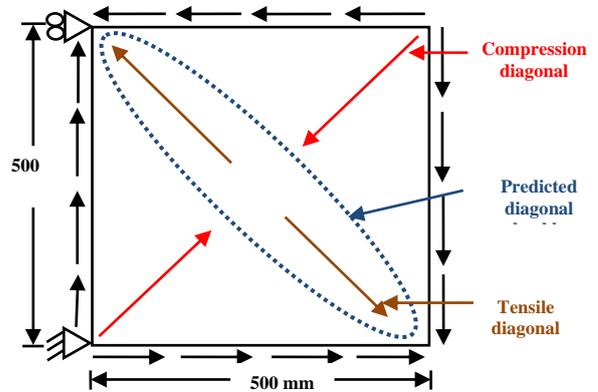


Figure 3 Predicted mode of out-of-plane diagonal buckling of plate

### C. Failure mode

The simply supported plate subjected to shear shall fail by yielding in the shear. The possible mode of the failure of the plate has accordingly been predicted and is shown in Fig 4.

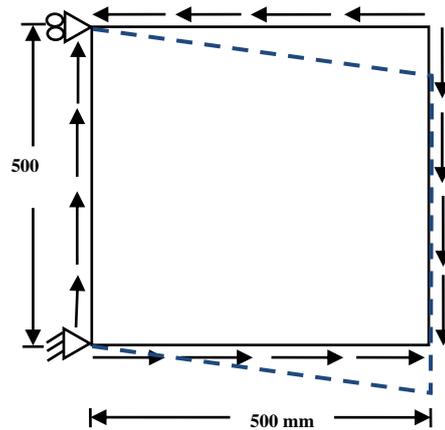


Figure 4 Predicted failure mode of plate

## IV. FINITE ELEMENT ANALYSES

### A. Analyses

Following three FE analyses were carried out for the simply supported steel plate using the LUSAS FE program, Version 14.3 [8].

- Linear elastic analyses at an applied shear load of 80 kN to predict the maximum vertical displacement at the unsupported bottom corner of the plate.
- Eigenvalue analyses to determine the elastic critical load.
- Nonlinear analyses to obtain the ultimate plastic load. Material nonlinearity was only modelled.

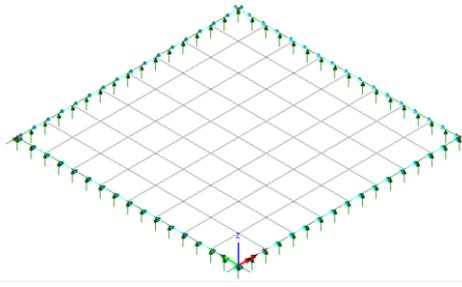


Figure 5 FE model of simply supported steel plate

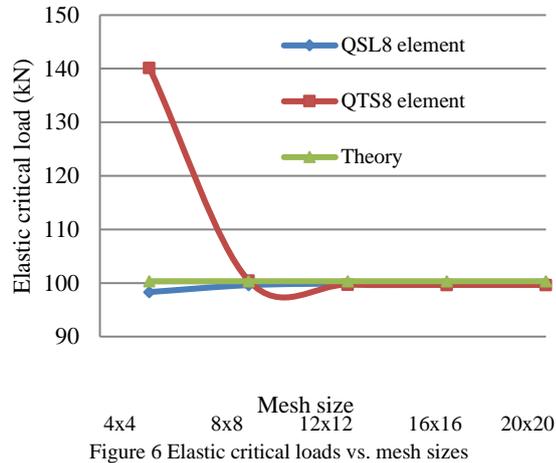


Figure 6 Elastic critical loads vs. mesh sizes

**B. Material modeling, loading and boundary conditions**

The steel was modeled as an isotropic material; the properties are given in Table 1. Loading and boundary conditions of the plate are shown in Fig 5.

**C. Element type**

The 8-node semiloof curved thin shell element QSL8 and 8-node thick shell element QTS8 were used.

**D. Mesh convergence**

Five uniform mesh sizes 4x4, 8x8, 12x12, 16x16 and 20x20 were used to check the convergence of the results of the eigenvalue FE analyses. Fig 6 shows the elastic critical load plotted versus each of the five mesh sizes. The results for the elastic critical load using the QSL8 element converged for a mesh of 8x8 and those using the QTS8 element converged for the mesh of 12x12.

**V. RESULTS OF THEORETICAL PREDICTIONS AND FINITE ELEMENT ANALYSES**

Details of the calculations for theoretical predictions are given in Appendix 1.

**A. Elastic critical and ultimate loads**

Table 2 shows the results of the linear and nonlinear FE analyses compared to the theoretical predictions.

TABLE 2

COMPARISON OF THEORETICAL PREDICTIONS AND RESULTS OF FINITE ELEMENT ANALYSIS

Type of analysis	Theory	Element type (mesh size)	
		QSL8 (8x8)	QTS8 (12x12)
Linear elastic analyses			
Maximum vertical displacement (mm) at a load of 80 kN	0.338	0.338	0.338
Theory/ FEA	---	1.0	1.0
Eigenvalue analyses			
Elastic critical load (kN)	93.46	92.8	92.8
Theory/ FEA	---	1.01	1.01
Nonlinear analyses			
Ultimate plastic load (kN)	259.8	259.8	259.8
Theory/ FEA	---	1.0	1.0

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Nonlinear analyses			
Ultimate plastic load (kN)	259.8	259.8	259.8
Theory/ FEA	---	1.0	1.0

**B. Maximum vertical displacement**

Fig 7 shows the distribution of linear vertical displacements of steel plate obtained from the linear FE analyses using QSL8 element with a 8x8 mesh.

**C. Buckling mode**

Fig 8 shows the first buckling mode of steel plate obtained from the linear FE analyses using QSL8 element with a mesh of 8x8.

**D. Failure mode**

Fig 9 shows mode of the failure of steel plate obtained from the nonlinear FE analyses using QSL8 element with a mesh of 8x8.

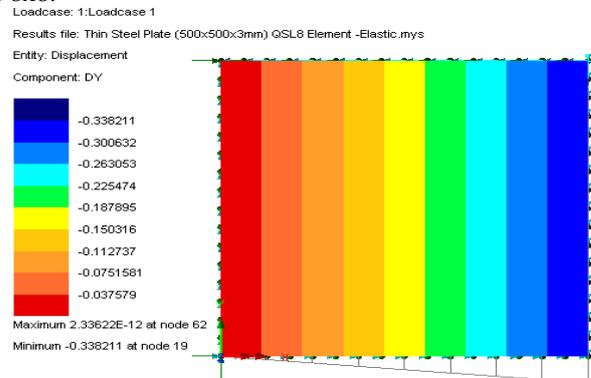


Figure 7 Linear vertical displacements of steel plate (QSL8 element with 8x8 mesh)

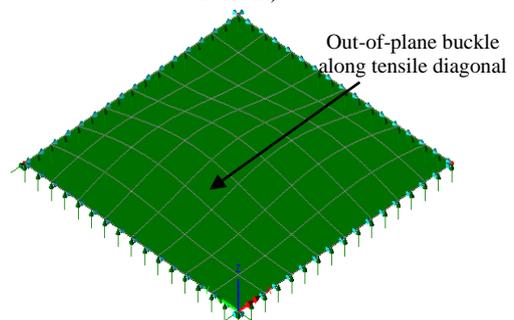


Figure 8 Buckling mode of steel plate (QSL8 element with 8x8 mesh)

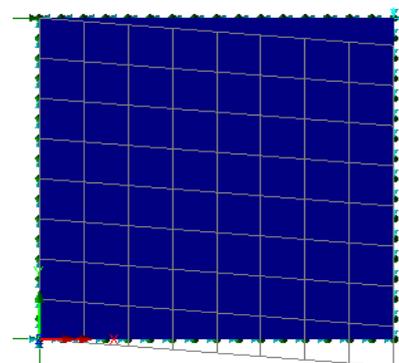


Figure 9 Failure mode of steel plate (QSL8 element with 8x8 mesh)

VI. DISCUSSION OF RESULTS

1. At an applied shear load of 80 kN, the theoretical prediction for the maximum elastic vertical displacement at the un-supported bottom corner of the steel plate was in very good agreement with that obtained from linear FE analyses using each of the QSL8 and QTS8 elements.
2. The theoretical elastic critical load of the plate was also in very good agreement with the load at the first buckling mode obtained from eigenvalue FE analyses using either of QSL8 and QTS8 elements.
3. The first buckling mode of the plate predicted by eigenvalue FE analysis as shown in Fig 8 was in agreement with that of the theoretical prediction shown in Fig 3.
4. The theory and the FEA ultimate plastic loads of the plate using the QSL8 and QTS8 elements were in good agreement.
5. The failure mode of the plate predicted by nonlinear FE analysis as shown in Fig 9 was in good agreement with that of theoretical prediction shown in Fig 4.
6. At the ultimate plastic load of the plate, the shear stresses were uniform over whole area of the plate surface and were equal to the yield stress in shear,  $\tau_y = \sigma_y/\sqrt{3} = 0.1732$  GPa, of the steel.

VII. CONCLUSIONS

The conclusions of the work reported in this paper can be summarized as under:

1. The thin shell element QSL8 with an 8x8 mesh and the thick shell element QTS8 with a 12x12 mesh were found to give satisfactory results for the steel plate model subjected to pure shear.
2. A comparison of the results shows that the elastic critical load and ultimate plastic load obtained from the FE analyses were in very good agreement with those of the theoretical predictions.
3. The FE analyses also predicted correctly the elastic buckling as well as the plastic failure modes of the plate.

VIII. SYMBOLS AND ABBREVIATIONS

- a, L = length of plate (mm)
- d = depth of web/ plate (mm)
- E = Young's modulus of elasticity (GPa)
- G = shear modulus (GPa)
- K = shear buckling co-efficient
- t = thickness of web/ plate (mm)
- $\delta_{s \max}$  = maximum vertical displacement due to shear (mm)
- $\sigma_y$  = yield strength of steel (MPa)
- $\tau_{cr}$  = elastic buckling shear stress (GPa)
- $\tau_y$  = yield strength of steel in shear (GPa) =  $\sigma_y/\sqrt{3}$
- $\vartheta$  = Poisson's ratio
- $V_p$  = ultimate plastic load (kN)
- $V_{cr}$  = elastic critical load (kN)

APPENDIX 1

CALCULATIONS FOR THEORETICAL PREDICTIONS OF SIMPLY SUPPORTED STEEL PLATE

Formula	Result
Linear elastic analyses	
Applied shear load, F (kN)	80
Area of cross-section of plate, A = td	1500
$\delta_{s \max} = FL/ AG$ (mm)	0.338
Shear buckling constant 'K' (for an square plate with simply supported boundary conditions)	9.34
$\tau_{cr} = \frac{\pi^2 KE}{12(1 - \vartheta^2)} \frac{t^2}{d^2}$	0.06231
Elastic critical load, $V_{cr} = \tau_{cr} A$	93.465
Nonlinear plastic analyses	
Yield stress in shear, $\tau_y = \sigma_y/\sqrt{3}$ (GPa)	0.1732
Ultimate plastic load, $V_p = \tau_y A$ (kN)	259.8

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