

Dynamic Stochastic Model to Forecast Non-Stationary Electricity Demand

Mohammad Anwar Rahman

Abstract—This paper presents a dynamic stochastic model to forecast the pattern of residential electricity consumption of a rapidly developing industrial nation. Electricity usage is essential for continuous economic development and urbanization. Long term projection of residential electricity demand is vital for decision makers to develop strategic resource planning and energy policy. In this forecasting model, electricity demand is a function of the price of electricity, household electric appliances, real personal income, number of households, and urban conditions. We propose the Bayesian statistical technique on a dynamic linear model to predict the parameters of the demand model. We apply the model to a time series of a nonlinear, non-stationary household electricity demand. The forecast is generated from the inference of marginal posterior distribution of the model parameters obtained with a Markov Chain Monte Carlo simulation method. The forecast result is tested and compared with actual data and two alternate models. The Bayesian model is proven to be an effective forecasting method with the flexibility to solve multi-dimensional time series models and update estimated parameters as the demand changes over time. Test results indicate that Bayesian model is preferred over the classical artificial neural networks and the regression models due to its capacity to predict parameters of large-scale multivariate models.

Index Terms—Bayesian statistical model, classical artificial neural network, dynamic linear model, electricity load data, forecast validation.

I. INTRODUCTION

Residential electricity consumption has dramatically increased due to continuing economic growth and changes of demographic factors such as population and per capita income growth. The innovation of modern electrical appliances, abundant availability of electronic products in the market place, increase in household income and desire for modern ways of living, together are further reasons for rapid growth of electricity demand. Sufficient and adequate production of electricity is a necessary condition for achieving sustainable economic development [1]. Managing electricity supply is a task that requires accurate forecasting and sustainable power generation strategy. The ability to predict long-term residential electricity demand is critically important to government agencies, corporations and many associated companies. Since the electricity generation process is capital intensive and requires large-scale production systems, it is considered vital for management decision making. Energy planning is particularly important for industrially developing countries to enable sufficient and cost effective energy sourcing in the midst of growing electricity needs. Central to any energy planning and control strategy is the statistical forecast.

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A large variety of forecasting models have attempted to project electricity consumption demand including extrapolating methods such as multiple regression models [2-4], econometric models such as Box Jenkins methods and artificial neural networks [5], and artificial neural networks [6, 7]. A short-term load forecasting model has been developed using seasonal auto-regression and exponential smoothing forecasting methods employing an hourly time series load data [5]. A dynamic multivariate periodic regression model generates time-varying coefficients for trend, seasonal and regression coefficients in an hourly time series data [3]. Recently many developing countries focus on future energy demand for effective supply-demand management programs and making optimal capital investment decisions. Researchers have estimated energy demand and production needs in Spain using a Box-Jenkins time-series ARIMA model [9]. The multiple linear regression model is one of the popular models used in developing forecasts for electricity consumption integrated with population growth and per capita consumption rates in Turkey [10]. In a related study, energy resources policy, electric energy production and consumption rates in Turkey has been investigated and compared with that of France, Germany and Switzerland [11]. The study of electricity demand in Namibia has revealed that their electricity consumption is driven by income growth and negatively to changes in electricity price and air temperature [12]. The study of industrial, service, and residential electricity demand in Kazakhstan has shown that price elasticity of demand is low [13]. These results suggest that there is considerable room for price increases to make necessary financial adjustments and future system upgrading. In a linear regression model, the energy demand in Sri Lanka has been expressed as a linear function of price and income to examine economic growth and electricity consumption rate [14]. Another study has investigated the impact of privatizing the power sector in Nigeria [15], revealing that privatization may reduce electricity demand if the power supply line is efficiently managed. However, the benefit of privatization may be jeopardized if the responsibility goes to inexperienced and underdeveloped private sectors. A dynamic autoregressive model has estimated short and long run electricity revenue and price in South Africa [16]. This study revealed that income significantly affects electricity demand but the electricity price rates have no effect on electricity demand. An aggregate electricity demand has been studied in South Africa [17] and the authors have concluded that growth in electricity consumption was mainly due to increased household income and improvement in the production handling factors. Modeling household energy consumption is more complex due to high seasonal variation and constant new urbanization.

Most of the models used for electricity demand forecast are either not capable of predicting complex demand due to high levels of uncertainty, or it requires large sets of data to trace the demand trend and forecast. For example, statistical techniques such as multiple linear regression models use multiple input variables to predict random demand for the consumables, but the parameters of the models are static. A neural network model requires large amounts of data to train the model for generating sensible parameter values. A prediction model works well if the parameters of the model are dynamically updated as the demand information becomes available over time. The Bayesian technique is a statistical modeling approach recently attracting many researchers, which allows wide flexibility in the input variables and accommodates any pattern or behavior of variables [18-20]. Bayesian procedures are commonly adopted to forecast electricity load [18]. In more recent illustrations, the Bayesian technique is applied to forecast high-dimensional parameter vectors from a multivariate time series [19, 20].

Accurate energy demand forecast is very important in developing countries for sustainable economic development. In this paper, we have two goals. First, we present a Bayesian statistical technique to forecast long term residential electricity consumption demand in an industrially developing country. The second goal is to test the effectiveness of the Bayesian model by comparing it with actual data and two known alternate models. The forecast results are compared with the forecast produced by a classical artificial neural network (C-ANN) model and a dynamic linear regression (DLR) model. The activity flows in this paper comprises collecting multivariate residential electricity time series data, the Bayesian forecasting technique and model validation is illustrated in Fig. 1.

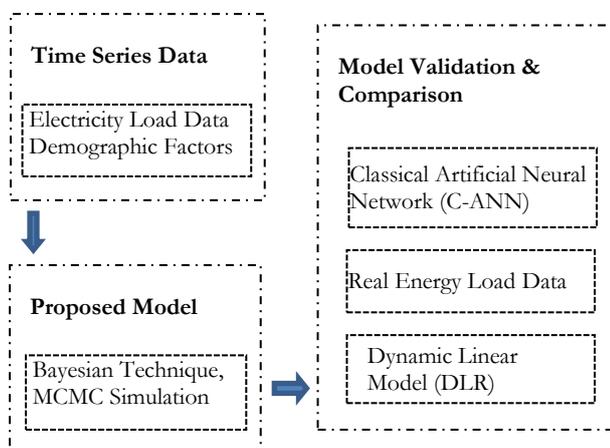


Fig. 1: Model Flow Process and Forecasting Validation

II. BAYESIAN FORECASTING METHOD

The energy consumption pattern has a rising trend for most of the countries. This section presents a framework for an application of the Bayesian statistical technique on a dynamic linear model to forecast the residential electricity demand. In most cases, Bayesian inference requires numerical approximation of analytically intractable integrals [20]. Bayesian inference provides estimates of the values of unknown model parameters using some prior approximations of the parameters. These prior approximations are often expressed with the probability density functions. In Bayesian techniques, the investigators are required to specify a number of subjective choices for the prior parameters. These can be

determined through the iterative processes. The prior belief about the test parameters is approximated before data are collected. The choices of density models of the prior parameters have limited effect in determining the values of the posterior parameters. An equally likely prior density function, known as flat prior, can be used for all parameters. However, alternative priors are often used to produce different sets of posterior distributions for the test parameters. An advantage of the Bayesian approach is the ability to take into account the uncertainty of parameters in the model [21].

Bayesian statistical inference can be used to estimate unknown model parameters by combining prior knowledge and observed data to provide a probability distribution, known as the posterior distribution. The values of the posterior distribution describe the updated parameters in the model. In the Bayesian method, the values of unknown model parameters, $\alpha (\alpha_1, \dots, \alpha_p)$ are expressed with probability density functions, $f(\alpha)$, which reflects the investors prior beliefs before collecting any new data. The dependence of observations $\mathbf{X} = (x_1, \dots, x_i)$ on model parameters α can be expressed with the probability density function as $L(\mathbf{X}|\alpha)$, which is known as the likelihood function. The prior probability density functions are modified by the likelihood function as the new data becomes available and yield to the posterior distribution function. Using Bayes' theorem, the updating is performed as follow:

$$f(\alpha|X) = \frac{f(\alpha)L(X|\alpha)}{\int_{\alpha} f(\alpha)L(X|\alpha)d\alpha} \tag{1}$$

where $f(\alpha|X)$ is the posterior distribution and expresses the probability of the parameters after observing new data. Once the posterior distribution is available, any features of θ , such as the marginal distributions or means and variances of the individual parameter α_i , as well as the predictive distribution of future observations, require integrating over the posterior distribution. For example, the marginal posterior distribution of an individual α_i can be calculated as $f(\alpha_i|X) = \int_{\alpha_{-i}} f(\alpha|X) d\alpha$, where α_{-i} represents all α 's except α_i [23]. Therefore, the forecast results in the Bayesian models is expressed by the expectation of a function of interest, $f(\alpha|X)$, evaluated over the posterior distribution as

$$E(f(\alpha|X)) = \int_{\theta_{-1}} f(\alpha|X)f(\alpha)d\alpha \tag{2}$$

where E denotes the expectation operator. In scientific literature, the limitation of Bayesian methods is the analytical solutions for the required integrations over the likelihood and the prior density functions. For Bayesian inference, limited combinations of closed form integrals for demand models and the conjugate prior probability functions are available. For most nonlinear and high-dimensional models, the inability to solve such integrals made the implementation of Bayes theorem increasingly difficult. However, with the advent of computer technology, software and the development of new methods made solving the complex integrals possible through the posterior Bayesian inference [9]. The Bayesian parameter estimation method now can be applied when the closed-form solution to a function is not available [22-23].



The method uses sampling-based approaches, particularly the Markov Chain Monte Carlo (MCMC) method and Gibbs Sampling approach to solve this problem. The flow diagram in Fig. 2 shows the Bayesian process using sampling-based MCMC method.

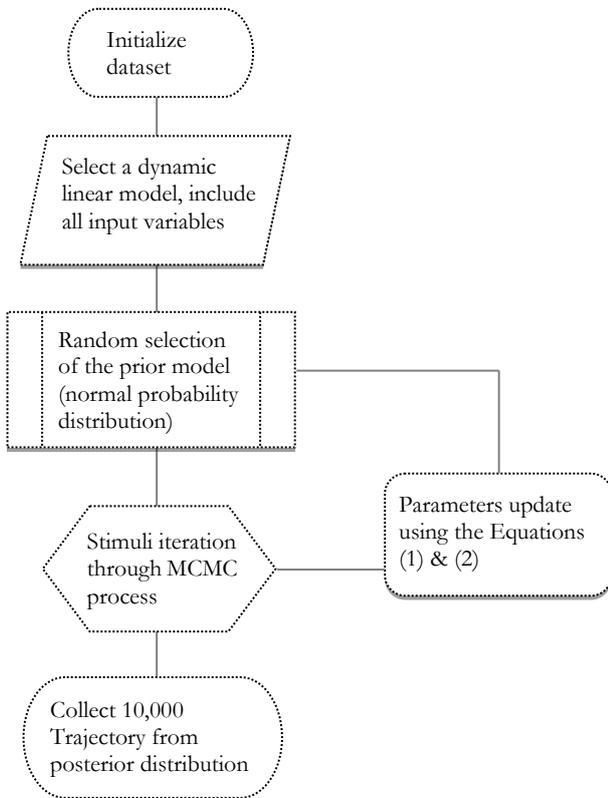


Fig. 2: Bayesian Sampling-Based MCMC Method

In the Bayesian process, the method begins by initializing the dataset, then selecting a dynamic linear model and input model variables. The process requires prior probability distribution for each input variable, and then, integrates over the likelihood and the prior probability functions using the Equations (1) and (2) through the MCMC process. Input parameters are updated through simulation iteration collecting 10,000 trajectories from the posterior distribution for the values of each input variable.

III. CASE STUDY

We use real time series data of residential electricity demand presented by [7] to develop the forecast using the Bayesian model. Generally, electricity demands are lumpy and vary greatly with the seasons. The demand model is integrated with five input variables: (1) electricity prices, (2) TV price index, (3) refrigerator price index, (4) urban household size and (5) urban household income. The residential electricity demand is a function of input variables, $f(x_1, x_2, \dots, x_5)$, where x_1, x_2, \dots, x_5 is assigned to each input variable. The time series dataset from 1974 to 1995 is used to develop the Bayesian forecast models, and a dataset from 1996 to 2003 is used to test the efficiency of the models. The flexibility of the Bayesian method is the capacity to use non-static parameters and accessibility to insert new data (or information) into the model as it enables the estimation process to improve continuously. The Bayesian approach

combines the observed data and the uncertainty of the unknown parameters to provide posterior information. The focus here is to get the posterior or marginal probability $f(\alpha|X)$ based on the likelihood $L(X|\alpha)f(\alpha)$ and the observed data. The input variables are related with the model through a link function, indicated by c_i for $i = 1, \dots, 5$. The training model for period t , from 1973 to 1995 is expressed as follows:

$$f(x_1, x_2, x_3, x_4, x_5) = \text{Normal}(\mu[t], \tau) \quad (3)$$

where $\mu[t] = \alpha_0 + \alpha_1 x_1(t) + \alpha_2 x_2(t) + \alpha_3 x_3(t) + \alpha_4 x_4(t) + \alpha_5 x_5(t)$, and τ is the precision, reciprocal of variance.

In the Bayesian method when prior information is not available, a common approach is to apply highly diffused probability distributions called non-informative priors. A Normal distribution with mean value 0 and inverse of variance $10^4 - 10^8$ can be considered a diffused prior. Initially little knowledge is available about the coefficient of the input variables (c_1, c_2, \dots, c_5). We assume $\text{Normal}(0, 10^4)$ is the prior probability density function for each of these coefficients. The parameter τ follows a chi squared distribution with one degree of freedom. The choice of prior distribution is followed by [6, 8]. As noted earlier, the coefficients of the input variables are estimated based on the likelihood function using the dataset from 1973 to 1995 and the prior distributions. After deriving the values for each of the coefficients (c_1 to c_5), the model uses the dynamic form of the parameters to forecast electricity demand from 1996 to 2003. The forecasting model for period t , from 1996 to 2003 is expressed as follows:

$$f(x_1, x_2, x_3, x_4, x_5) = \text{Normal}(\mu.\text{new}[t], \tau) \quad (4)$$

where $\mu.\text{new}(t) = \alpha_{0.\text{new}} + \alpha_{1.\text{new}} x_1(t) + \alpha_{2.\text{new}} x_2(t) + \alpha_{3.\text{new}} x_3(t) + \alpha_{4.\text{new}} x_4(t) + \alpha_{5.\text{new}} x_5(t)$ $c_{i.\text{new}} = \text{Normal}(c_i; [t], \tau_i)$, for $i \in (1, 5)$.

IV. FORECAST RESULTS

Modeling household energy consumption is more complex than the regional energy consumption model due to demographic factors, modern household electronic appliances and growing income. Bayesian approaches are the preferred methods capable of modeling nonlinear and non-stationary time series demand. In this forecasting model, given the time series data, a Monte Carlo Markov Chain (MCMC) is constructed using the Gibbs sampler algorithm for obtaining posterior inference of input parameters through simulation. The summary statistics such as mean, median, standard deviation and Bayesian confidence intervals for each parameter are estimated using the MCMC samples from the posterior distributions in the Bayesian procedure. The sample mean, standard deviation (SD) and 95% confidence level are reported in Table 1.

Table I: Forecast Model Parameter Estimates

Parameters	α_0	α_1	α_2	α_3	α_4	α_5
Mean	4.33	143.9	-0.619	78.03	230	108.5



SD	31.72	128	122.6	106.6	151	109
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The estimates obtained with the Bayesian model is the difference between the mean of the sampled values (estimate of the posterior mean for each parameter) and the true posterior mean [20]. Using the parameter values presented in Table 1 and corresponding density functions, the forecast obtained by posterior function is shown in Table 2.

Table II: Forecast Results for Eight years from 1996 to 2003 Using Bayesian Model

Year	Mean	SD	MC error	2.50%	Median	97.50%
1996	27770	3249	166.3	21660	27770	34320
1997	26920	3005	91.79	21060	26860	32930
1998	27750	2955	79.61	21740	27690	33610
1999	30780	3374	98.17	24020	30760	37620
2000	31730	3397	70.44	25070	31730	38140
2001	32940	3392	73.43	26370	32930	39730
2002	35110	3441	68.09	28250	35160	41740
2003	37680	3970	102.5	29680	37740	45280

A public domain software package, WinBUGS, [22] is used to generate feasible posterior samples via MCMC simulations in minimum computational time. The convergence of the MCMC algorithm is checked by the ANOVA-based method integrated in WinBUGS [22].

V. MODEL COMPARISON

The proposed model is tested with actual data and compared with C-ANN and DLR model forecasts. A result of C-ANN is borrowed from [7]. Table 3 details the forecast results including forecast errors, percentage error (PE), mean absolute deviation (MAD), tracking signals (TS) and standard error (SE) (appendix).

Table III: Forecast Results Comparison

Model	Estimate	Actual	PE	MAD	TS	SE	
Proposed (Bayesian Model)	1996	27770	23993	0.16	3777	-1.00	1.43E+07
	1997	26920	26523	0.01	2087	-0.19	1.58E+05
	1998	27750	28686	0.03	1703	0.55	8.76E+05
	1999	30780	29754	0.03	1534	-0.67	1.05E+06
	2000	31730	31266	0.01	1320	-0.35	2.15E+05
	2001	32940	32891	0.00	1108	-0.04	2.40E+03
	2002	35110	34946	0.00	1363	-0.12	2.69E+04
C-ANN [7]	2003	37680	37967	0.01	1183	0.24	8.24E+04
	1996	25083	23993	0.05	1090	-1.00	1.19E+06
	1997	28146	26523	0.06	1357	-1.20	2.63E+06
	1998	32171	28686	0.12	2066	-1.69	1.21E+07
	1999	29426	29754	0.01	1631	0.20	1.07E+05
	2000	31651	31266	0.01	1382	-0.28	1.49E+05
	2001	32764	32891	0.00	1173	0.11	1.60E+04
DLR Model	2002	35564	34946	0.02	1531	-0.40	3.82E+05
	2003	36964	37967	0.03	1443	0.70	1.01E+06
	1996	25277	23993	0.05	1284	-1.00	1.65E+06
	1997	32225	26523	0.21	3493	-1.63	3.25E+07

1998	32442	28686	0.13	3580	-1.05	1.41E+07
1999	33925	29754	0.14	3728	-1.12	1.74E+07
2000	34703	31266	0.11	3670	-0.94	1.18E+07
2001	39913	32891	0.21	4229	-1.66	4.93E+07
2002	39824	34946	0.14	6050	-0.81	2.38E+07
2003	35354	37967	0.07	5477	0.48	6.83E+06

The results of both Bayesian and C-ANN models are impressive and closely emerged. Forecast by the DLR method is not comparable with other results. However, the Bayesian model appears to be flexible, comprehensive and accessible to incorporate the judgment of experts about future demand with observed data to predict the future. The results derived by the Bayesian model are marginally better than those from the C-ANN model, and are suitable for enterprise forecast practices. Numerical values of the forecasts show the mean square error (MSE), mean absolute percentage error (MAPE), and mean absolute error (MAE) (appendix). The Bayesian model displays less error than of the C-ANN model and the DLR model. Table 4 illustrates the performance comparison between the Bayesian, C-ANN and DLR models.

Table IV: Predictive Performance Comparison

Models	MSE	MAPE	MAE
Bayesian	0.0208	0.0335	0.0089
C-ANN	0.022	0.0351	0.0108
DLR	0.1968	0.1339	0.0411

A. Error Statistics

SE	Square error	$\sum_{t=1}^T e_t^2$
MSE	Mean square error,	$1/T \sum_{t=1}^T e_t^2$
MAE	Mean absolute error	$1/T \sum_{t=1}^T e_t $
MAD	Mean absolute deviation	$1/T \sum_{t=1}^T e_t $
MAPE	Mean absolute percentage error	$1/T \sum_{t=1}^T e_t * 100$
TS	Tracking signals	$1/T \sum_{t=1}^T e_t / e_t $

where $e_t = (a_t - f_t)$, or error = actual - estimate.

The tracking signal indicates a bias in the forecast. It measures whether the forecast reflects the actual demand with respect to the level and trend in the demand profile. Similar to statistical process control, tracking signal measures when the forecasting process is going out of control and needs intervention for adjustment. Tracking signal also indicates if there is a persistent tendency for forecasts to be higher or lower than the actual demand. If the forecast is consistently lower than the actual demand, then there is under forecasting and the tracking signal will be positive. The tracking signal in the best forecast moves along the zero axes, going upward and downward. The tracking signal should pass a threshold test to be significant. If the tracking signal is above 3.75 or less than -3.75 then the forecast implies a bias. The comparison of the tracking signal is shown in Figure 3.



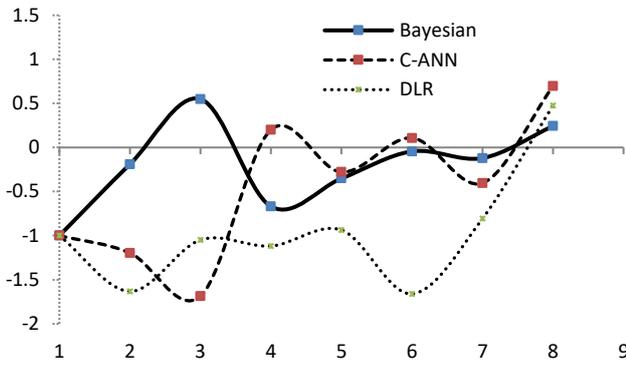


Fig. 3: Forecasts Tracking Signals

The focus here is to observe the forecast performance of the Bayesian model in comparison with the forecast of C-ANN and DLR models. The figure shows that the Bayesian model forecast is unbiased relative to the C-ANN model. Clearly, the statistical DLR model is biased with respect to actual data. The application of the Bayesian process in a time series of residential electricity consumption is fundamental better with respect to the classical statistical method. The Bayesian model is efficient to predict multi-dimensional non-stationary data and the generated forecast could enhance the framework of suitable energy policy. Therefore, Bayesian models are a viable alternative to predict future electricity requirements.

VI. CONCLUSION

Forecast of rapidly growing residential electricity demand is a complex task, particularly in industrially developing countries. The need for accurate projection of electricity demand is crucial; hence, it is of vital interest to choose the best forecasting model. The government sector, electric power industry and energy divisions need to develop advanced plans to manage energy supplies for wider usages. The proposed model uses the Bayesian technique to illustrate residential electricity demand forecasting problem. The study explored the potential advantage of the Bayesian technique over artificial intelligence methods and classical statistical forecasting models with respect to complex demand data. For an 8-year forecast period, the results clearly indicate that values forecasted by the Bayesian model are fairly close to the real values, compared to C-ANN and DLR models. The dataset used shows annual changes of demand pattern and nonlinear trends, however, the Bayesian model is capable of predicting complex data patterns with wider ranges of variability. The results illustrate that the Bayesian technique is an effective tool to model dynamic demand patterns and project the future trend of energy usages and demand comprising multiple input variables, demographic factors and latest consumer data.

Appropriate forecast procedure, software user-friendliness and precise prediction capacity have been an issue in selection of software and method in predicting changing electricity demand. There is only a handful of forecasting methods capable of predicting rapidly growing and large fluctuating demand. In most cases, practitioners do not follow concurrent software or technological advancement. Software packages for Bayesian statistical application are mostly open sourced. It is commonly perceived that these software are user friendly, easier to implement in developing a complex model and capable of combining expert judgment with the latest data information in the modeling process to find the

best forecast solutions. In light of current practices, Bayesian techniques are methods suitable for enterprise forecasters.

It is understood that judgment-based forecasting procedures are not adequate to draw a fair forecast conclusion for a dataset with multiple variables. In contrast, simply replacing the management judgment about future events with advanced data extrapolation methods, such as C-ANN or DLR methods, with no judgmental input is also unrealistic. Although the Bayesian model requires subjective choices, one can find more subjective choices in developing a classical ANN model than a Bayesian model. The Bayesian model requires less data compared to many econometric and C-ANN models. The Bayesian method is increasingly applied to many complex forecasting processes and applications. The Bayesian model has advantages over existing statistical forecasting techniques due to the ability to react with any changes and the adjustments with new occurrences outside of the routine model. For example, if there is a change in the electricity consumption demand due to various reasons such as new urbanization, government policy or natural disasters, subjective view can be incorporated in the model by using the judgment of experts about the new situation. Thereby, an immediate override can be made to adjust the forecasts. When time series shows significant changes and alterations in their (general) pattern, most models are not sufficient in picking up these changes. Thereby it causes a higher level of forecast errors and variances. Since the Bayesian model captures these changes, it is better than other models. It significantly reduces the variances, and the forecast errors. Although there are many favorable features available in the Bayesian method, the model is still being used much less frequently for forecasting electricity demand.

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