International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-2 Issue-6, August 2013

Analysis of Orthotropic Plate By Refined Plate Theory

S.B.Chikalthankar, I.I.Sayyad, V.M.Nandedkar

Abstract- In this paper a Trigonometric Shear Deformation Theory (TSDT) for the analysis of orthotropic plate, taking into account transverse shear deformation effect is presented. Present theory exactly satisfies stress boundary conditions on the top and bottom of the plate. In this displacement-based, trigonometric shear deformation theory, the in-plane displacement field uses sinusoidal function in terms of thickness coordinate to include the shear deformation effect. The theory obviates the need of shear correction factor like other higher order or equivalent shear deformation theories. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. Results obtained for static flexural analysis of simply supported thick orthotropic plates for uniformly distributed loading case is compared with those of other refined theories and exact solution from theory of elasticity.

Key words: Orthotropic thick plates, Shear deformation, trigonometric shear deformation theory.

I. INTRODUCTION

In current scenario composite structures are used in many engineering applications like aerospace, nuclear power plants, automotive, submarines, sport and health instruments etc. This is because of advent of composite materials and many composite materials exhibit high strength-to-weight and stiffness-to-weight ratios, which make them ideally suited for use in weight-sensitive structures. In order to analyze the composite thick plate various plate deformation theories developed.

Plate deformation theories can be divided in to two groups: stress based and displacement based theories. Since in the present study a new displacement based theory will be analyzed. Displacement based theories can be divided in to two parts: The classical plate theory and shear deformation plate theories. Transverse shear stress components are neglected in the classical plate theory where it is included in the shear deformation theories. The classical plate theory can be used only for thin plates since it gives improper results for thick plates especially made from advanced composites. Kirchhoff [1-7] developed classical plate theory (CPT) which is based on hypothesis that straight lines normal to the undeformed midplane remain straight and normal to the deformed midplane. Since the transverse shear deformation is neglected in Kirchhoff theory, it cannot be adequate for the analysis of shear flexible thick plates wherein shear deformation effects are more significant.

Manuscript published on 30 August 2013.
***** Correspondence Author (s)
Imran Sayyad, Department of Mechanical Engineering, Government college of Engineering, Aurangabad, Maharashtra, India.
Sanjay Chikalthankar, Department of Mechanical Engineering, Government college of Engineering, Aurangabad, Maharashtra, India.
V.M.Nandedkar, Department of Production Engineering, SGGS Institute of Eng-neering and Technology, Nanded, India.

The first order shear deformation theory (FSDT) is considered as improvement over the classical plate theory. This is achieved by including gross transverse shear deformation in the kinematic assumptions. Reissner [2, 3] was the first to provide a consistent stress based plate theory which incorporates the effect of shear deformation. Mindlin [4] has provided displacement based theory. In this theory, the transverse shear strain is assumed to be constant across the thickness, and thus shear correction factor to correct the strain energy of shear deformation is required. These factors are problem dependent.

The second order shear deformation theories by Naghdi [5], Pister and Westmann [6], Whitney and Sun [7], Nelson and Lorch [8] give marginally improved results over FSDT, but suffer from the same drawbacks of FSDT. The limitations of classical plate theory and first order shear deformation theory forced the development of higher order shear deformation theories to avoid the use of shear correction factors, to include correct cross sectional warping and to get the realistic variation of the transverse shear strain and stresses through the thickness of plate. The higher order theories based on series expansions are developed by Reissner [9], Provan and Koeller [10], Lo et al. [11, 12] and are modified by Levinson [13], Reddy [14] to get the parabolic shear stress distribution through the thickness of plate and to satisfy the shear stress free surface conditions on the top and bottom surfaces of the plate to avoid the need of shear correction factor. Sayyad and Ghugal[15,16] has carried out bending,buckling and free vibration analysis using exponential shear deformation theory.In the present work, emphasis has been laid on specific development of Trigonometric Shear Deformation Theories (TSDT) for plate analysis. The displacement models contain trigonometric terms in addition to classical plate theory terms. This displacement models is different from the generalized model of LO and Christensen [21-22] which contains only polynomial terms. Use of trigonometric functions to describe the plate behavior in thickness direction was first proposed by Stein [23-25] and was used for laminated beam and post buckling analysis of plates. However, the present TSDT model differs from that of Stein. In Stein's the top and bottom shear stress conditions are not satisfied. Results obtained for uniformly distributed loading case and are compared with those of refined theories and exact elasticity theory [29] available in the literature.

II. THEORETICAL FORMULATION

2.1 Plate under consideration

The plate under consideration occupies in $O - x - y - z$ Cartesian coordinate system the region:

$$0 \le x \le a, \quad 0 \le y \le b, \quad -h/2 \le z \le h/2, \tag{1}$$

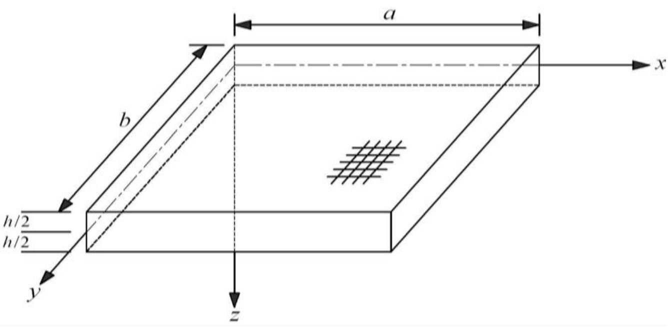

Fig.1. Geometry of thick plate.

Where *x, y, z* are Cartesian coordinates, *a* and *b* are the edge lengths in the *x* and *y* directions respectively, and *h* is the thickness of the plate. The plate is made up of homogeneous, linearly elastic orthotropic material with the principal material axes parallel to the *x* and *y* axes in the plane of plate. The plate material obeys generalized Hooke's law.

Assumptions made in the theoretical formulation:

1. The displacements are small and, therefore, strains involved are infinitesimal.
2. The inplane displacement *u* in *x* direction as well as displacement *v* in *y* direction each consists of following parts:
   a) Displacement component analogous to the displacement in classical plate theory of bending.
   b) Displacement component due to shear deformation, which is assumed to be hyperbolic sine in nature with respect to thickness coordinate, such that the maximum shear stress occurs at neutral axis.
3. The transverse displacement *w* in *z* direction is assumed to be a function of *x* and y coordinates only.
4. The plate can be subjected to transverse as well as inplane loads.

2.2 The Displacement Field

Based on the before mentioned assumptions, the displacement field of the present refined plate theory can be expressed as follows:

$$u = -z\frac{\partial w}{\partial x} + \frac{h}{\pi}\sin\frac{\pi z}{h}\phi(x,y),$$
$$v = -z\frac{\partial w}{\partial y} + \frac{h}{\pi}\sin\frac{\pi z}{h}\psi(x,y), \tag{2}$$
$$w = w(x,y),$$

Here *u* and *v* are the in plane displacement components in the *x* and *y* directions respectively, and *w* is the transverse displacement in the *z* direction. The trigonometric function in terms of thickness coordinate in both the displacements *u* and *v* is associated with the transverse shear stress distribution through the thickness of plate and the functions $\phi(x,y)$ and $\psi(x,y)$ are the unknown functions associated with the shear slopes.

2.3 Strain-Displacement Relationships

Normal Strain:

Normal and shear strains are obtained within the framework of linear theory of elasticity using the displacement field given by Eq. (.2). These relationships are given as follows:

$$\varepsilon_x = \frac{\partial u}{\partial x}, \ \varepsilon_y = \frac{\partial v}{\partial y}, \ \varepsilon_z = \frac{\partial w}{\partial z}, \tag{3}$$

Shear Strains:

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \ \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x},$$
$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \tag{4}$$

Stress-Strain Relationships:

For a linearly elastic orthotropic material, stresses τ_{xy}, σ_x and σ_y are related to strains $\gamma_{xy}, \varepsilon_x$ and ε_y shear stresses are related to shear strains by the following constitutive relations:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \tag{5}$$

where σ_x, σ_y and τ_{xy} are the in plane stresses and τ_{yz}, τ_{zx} are the transverse shear stresses. They $(\varepsilon_x, \varepsilon_y)$ are normal strain component and $(\gamma_{xy}, \gamma_{yz}, \gamma_{zx})$ are the shear strain components. And stiffnes coefficients are

$$Q_{11} = [E_1/1-(\mu_{12}.\mu_{21})], Q_{12} = [\mu_{12}.E_1/1-(\mu_{12}.\mu_{21})],$$
$$Q_{22} = [E_2/1-(\mu_{12}.\mu_{21})],$$
$$Q_{44} = G_{23}, Q_{55} = G_{13}, Q_{66} = G_{12},$$

2.4 Governing Equations and Boundary Conditions

Using the expressions for strains and stresses through (5) and using the principle of virtual work, variationally consistent governing differential equations and boundary conditions for the plate under consideration can be obtained. The principle of virtual work when applied to the plate leads to:

$$\int_{z=-h/2}^{z=h/2}\int_{y=0}^{y=b}\int_{x=0}^{x=a}\begin{bmatrix}\sigma_x\,\delta\varepsilon_x + \sigma_y\,\delta\varepsilon_y + \sigma_z\,\delta\varepsilon_z \\ +\tau_{yz}\,\delta\gamma_{yz} + \tau_{zx}\,\delta\gamma_{zx} + \tau_{xy}\,\delta\gamma_{xy}\end{bmatrix}dxdydz$$
$$-\int_{y=0}^{y=b}\int_{x=0}^{x=a} q(x,y)\,\delta w\,dxdy = 0, \tag{6}$$

where symbol δ denotes the variation operator.

Employing Green's theorem in Eq. (6) successively, we obtain the coupled Euler-Lagrange equations, which are the governing differential equations and the associated boundary conditions of the plate.

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

The governing differential equations obtained are as follows:

$$
\begin{aligned}
&Q_{11}A\frac{\partial^4 w}{\partial x^4}+(2Q_{12}A+4Q_{66}A)\frac{\partial^4 w}{\partial x^2\partial y^2}+Q_{22}A\frac{\partial^4 w}{\partial y^4}-Q_{11}B\frac{\partial^3\phi}{\partial x^3}\\
&-(Q_{12}B+2Q_{66}B)\frac{\partial^3\phi}{\partial x\partial y^2}-Q_{22}B\frac{\partial^3\psi}{\partial y^3}-(Q_{12}B+2Q_{66}B)\frac{\partial^3\psi}{\partial x^2\partial y}=q,\\
&Q_{11}B\frac{\partial^3 w}{\partial x^3}+(Q_{12}B+2Q_{66}B)\frac{\partial^3 w}{\partial x\partial y^2}-Q_{11}C\frac{\partial^2\phi}{\partial x^2}\\
&-Q_{66}C\frac{\partial^2\phi}{\partial y^2}-(Q_{12}C+Q_{66}C)\frac{\partial^2\psi}{\partial x\partial y}+Q_{55}D\phi=0,\\
&Q_{22}B\frac{\partial^3 w}{\partial y^3}+(Q_{12}B+2Q_{66}B)\frac{\partial^3 w}{\partial x^2\partial y}-(Q_{12}C+Q_{66}C)\frac{\partial^2\phi}{\partial x\partial y}\\
&-Q_{66}C\frac{\partial^2\psi}{\partial x^2}-Q_{22}C\frac{\partial^2\psi}{\partial y^2}+Q_{44}D\psi=0. \qquad (7)
\end{aligned}
$$

The associated consistent boundary conditions obtained are as below:

Along the edge $x = 0$ and $x = a$

$$
\begin{aligned}
&Q_{11}A\frac{\partial^2 w}{\partial x^2}-Q_{11}B\frac{\partial\phi}{\partial x}+Q_{12}A\frac{\partial^2 w}{\partial y^2}\\
&-Q_{12}B\frac{\partial\psi}{\partial y}=0 \text{ or } \frac{\partial w}{\partial x}\textit{ is prescribed},
\end{aligned}
$$

$$
\begin{aligned}
&-Q_{11}A\frac{\partial^3 w}{\partial x^3}+Q_{11}B\frac{\partial^2\phi}{\partial x^2}-Q_{12}A\frac{\partial^3 w}{\partial x\partial y^2}+Q_{12}B\frac{\partial^2\psi}{\partial x\partial y}\\
&-4Q_{66}A\frac{\partial^3 w}{\partial x\partial y^2}-4Q_{66}A\frac{\partial^3 w}{\partial x^2\partial y}+2Q_{66}B\frac{\partial^2\phi}{\partial y^2}\\
&+2Q_{66}B\frac{\partial^2\phi}{\partial x\partial y}+2Q_{66}B\frac{\partial^2\psi}{\partial x\partial y}=0 \quad \text{or w is prescribed,}
\end{aligned}
$$

$$
\begin{aligned}
&-Q_{11}B\frac{\partial^2 w}{\partial x^2}+Q_{11}C\frac{\partial\phi}{\partial x}-Q_{12}B\frac{\partial^2 w}{\partial y^2}\\
&+Q_{12}C\frac{\partial\psi}{\partial y}=0 \text{ or } \phi \text{ is prescribed,}
\end{aligned}
$$

$$
-2Q_{66}B\frac{\partial^2 w}{\partial x\partial y}+Q_{66}C\frac{\partial\phi}{\partial y}+Q_{66}C\frac{\partial\psi}{\partial y}=0 \text{ or } \psi \text{ is prescribed.} \qquad (8)
$$

Along the edge $y = 0$ and $y = b$:

$$
\begin{aligned}
&Q_{12}A\frac{\partial^2 w}{\partial x^2}-Q_{12}B\frac{\partial\phi}{\partial y}+Q_{22}A\frac{\partial^2 w}{\partial y^2}\\
&-Q_{22}B\frac{\partial\psi}{\partial y}=0 \text{ or } \frac{\partial w}{\partial y}\textit{ is prescribed},
\end{aligned}
$$

$$
\begin{aligned}
&-Q_{12}A\frac{\partial^3 w}{\partial x^2\partial y}+Q_{12}B\frac{\partial^2\phi}{\partial x\partial y}-Q_{22}A\frac{\partial^3 w}{\partial y^3}+Q_{22}B\frac{\partial^2\psi}{\partial y^2}\\
&-4Q_{66}A\frac{\partial^3 w}{\partial x^2\partial y}+2Q_{66}B\frac{\partial^2\phi}{\partial x\partial y}+2Q_{66}B\frac{\partial^2\psi}{\partial x^2}\\
&=0 \text{ or w is prescribed,}
\end{aligned}
$$

$$
\begin{aligned}
&-2Q_{66}B\frac{\partial^2 w}{\partial x\partial y}+Q_{66}C\frac{\partial\phi}{\partial y}\\
&+Q_{66}C\frac{\partial\psi}{\partial x}=0 \text{ or } \phi \text{ is prescribed,}
\end{aligned}
$$

$$
\begin{aligned}
&-Q_{12}B\frac{\partial^2 w}{\partial x^2}+Q_{12}C\frac{\partial\phi}{\partial x}-Q_{22}B\frac{\partial^2 w}{\partial y^2} \qquad (9)\\
&+Q_{22}C\frac{\partial\psi}{\partial y}=0 \text{ or } \psi\textit{ is prescribed},
\end{aligned}
$$

The values of integration constants A, B, C and D are mentioned below.

$$
\begin{aligned}
&A=\int_{-h/2}^{h/2} z^2 dz=\frac{h^3}{12}\ ,\ C=\int_{-h/2}^{h/2} f^2(z)\,dz=\frac{h^3}{2\pi^2},\\
&B=\int_{-h/2}^{h/2} z\,f(z)dz = 2\left(\frac{h}{\pi}\right)^3,\\
&D=\int_{-h/2}^{h/2}\left[\frac{d\,f(z)}{dz}\right]^2 dz=\frac{h}{2}.
\end{aligned}
$$

$$
\text{where } f(z)=\frac{h}{\pi}\sin\frac{\pi z}{h} \qquad (10)
$$

The flexural behavior of the plate is described by the solution satisfying these equations and the associated boundary conditions at each edge and corner of the plate.

III. ILLUSTRATIVE EXAMPLES

Example 1: A simply supported rectangular plate subjected to uniformly distributed load. The rectangular plate occupying the region given by the Eq.(1) is considered, the plate is subjected to uniformly distributed transverse load, q(x,y) on surface z = - h/2 acting in the downward z-direction as given below:

$$
q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \qquad (11)
$$

Where, q_{mn} are the coefficients of Fourier expansion of load, which are given by,

$$
q_{mn}=\frac{16q_0}{mn\pi^2} \quad for\ m=1,3,5,...,\ and\ n=1,3,5,...,
$$

$$
q_{mn}=0 \quad for\ m=2,4,6,...,\ and\ n=2,4,6,..., \quad (12)
$$

The governing differential equations and the associated boundary conditions for static flexure of rectangular plate under consideration can be obtained directly from Eqs. (7) through (9).

Navier Solution

The following is the solution form for $w(x,y)$, $\phi(x,y)$, and $\psi(x,y)$ satisfying the boundary conditions given by the equations (8) and (9) perfectly for a plate with all the edges simply supported:

$$
\begin{aligned}
&w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},\\
&\phi(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \phi_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \qquad (13)\\
&\psi(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \psi_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}.
\end{aligned}
$$

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

where w_{mn}, ϕ_{mn}, and ψ_{mn} are coefficients, which can be easily evaluated after substitution of Eq. (10) in the set of three governing differential equations (7) and solving the resulting simultaneous equations . Having obtained the values of w_{mn}, ϕ_{mn}, and ψ_{mn} one can then calculate all the displacement and stress components within the plate.
Displacements:

Substituting the final solution for $w(x,y)$, $\phi(x,y)$, and $\psi(x,y)$ in the displacement field, the final displacements can obtained.The displacements are obtained as follows.

$$
\begin{aligned}
u &= -z\left[\bar{w}q_{mn}\left(\frac{m\pi}{a}\right)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\right] \\
&+ f(z)\bar{\phi}.q_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}, \\
v &= -z\left[\bar{w}q_{mn}\left(\frac{n\pi}{b}\right)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\right] \\
&+ f(z)\bar{\psi}.q_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \\
w &= \bar{w}q_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}.
\end{aligned} \qquad (14)
$$

Stresses:-

$$
\sigma_x = \left\{\begin{aligned} &Q_{11}\left[z\bar{w}\left(\frac{m\pi}{a}\right)^2 - f(z)\bar{\phi}\left(\frac{m\pi}{a}\right)\right] \\ &+Q_{12}\left[z\bar{w}\left(\frac{n\pi}{b}\right)^2 - f(z)\bar{\psi}\left(\frac{n\pi}{b}\right)\right]\end{aligned}\right\} q_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},,
$$

$$
\sigma_y = \left\{\begin{aligned} &Q_{12}\left[z\bar{w}\left(\frac{m\pi}{a}\right)^2 - f(z)\bar{\phi}\left(\frac{m\pi}{a}\right)\right] \\ &+Q_{22}\left[z\bar{w}\left(\frac{n\pi}{b}\right)^2 - f(z)\bar{\psi}\left(\frac{n\pi}{b}\right)\right]\end{aligned}\right\} q_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},
$$

$$
\tau_{xy} = \left\{-Q_{66}\left[\begin{aligned} &2z\bar{w}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) \\ &+f(z)\left[\bar{\phi}\left(\frac{n\pi}{b}\right)+\bar{\psi}\left(\frac{m\pi}{a}\right)\right]\end{aligned}\right]\right\} q_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b},
$$

$$
\tau_{zx} = Q_{55}f'(z)\bar{\phi}q_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}. \qquad (15)
$$

IV. NUMERICAL RESULTS AND DISCUSSION

Results obtained for analysis of simply supported orthotropic and plate and now be compared and discussed with the corresponding results of refined plate theory (RPT),Higher order shear deformation theory (HSDT) of Reddy, classical plate theory (CPT) of Kirchhoff, first order shear deformation theory (FSDT) of Reissner and an exact solution for analysis of plate Srinivas.

The percentage of error is calculated as:-

$$
\%\ \text{error} = \frac{\text{value by particular theory} - \text{value by exact elasticity solution}}{\text{value by exact elasticity solution}}\ \text{X}\ 100,
$$

V. CONCLUSIONS

From the numerical results it is concluded that, deflection and stresses predicted by present theory are in close agreement with the exact solution and other higher order theories. Present theory exactly satisfies transverse shear stress boundary condition on the top and bottom of the plate. An author is put forth the result of noarmal shear stress $\bar{\tau}_{xy}$ by this theory for researcher. From the results it is concluded that when the a/b ratio of plate is incresed the value of shear stresses reduces. Through thickness variation of shear stress obtained by present theory is alsmost similar to the other higher order theories. Hence the theory does not require shear correction factor as this was taken by the first order shear deformation theory. This will give the efficacy and crediability of present theory.

REFRENCES

1. Timoshenko, S. P. And Krieger, S. W., “Theory of Plates and Shell”, 1959,McGraw Hill, New York.
2. Reissner, E. and Stavsky, Y., “Bending and stretching of certain type of heterogeneous aelotropic elastic plates”, Journal of App. Mech., 28, 1961, pp 402-408.
3. Kirchhoff, G.R., Uber das gleichgewicht und die bewegung einer elastischen Scheibe, *Journal für die reine und angewandte Mathematik (Crelle's Journal)* 1850, 40: 51-88.
4. Lekhnitskii, S. G., “Anisotropic plates”, Gorden and Breach, New York, 1968.
5. Whitney, J. M. and Leissa A.W., “Analysis of heterogeneous anisotropic plates”, Journal of App. Mech., 36, 1969, pp 261-266.
6. Szilard, R., “Theory and analysis of plates-classical and numerical methods”, Prentice-Hall Inc., Englewood Cliffs, New Jersey.
7. Whitney, J. M. and Pagano N.J., “Shear Deformation in Heterogeneous Anisotropic plate,” American Society of Mechanical Engineers. Journal Applied Mechanics, 1970,Vol. 37, pp. 1031-1036.
8. Jones, R. M., “Mechanics of composite Material”, 1975,McGraw Hill Kogakusha, Ltd.,
9. Reissner, E.,“The Effect of transverse Shear Deformation on the Bending Elastic Plate,” Transactions of the American society of Mechanical Engineers, Journal of Applied Mechanics, 1945, Vol. 12, pp. 69-77.
10. Mindlin, R. D., “Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic, Elastic Plates,” ASME Journal of Applied Mechanics, 1951, Vol. 18, pp.31-38.
11. Whitney, J.M.,“Shear Correction Factors for Orthotropic Laminates under Static Load”, ASME Journal of Applied Mechanics, 1973, Vol.40, pp.303-304. 1973.
12. Bert, C. W., “A critical evaluation of new plate theories applied to laminated composites, Composite Structure., 2, 1984, pp 329-347.
13. Noor, A.K. and Burton, W.S., “Assessment of Shear Deformation Theories for Multilayered Composite Plates,” Applied Mechanics Reviews, 1989, Vol. 42, pp.1-13.
14. Kapania, R. K. and Reciti, S., “Recent advances in analysis of laminated Beams and Plates: Part-II”, AIAA Journal., 27, 1989, pp 935-946.
15. Sayyad, A.S.,Ghugal,Y.M.,“Buckling analysis of thick isotropic plates by using exponential shear deformation theory”, Applied and Computational mechanics 6(2012) 185-196.
16. Sayyad, A.S.,Ghugal,Y.M.,“Bending and Free Vibration Analysis of Thick Isotropic Plates by using Expoential Shear deformation Theory”, Applied and Computational mechanics 6(2012) 65-77.
17. Reddy, J. N., “On the generalization of displacement-based laminate theories”, Applied Mech. Rev., 42 1989, S213-S222.
18. Mallikarjuna, and Kant, T., “A critical review and some results recently developed refine theories of fibre reinforced laminated composites and sandwiches”, Composite Structures, 26, 1993, 293-312.
19. Reddy, J. N. and Robbins, Jr. D. H., “Theories and computational Models for composite laminates”, Applied Mech. Rev., 47, no 6, 1994, pp 147-169.
20. Noor, A. K., Burton, W.S. and Bert, C. W., “Computational models for sandwich panels and shells”, Applied Mech. Rev., 49, 1996, pp 155-199
21. Lo, K. H., Christensen, R.M. and. Wu, E.M, “A High-Order Theory of Plate Deformation, Part-1: Homogeneous Plates,” ASME Journal of Applied Mechanics, 1977, Vol. 44, pp.663-668.

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

22. Lo, K.H., Christensen, R. M. and Wu, E.M., "A High-Order Theory of Plate Deformation, Part-2: Laminated Plates," ASME Journal of Applied Mechanics, 1977, Vol. 44, pp.669-676.
23. Stein, M., "Nonlinear Theory for Plates and Shells Including the Effects of Transverse Shearing," AIAA Journal, 1986, Vol. 24, pp.1537-1544.
24. Stein, M. Jegley, D. C., "Effect of Transverse Shearing on cylindrical bending, vibration buckling of laminated plates," AIAA Journal, Vol. 25, no. 1, 1997, pp.123-129.
25. Stein, M., "post buckling behavior of longitudinally compressed orthotropic plates with transverse shearing flexibility," AIAA Journal, Vol. 28, no. 5, 1990, pp.892-895
26. Gordaninejad, F. and Bert, C. W., "A new theory of bending of thick sandwich beams, Int. Journal of Mech. Sci., 31, no 11/12, 1989, pp. 925-934.
27. Heur, R., "Static and dynamic analysis of transversely isotropic, moderately thick sandwich beams by analogy", Acta Mechanica, 91, 1992, pp., 1-9.
28. Krishna Murty, "Toward a Consistent Plate Theory," 1986, AIAA Journal, Vol. 24, pp.1047-1048.
29. Srinivas, S.,Joga Rao, C.V., A.K. Rao, Bending, Vibration and buckling simply supported thick orthotropic rectangular plates and laminates, International Journal of Solids and Structures 6:1463-1481.
30. Krishna Murty, A. V., "Higher Order Theory for Vibrations of Thick Plates," 1977, AIAA Journal, Vol. 15, pp.1823-1824.
31. Ghugal, Y.M., Sayyad, A.S., "A static flexure of Thick Isotropic Plates Using Trigonometric Shear Deformation Theory",Journal of Solid Mechanics, Vol.2, 2010, pp.79-90.
32. Reddy, J. N., "A simple Higher-order Theory for laminated Composites Plate," Transaction of the American Society of Mechanical Engineers, Journal Applied Mechanics, 1984, Vol. 51, pp. 745-752.
33. Reddy, J., N., and Phan, N.D., "Stability and Vibration of isotropic, orthotropic and laminated plates according to higher order shear deformation theory," Journal of Sound and Vibration, 1985,Vol. 98, pp. 157-170.

Table 1

Comparison of deflection $\bar{w}$ at (x=a/2, y=b/2,z=0), in plane normal stresses $\bar{\sigma}_x$ at (x=a/2, y=b/2,z=h/2), $\bar{\sigma}_y$ at (x=a/2, y=b/2,z=h/2), in-plane shear stress $\bar{\tau}_{xy}$ (x=0, y=0,z=h/2) and transverse shear stress $\bar{\tau}_{zx}$ at (x=0, y=b/2,z=0) in rectangular orthotropic plate subjected to uniformly distributed load.

a/b	***h/a***	***Theory***	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
0.5	0.1	Present	1443.2605	66.0348	20.0650	25.3671	7.2108	6.9190
		Ghugal and Sayyad	1408.4	65.90	21.22	…….	7.187	6.800
		Shrinivas and Rao	1408.5	65.97	20.20	…….	6.927	…….
		Reddy (HSDT)	1408.5	65.95	19.932	…….	6.958	7.163
		Mindlin (FSDT)	1408.4	65.38	20.00	…….	7.061	7.048
		Kirchoff (CPT)	1326.0	65.55	19.78	…….	7.1328	7.133
		Exact	1408.5	65.975	20.204	…….	6.9266	……..

Table2

Comparison of deflection $\bar{w}$ at (x=a/2, y=b/2,z=0), in plane normal stresses $\bar{\sigma}_x$ at (x=a/2, y=b/2,z=h/2), $\bar{\sigma}_y$ at (x=a/2, y=b/2,z=h/2), in-plane shear stress $\bar{\tau}_{xy}$ (x=0, y=0,z=h/2) and transverse shear stress $\bar{\tau}_{zx}$ at (x=0, y=b/2,z=0) in square orthotropic plate subjected to uniformly distributed load.

a/b	***h/a***	***Theory***	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
1	0.1	Present	689.31	36.0772	22.1431	20.1070	5.5738	5.4487
		Ghugal and Sayyad	688.50	35.940	22.33	…….	5.546	5.191
		Shrinivas and Rao	688.57	36.021	22.21	…….	5.341	…….
		Reddy (HSDT)	689.50	36.01	22.08	…….	5.382	5.320
		Mindlin (FSDT)	688.37	35.578	22.04	…….	5.426	5.461
		Kirchoff (CPT)	640.39	36.098	21.62	…….	……	5.564
		Exact	688.57	36.021	22.21	…….	5.3411	…..

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

From table 1 it is observed that the present theory gives 2.46% error in deflection ($\bar{w}$) where as 0.09% error in normal stress ($\bar{\sigma}_x$) for (a/b) = 0.5 and aspect ratio 10. Similarly from table 2 the error in deflection ($\bar{w}$) is 0.107% 2.01% error in normal shear stress ($\bar{\tau}_{zx}$) for (a/b) = 0.1 and aspect ratio 10. It is also observed that by using equilibrium equation the value of shear stress ($\bar{\tau}_{zx}$) obtained by this theory is very close towards the exact solution.

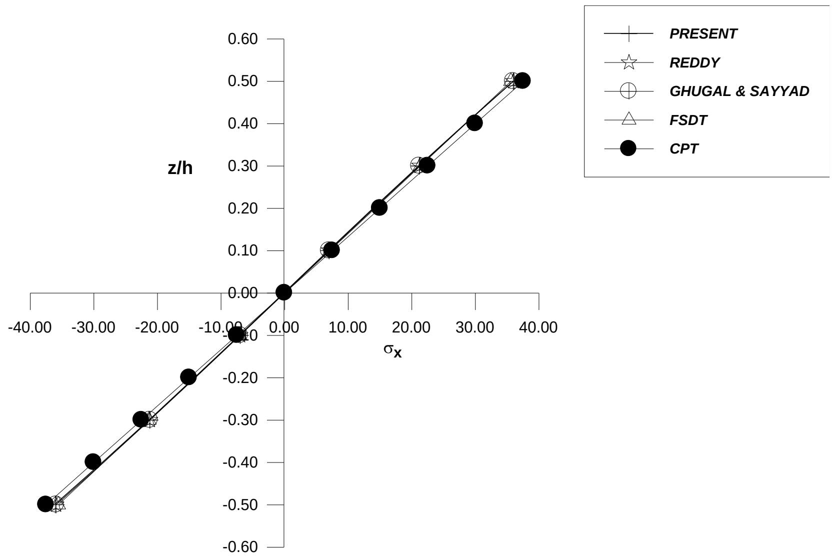

Fig.2. Variation of naormal stress $\bar{\sigma}_x$ through thickness of plate subjected to uniformly distributed load.

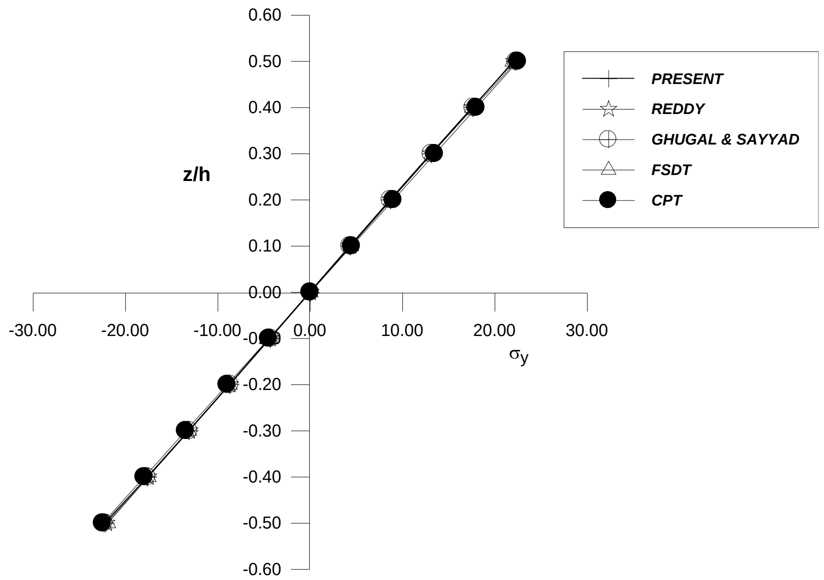

Fig.3. Variation of naormal stress $\bar{\sigma}_y$ through thickness of plate subjected to uniformly distributed load.

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

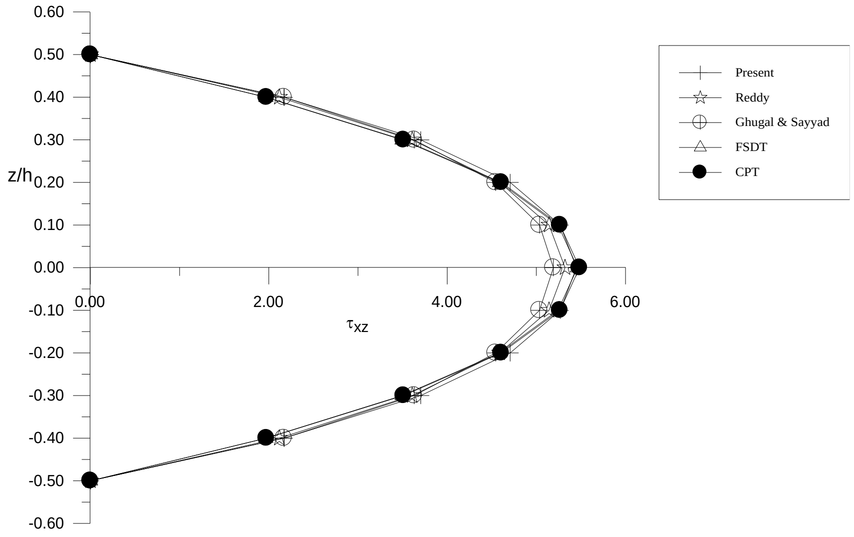

Fig.4. Variation of shear stress τ_{zx} through thickness of plate subjected to uniformly distributed load.

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)