Analysis of Orthotropic Plate By Refined Plate Theory

S.B.Chikalthankar, I.I.Sayyad, V.M.Nandedkar

Abstract- In this paper a Trigonometric Shear Deformation Theory (TSDT) for the analysis of orthotropic plate, taking into account transverse shear deformation effect is presented. Present theory exactly satisfies stress boundary conditions on the top and bottom of the plate. In this displacement-based, trigonometric shear deformation theory, the in-plane displacement field uses sinusoidal function in terms of thickness coordinate to include the shear deformation effect. The theory obviates the need of shear correction factor like other higher order or equivalent shear deformation theories. Governing equations and boundary conditions of the theory are obtained using the principle of virtual work. Results obtained for static flexural analysis of simply supported thick orthotropic plates for uniformly distributed loading case is compared with those of other refined theories and exact solution from theory of elasticity.

Key words: Orthotropic thick plates, Shear deformation, trigonometric shear deformation theory.

I. INTRODUCTION

In current scenario composite structures are used in many engineering applications like aerospace, nuclear power plants, automotive, submarines, sport and health instruments etc. This is because of advent of composite materials and many composite materials exhibit high strength-to-weight and stiffness-to-weight ratios, which make them ideally suited for use in weight-sensitive structures. In order to analyze the composite thick plate various plate deformation theories developed. Plate deformation theories can be divided in to two groups: stress based and displacement based theories. Since in the present study a new displacement based theory will be analyzed. Displacement based theories can be divided in to two parts: The classical plate theory and shear deformation plate theories. Transverse shear stress components are neglected in the classical plate theory where it is included in the shear deformation theories. The classical plate theory can be used only for thin plates since it gives improper results for thick plates especially made from advanced composites. Kirchhoff [1-7] developed classical plate theory (CPT) which is based on hypothesis that straight lines normal to the undeformed midplane remain straight and normal to the deformed midplane. Since the transverse shear deformation is neglected in Kirchhoff theory, it cannot be adequate for the analysis of shear flexible thick plates wherein shear deformation effects are more significant.

The first order shear deformation theory (FSDT) is considered as improvement over the classical plate theory. This is achieved by including gross transverse shear deformation in the kinematic assumptions. Reissner [2, 3] was the first to provide a consistent stress based plate theory which incorporates the effect of shear deformation. Mindlin [4] has provided displacement based theory. In this theory, the transverse shear strain is assumed to be constant across the thickness, and thus shear correction factor to correct the strain energy of shear deformation is required. These factors are problem dependent. The second order shear deformation theories by Naghdi [5], Pister and Westmann [6], Whitney and Sun [7], Nelson and Lorch [8] give marginally improved results over FSDT, but suffer from the same drawbacks of FSDT. The limitations of classical plate theory and first order shear deformation theory forced the development of higher order shear deformation theories to avoid the use of shear correction factors, to include correct cross sectional warping and to get the realistic variation of the transverse shear strain and stresses through the thickness of plate. The higher order theories based on series expansions are developed by Reissner [9], Provan and Koeller [10], Lo et al. [11, 12] and are modified by Levinson [13], Reddy [14] to get the parabolic shear stress distribution through the thickness of plate and to satisfy the shear stress free surface conditions on the top and bottom surfaces of the plate to avoid the need of shear correction factor. Sayyad and Ghugale[15,16] has carried out bending, buckling and free vibration analysis using exponential shear deformation theory.In the present work, emphasis has been laid on specific development of Trigonometric Shear Deformation Theories (TSDT) for plate analysis. The displacement models contain trigonometric terms in addition to classical plate theory terms. This displacement models is different from the generalized model of LO and Christensen [21-22] which contains only polynomial terms. Use of trigonometric functions to describe the plate behavior in thickness direction was first proposed by Stein [23-25] and was used for laminated beam and post buckling analysis of plates. However, the present TSDT model differs from that of Stein. In Stein’s the top and bottom shear stress conditions are not satisfied. Results obtained for uniformly distributed loading case and are compared with those of refined theories and exact elasticity theory [29] available in the literature.

II. THEORETICAL FORMULATION

2.1 Plate under consideration

The plate under consideration occupies in \( O - x - y - z \) Cartesian coordinate system the region:
\[
0 \leq x \leq a,
0 \leq y \leq b,
-h/2 \leq z \leq h/2,
\]

(1)
Fig. 1. Geometry of thick plate.

Where x, y, z are Cartesian coordinates, a and b are the edge lengths in the x and y directions respectively, and h is the thickness of the plate. The plate is made up of homogeneous, linearly elastic orthotropic material with the principal material axes parallel to the x and y axes in the plane of plate. The plate material obeys generalized Hooke’s law.

Assumptions made in the theoretical formulation:
1. The displacements are small and, therefore, strains involved are infinitesimal.
2. The in-plane displacement u in x direction as well as displacement v in y direction each consists of following parts:
   a) Displacement component analogous to the displacement in classical plate theory of bending.
   b) Displacement component due to shear deformation, which is assumed to be hyperbolic sine in nature with respect to thickness coordinate, such that the maximum shear stress occurs at neutral axis.
3. The transverse displacement w in z direction is assumed to be a function of x and y coordinates only.
4. The plate can be subjected to transverse as well as inplane loads.

2.2 The Displacement Field

Based on the before mentioned assumptions, the displacement field of the present refined plate theory can be expressed as follows:

\[
\begin{align*}
  u &= -z \frac{\partial w}{\partial x} + \frac{h}{\pi} \sin \frac{\pi z}{h} \phi(x, y), \\
  v &= -z \frac{\partial w}{\partial y} + \frac{h}{\pi} \sin \frac{\pi z}{h} \psi(x, y), \\
  w &= w(x, y)
\end{align*}
\]  
(2)

Here u and v are the in plane displacement components in the x and y directions respectively, and w is the transverse displacement in the z direction. The trigonometric function in terms of thickness coordinate in both the displacements u and v is associated with the transverse shear stress distribution through the thickness of plate and the functions \( \phi(x, y) \) and \( \psi(x, y) \) are the unknown functions associated with the shear slopes.

2.3 Strain-Displacement Relationships

Normal Strain:

Normal and shear strains are obtained within the framework of linear theory of elasticity using the displacement field given by Eq. (2). These relationships are given as follows:

\[
\begin{align*}
  \varepsilon_x &= \frac{\partial u}{\partial x}, \\
  \varepsilon_y &= \frac{\partial v}{\partial y}, \\
  \varepsilon_z &= \frac{\partial w}{\partial z}
\end{align*}
\]  
(3)

Shear Strains:

\[
\begin{align*}
  \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \\
  \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \\
  \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}
\end{align*}
\]  
(4)

Stress-Strain Relationships:

For a linearly elastic orthotropic material, stresses \( \tau_{xy}, \sigma_x \) and \( \sigma_y \) are related to strains \( \gamma_{xy}, \varepsilon_x \) and \( \varepsilon_y \) shear stresses are related to shear strains by the following constitutive relations:

\[
\begin{align*}
  \begin{bmatrix}
    \sigma_x \\
    \sigma_y \\
    \tau_{xy}
  \end{bmatrix} &=
  \begin{bmatrix}
    Q_{11} & Q_{12} & 0 & 0 & 0 \\
    Q_{12} & Q_{22} & 0 & 0 & 0 \\
    0 & 0 & Q_{66} & 0 & 0
  \end{bmatrix}
  \begin{bmatrix}
    \varepsilon_x \\
    \varepsilon_y \\
    \gamma_{xy}
  \end{bmatrix}
\end{align*}
\]  
(5)

where \( \sigma_x, \sigma_y \) and \( \tau_{xy} \) are the in-plane stresses and \( \tau_{yz}, \tau_{zx} \) are the transverse shear stresses.

They \( \varepsilon_x, \varepsilon_y, \gamma_{xy} \) are normal strain components and \( \gamma_{xz}, \gamma_{yz} \) are the shear strain components. And stiffness coefficients are

\[
Q_{11} = \left[E_1 / (1 - \mu_{12}, \mu_{21}) \right], Q_{12} = \left[\mu_{12}, E_1 / (1 - \mu_{12}, \mu_{21}) \right],
\]

\[
Q_{22} = \left[E_2 / (1 - \mu_{12}, \mu_{21}) \right],
\]

\[
Q_{44} = G_{25}, Q_{55} = G_{13}, Q_{66} = G_{12}.
\]

2.4 Governing Equations and Boundary Conditions

Using the expressions for strains and stresses through (5) and using the principle of virtual work, variationally consistent governing differential equations and boundary conditions for the plate under consideration can be obtained. The principle of virtual work when applied to the plate leads to:

\[
\int_{z=-h/2}^{z=h/2} \int_{y=-h/2}^{y=h/2} \int_{x=-h/2}^{x=h/2} \left[ \sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \sigma_z \delta \varepsilon_z + \tau_{xy} \delta \gamma_{xy} + \tau_{xz} \delta \gamma_{xz} + \tau_{yz} \delta \gamma_{yz} \right] dx dy dz
\]

\[
- \int_{y=-h/2}^{y=h/2} \int_{x=-h/2}^{x=h/2} q(x, y) \delta w dx dy = 0,
\]  
(6)

where symbol \( \delta \) denotes the variation operator.

Employing Green’s theorem in Eq. (6) successively, we obtain the coupled Euler-Lagrange equations, which are the governing differential equations and the associated boundary conditions of the plate.

The governing differential equations obtained are as follows:
The associated consistent boundary conditions obtained are as below:

Along the edge \( x = 0 \) and \( x = a \):
\[
Q_i A \frac{\partial^2 w}{\partial x^2} - Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial w}{\partial y} \text{is prescribed},
\]
\[
- Q_i B \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial \psi}{\partial y} \text{is prescribed},
\]
\[
- Q_i A \frac{\partial^2 w}{\partial x^2} + Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial \phi}{\partial x} \text{is prescribed},
\]
\[
- Q_i A \frac{\partial^2 w}{\partial x^2} + Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} + Q_i A \frac{\partial^2 \psi}{\partial x^2} = 0 \quad \text{or} \quad \frac{\partial \psi}{\partial x} \text{is prescribed},
\]
\[
+ Q_i A \frac{\partial^2 w}{\partial x^2} + Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial \phi}{\partial x} = 0 \quad \text{is prescribed},
\]
\[
+ Q_i A \frac{\partial^2 w}{\partial x^2} + Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial \psi}{\partial x} = 0 \quad \text{is prescribed}.
\]

Along the edge \( y = 0 \) and \( y = b \):
\[
Q_i A \frac{\partial^2 w}{\partial x^2} - Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial w}{\partial y} = 0 \text{ is prescribed},
\]
\[
- Q_i B \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial \psi}{\partial y} = 0 \text{ is prescribed},
\]
\[
- Q_i A \frac{\partial^2 w}{\partial x^2} + Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} + Q_i A \frac{\partial^2 \psi}{\partial x^2} = 0 \quad \text{or} \quad \frac{\partial \psi}{\partial x} = 0 \text{ is prescribed},
\]
\[
+ Q_i A \frac{\partial^2 w}{\partial x^2} + Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial \phi}{\partial x} = 0 \text{ is prescribed},
\]
\[
+ Q_i A \frac{\partial^2 w}{\partial x^2} + Q_i B \frac{\partial \phi}{\partial x} + Q_i A \frac{\partial^2 w}{\partial y^2} = 0 \quad \text{or} \quad \frac{\partial \psi}{\partial x} = 0 \text{ is prescribed}.
\]

The values of integration constants \( A, B, C \) and \( D \) are mentioned below.
\[
A = \int_{h/2}^{h/2} z^2 \, dz = \frac{h^3}{12}, \quad C = \int_{h/2}^{h/2} z \, dz = \frac{h^3}{2\pi},
\]
\[
B = \int_{h/2}^{h/2} z \, f(z) \, dz = \frac{h^3}{\pi^2},
\]
\[
D = \int_{h/2}^{h/2} \left[ \frac{f(z)}{h} \right] \, dz = \frac{h}{2}.
\]

The flexural behavior of the plate is described by the solution satisfying these equations and the associated boundary conditions at each edge and corner of the plate.

III. ILLUSTRATIVE EXAMPLES

Example 1: A simply supported rectangular plate subjected to uniformly distributed load. The rectangular plate occupying the region given by the Eq.(1) is considered, the plate is subjected to uniformly distributed transverse load, \( q(x,y) \) on surface \( z = -h/2 \) acting in the downward z-direction as given below:
\[
q(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right).
\]

Where, \( q_{mn} \) are the coefficients of Fourier expansion of load, which are given by,
\[
q_{mn} = \frac{16q}{mn\pi^2} \quad \text{for} \ m=1,3,5,..., \ \text{and} \ n=1,3,5,...,
\]
\[
q_{mn} = 0 \quad \text{for} \ m=2,4,6,..., \ \text{and} \ n=2,4,6,.... \quad (12)
\]

The governing differential equations and the associated boundary conditions for static flexure of rectangular plate under consideration can be obtained directly from Eqs. (7) through (9).

Navier Solution

The following is the solution form for \( w(x,y), \phi(x,y), \) and \( \psi(x,y) \) satisfying the boundary conditions given by the equations (8) and (9) perfectly for a plate with all the edges simply supported:
\[
w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right),
\]
\[
\phi(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \phi_{mn} \cos \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right),
\]
\[
\psi(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \psi_{mn} \cos \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{b} \right),
\]

where \( w_{mn}, \phi_{mn}, \) and \( \psi_{mn} \) are coefficients, which can be easily evaluated after substitution of Eq. (10) in the set of three governing differential equations (7) and solving the resulting simultaneous equations. Having obtained the values of \( w_{mn}, \phi_{mn}, \) and \( \psi_{mn} \) one can then calculate all the displacement and stress components within the plate.

Displacements:

Substituting the final solution for \( w(x,y), \phi(x,y), \) and \( \psi(x,y) \) in the displacement field,
the final displacements can obtained. The displacements are obtained as follows.

\[ u = -\pi \left( \frac{m_\pi}{a} \cos \frac{m_\pi x}{a} \sin \frac{n_\pi y}{b} \right) \]

\[ f = \pi \left( \frac{n_\pi}{b} \right) \left( \frac{m_\pi}{a} \sin \frac{m_\pi x}{a} \cos \frac{n_\pi y}{b} \right) \]

\[ w = \pi \left( \frac{m_\pi}{a} \sin \frac{m_\pi y}{b} \right) \]

Stresses:

\[ \sigma_x = \begin{cases} \sigma_\alpha & = f(\pi)\left( \frac{m_\pi}{a} \cos \frac{m_\pi x}{a} \sin \frac{n_\pi y}{b} \right) \\ \frac{f(\pi)}{\pi} & \cdot q_{w_\pi} \sin \frac{m_\pi x}{a} \sin \frac{n_\pi y}{b} \end{cases} \]

\[ \sigma_y = \begin{cases} \sigma_\alpha & = f(\pi)\left( \frac{n_\pi}{b} \right) \left( \frac{m_\pi}{a} \sin \frac{m_\pi x}{a} \cos \frac{n_\pi y}{b} \right) \\ \frac{f(\pi)}{\pi} & \cdot q_{w_\pi} \cos \frac{m_\pi y}{b} \end{cases} \]

\[ \tau_{x\alpha} = \begin{cases} \tau_{\alpha} & = f(\pi)\left( \frac{m_\pi}{a} \sin \frac{m_\pi x}{a} \cos \frac{n_\pi y}{b} \right) \\ \frac{f(\pi)}{\pi} & \cdot q_{w_\pi} \cos \frac{m_\pi x}{a} \sin \frac{n_\pi y}{b} \end{cases} \]

IV. NUMERICAL RESULTS AND DISCUSSION

Results obtained for analysis of simply supported orthotropic and plate and now be compared and discussed with the corresponding results of refined plate theory (RPT).

Higher order shear deformation theory (HSDT) of Reddy, classical plate theory (CPT) of Kirchhoff, first order shear deformation theory (FSDT) of Reissner and an exact solution for analysis of plate Srinivas.

The percentage of error is calculated as:-

\[ \% \text{error} = \frac{\text{value by particular theory} - \text{value by exact elasticity solution}}{\text{value by exact elasticity solution}} \times 100 \]

V. CONCLUSIONS

From the numerical results it is concluded that, deflection and stresses predicted by present theory are in close agreement with the exact solution and other higher order theories. Present theory exactly satisfies transverse shear stress boundary condition on the top and bottom of the plate. An author is put forth the result of noamal shear stress \( \tau_{\alpha} \) by this theory for researcher. From the results it is concluded that when the \( a/b \) ratio of plate is incresed the value of shear stresses reduces. Through thickness variation of shear stress obtained by present theory is alomost similar to the other higher order theories. Hence the theory does not require shear correction factor as this was taken by the first order shear deformation theory. This will give the efficacy and credibility of present theory.

REFERENCES


### Table 1

Comparison of deflection $w$ at $(x=a/2, y=b/2, z=0)$, in-plane normal stresses $\sigma_x$ at $(x=a/2, y=b/2, z=h/2)$, $\sigma_y$ at $(x=a/2, y=b/2, z=h/2)$, in-plane shear stress $\tau_{xy}$ at $(x=0, y=0, z=h/2)$ and transverse shear stress $\tau_{zx}$ at $(x=0, y=b/2, z=0)$ in rectangular orthotropic plate subjected to uniformly distributed load.

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$h/a$</th>
<th>Theory</th>
<th>$w$</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\tau_{xy}$</th>
<th>$\tau_{zx}$CR</th>
<th>$\tau_{zx}$EE</th>
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<tbody>
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<td>0.5</td>
<td>0.1</td>
<td>Present</td>
<td>1443.2605</td>
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<td>Shrinivas and Rao</td>
<td>1408.5</td>
<td>65.97</td>
<td>20.20</td>
<td>……</td>
<td>6.927</td>
<td>……</td>
</tr>
<tr>
<td></td>
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<td>Reddy (HSDT)</td>
<td>1408.5</td>
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<td>Mindlin (FSDT)</td>
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### Table 2

Comparison of deflection $w$ at $(x=a/2, y=b/2, z=0)$, in-plane normal stresses $\sigma_x$ at $(x=a/2, y=b/2, z=h/2)$, $\sigma_y$ at $(x=a/2, y=b/2, z=h/2)$, in-plane shear stress $\tau_{xy}$ at $(x=0, y=0, z=h/2)$ and transverse shear stress $\tau_{zx}$ at $(x=0, y=b/2, z=0)$ in square orthotropic plate subjected to uniformly distributed load.

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$h/a$</th>
<th>Theory</th>
<th>$w$</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\tau_{xy}$</th>
<th>$\tau_{zx}$CR</th>
<th>$\tau_{zx}$EE</th>
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<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>Present</td>
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<td>36.0772</td>
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<td>689.50</td>
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</table>

From table 1 it is observed that the present theory gives 2.46% error in deflection ($w$) where as 0.09% error in normal stress ($\sigma_y$) for $(a/b) = 0.5$ and aspect ratio 10. Similarly from table 2 the error in deflection ($w$) is 0.107% 2.01% error in normal shear stress ($\tau_{zx}$) for $(a/b) = 0.1$ and aspect ratio 10. It is also observed that by using equilibrium equation the value of shear stress ($\tau_{zx}$) obtained by this theory is very close towards the exact solution.
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Fig. 2. Variation of normal stress $\sigma_x$ through thickness of plate subjected to uniformly distributed load.

Fig. 3. Variation of normal stress $\sigma_y$ through thickness of plate subjected to uniformly distributed load.

Fig. 4. Variation of shear stress $\tau_{zx}$ through thickness of plate subjected to uniformly distributed load.