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Agreement Results of Non-Associative Rings with Cyclic Property

C. Jaya Subba Reddy, T. Mahesh Kumar, L. Sreenivasulu Reddy

Abstract: ***This paper describes results on a non-associative ring R with the cyclic property: $x(yz) = y(zx) = z(xy)$ for all x, y, z in non-associative ring . Some of the results proved by many researchers like Klienfeld and Novikov etc., by using their own identities and/or conditions on non-associative rings.***

Key words: Non-associative ring, Cyclic Property .

I. INTRODUCTION

Schafer, Richard D how they were defined the Cyclic non associative ring is adopted their assumption [4] and in addition to consider the assumptions of [5]. Mainly, their papers shows when non associative ring R is a field or a skew field. Their work show me the way to derive some special results on their defined structures with some properties . Throughout this paper non –associative ring with cyclic property like in [2] instead of identities of Klienfeld and Novikov [1,3] etc . This paper is organized as follows, section 1: Introduction, section 2: our contribution.

II. OUR CONTRIBUTION:

Result 1: A left alternative ring R is flexible ring if elements of R satisfies the cyclic property.

Proof: Since R be a left alternative ring, So $(x,x,y) = 0$ $\forall$ $x,y \varepsilon$ R and Since R satisfies the cyclic property, so $x(yz) = y(zx) = z(xy)$, $\forall$ $x,y,z \varepsilon$ R .

Thus from above two equations , we have $(x,y,x) = 0$. Hence R is flexible ring.e

Result 2: A right alternative ring R is flexible ring if elements of R satisfies the cyclic property.

Proof: Since R be a right alternative ring , So $(x,y,y) = 0$ $\forall$ $x,y \varepsilon$ R and Since R satisfies the cyclic property, so $x(yz) = z(xy) = y(zx)$, $\forall$ $x,y,z \varepsilon$ R

Thus from above two equations , we $(y,x,y) = 0$ $\forall$ $x,y \varepsilon$ R. Hence R is flexible ring.

Result 3: Let R be an non- associative ring with cyclic property. Then R left alternative ring if and only if R is right alternative ring.

Proof:

Let R be an left alternative ring , So $(x,x,y) = 0$ $\forall$ $x,y \varepsilon$ R . From above equations , $(x,x,y) = 0 \Rightarrow x(xy) - (xx)y = 0 \Rightarrow y(xx) - (yx)x = 0 \Rightarrow (y,x,x) = 0$ $\forall$ $x,y \varepsilon$ R. Hence R is a right alternative ring.

Conversely , Let R be a right alternative ring , So $(x,y,y) = 0$ $\forall$ $x,y \varepsilon$ R . From above equations, $x(yy) - (xy)y = 0$ $\forall$ $x,y \varepsilon$ R$\Rightarrow$ $y(yx) - y(yx) = 0 \Rightarrow (y,y,x) = 0$ $\forall$ $x,y \varepsilon$ R. Hence R is a left alternative ring.

Result 4:

In every non – associative ring R with cyclic property, $(xy,z)y + x(yz,y) = 0$, $\forall$ $x,y,z \varepsilon$ R

Proof: From Left hand side expression, $(xy,z)y + x(yz,y) = ((xy)z - z(xy))y + x((yz)y - y(yz)) = (xy)zy - z(xy)y + x(yz)y - xy(yz) = (xy)zy - x(yz)y + x(yz)y - xy(yz)$ ($\because$ by cyclic property) $= 0$.

Result 5: In every non- associative ring with cyclic property, $(xy,z)y = x(y,zy)$, $\forall$ $x,y,z \varepsilon$ R.

Proof: $x(y,zy) = x(y(zy) - (zy)y) = xy(zy) - x(zy)y = xy(zy) - x(yz)y = xy(zy) - z(xy)y = ((xy)z - z(xy))y = (xy,z)y$.

Result 6: Let T = {t $\in$ R/ Rt = 0}. Then T is an ideal of R and T = 0 , where R be a non associative ring with cyclic identity.

Proof: Let t $\in$ T . For every $x \in$ R, $xt = 0$, So, $(xt)t = 0.t$ $\Rightarrow$ $(xt)t = 0$ $\Rightarrow$ $xt \in$ T, $\forall x \in$ R ,$\forall t \in$T$\Rightarrow$ RT $\subseteq$ T.

Since R satisfies the cyclic property, So $(y,t,x) = (x,y,t)$. But $(y,t,x) = y(tx) - (yt)x = y(tx) - 0.x$ ($\because$ Rt = 0) $= y(tx)$ i.e., $(y,t,x) = y(tx)$

Also $(x,y,t) = x(yt) - (xy)t = x.0 - 0$ ($\because$ Rt = 0) $= 0$ i.e., $(x,y,t) = 0$

Thus from above equations , we have $y(tx) = 0 \Rightarrow tx \in$ T, $\forall t \in$ T, $\forall x,y \in$ R

$\Rightarrow$ T R $\subseteq$ T .Thus T is an ideal in R . By hypothesis, T $\subseteq$ R $\Rightarrow$ T.T

$\Rightarrow$ $T^2 \subseteq 0$ ($\because$ by hypothesis) $\Rightarrow T^2 = 0 \Rightarrow$ T = 0 .Hence T be an ideal and T = 0.

Result 7: If R satisfies the cyclic property, (N, R, R) = 0 $\Leftrightarrow$ (R, N, R) = 0 $\Leftrightarrow$ (R, R, N) = 0.

Proof: Let n $\in$ N, $\forall x, y, z \in$ R .

(N, R, R) = 0 $\Leftrightarrow (n, x, y) = 0$, $\forall$ n $\in$ N, $\forall x, y, z \in$ R $\Leftrightarrow n(xy) - (nx)y = 0$

$\Leftrightarrow y(nx)(yn)x = 0 \Leftrightarrow (y, n, x) = 0$ $\forall$ n $\in$ N, $\forall x, y, z \in$ R $\Leftrightarrow$ (R, N, R) = 0

and (R, N, R) = 0 $\Leftrightarrow (n, x, y) = 0$, $\forall$ n $\in$ N, $\forall x, y, z \in$ R $\Leftrightarrow n(xy) - (nx)y = 0$

$\Leftrightarrow y(nx) - (yn)x = 0 \Leftrightarrow (y, x, n) = 0$ $\forall$ n $\in$ N, $\forall x, y, z \in$ R $\Leftrightarrow$ (R, R, N) = 0 Thus , we have (N, R, R) = 0 $\Leftrightarrow$ (R, N, R) = 0 $\Leftrightarrow$ (R, R, N) = 0.

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* Correspondence Author (s)

C. Jaya Subba Reddy, Deperatment of Mathematics, Sri Venkateswara University, Tirupati, A. P, India.

T. Mahesh Kumar, Deperatment of Mathematics, Sri Venkateswara University, Tirupati, A.P, India.

L. Sreenivasulu Reddy Deperatment of Mathematics, Sri Venkateswara University, Tirupati,A.P,India.

Result 8: Let C = { c $\in$N \ (C,R) = 0}. Where N is a nucleus and R is a non associative ring with cyclic property, then N = C.

Proof: Let $x \in$ N $\Leftrightarrow (x, n, y) = 0$, $\forall\, x, y \in$ R, $\forall$ n$\in$N ($\because$ (R,N, R) = 0) $\Leftrightarrow x(ny) - (xn)y = 0$
$\Leftrightarrow n(yx) - (yx)n = 0 \Leftrightarrow (n, R) = 0 \Leftrightarrow n \in$ C. Hence N = C.

Result 9: In a cyclic ring, Every left nucleus becomes a central nucleus and right nucleus.

Proof: **G**iven that (x,R,R)=0, $\forall$ x

Consider, (R,x,R)=(yx)Z – y(xz), $\forall$y, z$\in$R= (xz)y – x(zy) = (x, R, R), $\forall$x=0

again consider, (R, R, x)=(yz)x – y(zx)=(zx)y- x(yz)=(xy)z – x(yz)=(x, R, R), $\forall$ x=0

Hence , from above two , (x, R, R) = (R,x,R)=(R,R,x) = 0,$\forall$ x.

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