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Design of Unknown Inputs Multiple Observer for Uncertain Takagi-Sugeno Multiple Model

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Abstract*— This paper addresses the design of an unknown multiple observer for Takagi-Sugeno model subject to modelling and measures uncertainties. The proposed method in this paper is based on the development of an observer in presence of uncertainties. The specificity of this work is contained within the fact that a mathematical transformation which allows us to consider modelling and measures uncertainties in the form of unknown inputs is used. In so doing, a multiple observer based on the elimination of these unknown inputs is conceived. The synthesis conditions of that observer are expressed in Linear Matrix Inequalities (LMI) terms. An example of simulation is given to illustrate the validity of the proposed method.

Index Terms*— measure imprecision and modelling uncertainties, state estimation, Takagi-Sugeno model, unknown inputs and outputs.

I. Introduction

State estimation of nonlinear systems plays an important role in system theory and control engineering. For systems described by Linear Time Invariant (LTI) models, the state estimation can be successfully accomplished from their available inputs and outputs thanks to the Luenberger observer.

An observer is generally a dynamical system allowing the state reconstruction from the system model and the measurements of its inputs and outputs. In fact, the observer controlled by the same input signals that those applied to the system is able to provide the same output signals provided that the model employed reproduces with precision the behavior of the system to be supervised. But, the synthesis of the observer can become delicate according to the type and the complexity of the considered model.

Processes are often subjected to disturbances which have harmful effects on the normal behavior of the process. These disturbances, which have as origin fault of modeling and/or input uncertainties or fault of actuators and/or sensors , have harmful effects on the normal behavior of the process. They are called unknown inputs when they affect the input of the system and their presence can make difficult the state estimation.

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The state reconstruction of an uncertain system is a traditional problem of the automatic. The observer of Luenberger is not always sufficient for the fault detection, because the state estimation error given by this observer for an uncertain system does not converge inevitably towards zero.

In the case of linear systems, observers can be designed for unknown input systems [1] and uncertain system with timedelay perturbations [2]. However, in the majority of real cases the nonlinear nature of processes cannot be neglected. The assumption of linearity is checked only very locally around an operation point. Indeed, the physical systems present complex behaviors utilizing nonlinear laws. So, it is delicate to synthesize an observer for a nonlinear system. In order to represent the dynamic behavior of the nonlinear system, a global solution based on multiple models, was developed since several years, [3].

The idea of the multiple model approach is to apprehend the total behavior of a system by a set of local models. Each local model can be for example a linear time invariant system valid around an operation point. The local models are then aggregated by means of an interpolation mechanism. The global model is the sum of the local models weighted by activation functions associated to each one, [4]. These weighting functions quantify the relative contribution of each submodel to the global model according to the current operating point of the system. The Takagi-Sugeno structure is the most used in the multiple model approach. The main advantage of T-S structure is its simplicity because it originates from the interpolation between linear systems. Thus, analysis and design methods developed for linear systems can be generalized to nonlinear systems.

The problem of state estimation of nonlinear systems submitted to unknown inputs has received considerable attention [5], [6], [7]. Various studies dealing with the presence of unknown inputs acting on the system were published [8], [9], [10], [11], [12]. For state estimation, the suggested technique consists in associating to each local model a local unknown input observer. The multiple observer is the sum of the local observers weighted by their activation functions, which are the same than those associated with the submodels. Our contribution lies in the conception of a multiple observer of a nonlinear system described by a Takagi-Sugeno multiple model and influenced by modelling and measures uncertainties.

The main contribution in this paper is to conceive an observer for uncertain Takagi-Sugeno models. In order to consider the uncertainties as unknown inputs, a mathematical transformation that is proposed in the linear system case [12] is used.

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This paper proposes a method to extend the approach of design of observers with unknown inputs to taking into account the modelling and measures uncertainties.
The paper is organized as follows. A brief description of the Takagi-Sugeno model is presented in section 2. The state estimation problem using observers with unknown inputs is presented in section 3. The principle of the synthesis of a multiple observer with unknown inputs and outputs is described in section 4. Section 5 presents the main results concerning the design of a multiple observer based on multiple model submitted to modelling and measures uncertainties. In the last section, an academic example illustrates the state estimation of an uncertain Takagi-Sugeno model.

II. TAKAGI-SUGENO MULTIPLE MODEL

Each nonlinear dynamic system can be simply, described by a Takagi-Sugeno fuzzy model [13], [14]. The basis of the multiple model approach is the decomposition of the operating space of the system into a finite number of operating zones. Hence, the dynamic behaviour of the system inside each operating zone can be modelled using a simple submodel, for example a linear model. The relative contribution of each submodel is quantified with the help of a weighting function. Finally, the approximation of the system behavior is performed by associating the submodels and by taking into consideration their respective contributions. Note that a large class of nonlinear systems can accurately be modelled using multiple models.
The choice of the structure used to associate the submodels constitutes a key point in the multiple modelling framework. Indeed, the submodels can be aggregated using various structures [15]. Classically, the association of submodels is performed in the dynamic equation of the multiple model using a common state vector. This model, known as Takagi-Sugeno multiple model, has been initially proposed, in a fuzzy modelling framework, by Takagi and Sugeno [13] and in a multiple model modelling framework by Johansen and Foss [16]. This model has been largely considered for analysis, modelling, control and state estimation of nonlinear systems.
A Takagi-Sugeno multiple model is represented as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

$x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^p$ represents the measured output. $A_i \in \mathbb{R}^{n*n}$ is the state matrix $B_i \in \mathbb{R}^{n*m}$ is the matrix of input and $C \in \mathbb{R}^{p*n}$ is the output matrix of the system. Matrices A_i, B_i and C can be obtained by using the direct linearization of an a priori nonlinear model around operating points, or alternatively by using an identification procedure [17]. $\xi(t)$ represents the vector of decision depending on the input and/or the measurable state. M is the number of local models. It depends on the precision of desired modeling, the complexity of the nonlinear system and the choice of the structure of the weighting functions.
The normalized weighting functions $\mu_i(\xi(t))$ are nonlinear and depend on the decision variable $\xi(t)$. They are used to gradually quantify the membership of the current operation point of the system at a zone of operation. They are selected in order to check the following convex sum properties:

$$\sum_{i=1}^{M} \mu_i(\xi(t)) = 1 \quad \text{and} \quad 0 \le \mu_i(\xi(t)) \le 1 \quad (2)$$

The weighting functions can be obtained from normalised Gaussian functions:

$$\omega_i(\xi(t)) = \exp\left(\frac{-(\xi(t) - (\xi(t))^{(i)})^2}{\sigma^2}\right) \quad (3)$$

$$\mu_i(\xi(t)) = \frac{\omega_i(\xi(t))}{\sum_{j=1}^{M} \omega_j(\xi(t))} \quad (4)$$

III. ON MULTIPLE OBSERVER WITH UNKNOWN INPUTS

This part addresses the estimation of the state vector of a nonlinear system represented by a Takagi-Sugeno model and subject to the influence of unknown inputs, by using a multiple observer which is a linear combination of local observers. The synthesis of this multiple observer is based on the elimination of these unknown inputs.
Consider a nonlinear system modelled by the following Takagi-Sugeno model and dependent on unknown inputs:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + Rv(t)) \\ y(t) = Cx(t) \end{cases} \quad (5)$$

where $v(t) \in \mathbb{R}^q, q < n$ is the vector of unknown inputs. R is the matrix summarizing the influence of the unknown inputs.
The structure of the multiple observer results of the aggregation of local observers. It is described as follows [18]:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i z(t) + G_{i1} u(t) + G_{i2} + L_i y(t)) \\ \hat{x}(t) = z(t) - Ey(t) \end{cases} \quad (6)$$

$N_i \in \mathbb{R}^{n*n}$, $G_i \in \mathbb{R}^{n*m}$, $L_i \in \mathbb{R}^{n*p}$ is the gain of the ith local observer and E is a matrix transformation.
Let us define the state estimation error:

$$e(t) = x(t) - \hat{x}(t) \quad (7)$$
$$= (I + EC)x(t) - z(t) \quad (8)$$

The dynamic evolution of $e(t)$ is given by:

$$\begin{aligned} \dot{e}(t) = &\sum_{i=1}^{M} \mu_i(\xi(t))(P(A_i x(t) + B_i u(t) + Rv(t)) \\ &- \sum_{i=1}^{M} \mu_i(\xi(t))(N_i z(t) + G_i u(t) + L_i y(t)) \end{aligned} \quad (9)$$

where $P = I + EC$.
Replacing $y(t)$ and $z(t)$ by their expressions, expression (9) can be written as:

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$$\dot{e}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i e(t) + (PA_i - N_i - K_i C)x(t)) + \sum_{i=1}^{M} \mu_i(\xi(t))((PB_i - G_i)u(t) + PRv(t)) \quad (10)$$

with $K_i = L_i + N_i E$.

The state estimation error between the Takagi-Sugeno model (5) and its observer (6) converges towards zero, if all the pairs (A_i, C) are observables and if the following conditions are checked [18]:

$$N_i^T X + XN_i < 0 \quad (11a)$$
$$N_i = PA_i - K_i C \quad (11b)$$
$$P = I + EC \quad (11c)$$
$$PR = 0 \quad (11d)$$
$$L_i = K_i - N_i E \quad (11e)$$
$$G_i = PB_i \quad (11f)$$

where $X \in \mathbb{R}^{n*n}$ is a positive definite symmetric matrix.

IV. CONCEPTION OF MULTIPLE OBSERVER WITH UNKNOWN INPUTS AND OUTPUTS

This section deals with the design of a multiple observer with unknown inputs and outputs.

In the case of linear systems affected by unknown outputs, a mathematical transformation proposed in [12] and used in [19] is employed to consider these unknown outputs in the form of unknown inputs of an augmented system. This result is then extended to nonlinear systems represented by multiple model [20], [21], [22], [6]. In so doing, a multiple observer based on the elimination of these unknown inputs is conceived. The aim in this part is to extend that method to be employed to nonlinear systems modelled by Takagi-Sugeno models. Consider the following Takagi-Sugeno representation affected by an actuator fault and a sensor fault:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + Rv(t)) \\ y(t) = Cx(t) + D\bar{u}(t) \end{cases} \quad (12)$$

where $\bar{u}(t) \in \mathbb{R}^r$ is the sensor fault. The weighting functions $\mu_i(\xi(t))$ must satisfy the convexity conditions given by (2). Consider the new state $z(t)$ [20] that is a filtered version of $y(t)$ satisfying:

$$\dot{z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\,(-\bar{A}z(t) + \bar{A}Cx(t) + \bar{A}D\bar{u}(t)) \quad (13)$$

where $-\bar{A}$ is a p-dimensional stable matrix. Consider the augmented system

(14)

Which can be modelled as:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_{ai} X(t) + B_{ai} u(t) + D_r \gamma(t)) \\ Y(t) = C_a X(t) \end{cases} \quad (15)$$

where

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}C & -\bar{A} \end{bmatrix}, \; D_r = \begin{bmatrix} R & 0 \\ 0 & \bar{A}D \end{bmatrix}, \; B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix},$$

$$C_a = \begin{bmatrix} C & 0 \end{bmatrix}, \; \gamma(t) = \begin{bmatrix} v(t) \\ \bar{u}(t) \end{bmatrix}.$$

From the obtained results, sensor fault of the system (12) ppears as an actuator fault of the augmented system (15). In so doing, fault estimation strategy is similar to the method of conception of unknown inputs.

The structure of the multiple observer is choosen as follows:

$$\begin{cases} \dot{Z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i Z(t) + G_i u(t) + L_i Y(t)) \\ \hat{X}(t) = Z(t) - EY(t) \end{cases} \quad (16)$$

where $\hat{X}(t) \in \mathbb{R}^n$ and $Y(t) \in \mathbb{R}^p$ are respectively the state vector and the measured output. N_i , G_i , L_i is the gain of the local observer and E a matrix of transformation.

The augmented state estimation error is given by:

$$e_a(t) = X(t) - \hat{X}(t) \quad (17)$$

Using (16), we have:

$$\begin{aligned} e_a(t) &= (I + EC_a)X(t) - Z(t) \\ &= PX(t) - Z(t) \end{aligned} \quad (18)$$

with: $P = I + EC_a$

The dynamic of the augmented state estimation error is given as follows:

$$\dot{e}_a(t) = \dot{X}(t) - \dot{\hat{X}}(t) \quad (19)$$

that can be expressed as:

$$\dot{e}_a(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(P(A_{ai} X(t) + B_{ai} u(t) + D_r \gamma(t)) - \sum_{i=1}^{M} \mu_i(\xi(t))(N_i Z(t) + G_i u(t) + L_i Y(t)) \quad (20)$$

Using the expressions of $Z(t)$ and $Y(t)$ given respectively by (15) and (16), the equation (20) becomes:

$$\dot{e}_a(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i e_a(t) + (PB_{ai} - G_i)u(t)) + \sum_{i=1}^{M} \mu_i(\xi(t))((PA_{ai} - N_i - K_i C_a)X(t) + PD_r \gamma(t)) \quad (21)$$

where $K_i = L_i + N_i E$.

If the following conditions are fullfilled:

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$$\begin{cases} PD_r = 0 \\ P = I + EC_a \\ N_i = PA_{ai} - K_i C_a \\ L_i = K_i - N_i E \\ G_i = PB_{ai} \\ \sum_{i=1}^{M} \mu_i(\xi(t)) N_i \, stable \end{cases} \quad (22)$$

Then, the reconstruction error tends towards zero and the equation (21) is reduced to:

$$\dot{e}_a(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) N_i e_a(t) \quad (23)$$

Thus, the constraints (22) allow to synthesize the multiple observer of a system with unknown inputs and outputs.

A. Global convergence of the multiple observer

In this part, one will develop the sufficient conditions of the asymptotic global convergence of the state reconstruction. Let us consider the following Lyapunov function $V(t)$:

$$V(t) = e^T(t) X e(t) \quad (24)$$

where X denotes a positive definite symmetric matrix.
Its dynamic can be expressed as:

$$\dot{V}(t) = \dot{e}^T(t) X e(t) + e^T(t) X \dot{e}(t) \quad (25)$$

Substituting (23) in (25), one obtains:

$$\dot{V}(t) = \sum_{i=1}^{M} \mu_i(\xi(t)) e^T(t)(N_i^T X + X N_i) e(t) \quad (26)$$

Thus, the asymptotic convergence of the multiple observer is guaranteed and the state estimation error $e(t)$ converges towards zero, if the conditions (22) are verified and $\dot{V}(t) < 0$, that is $N_i^T X + X N_i < 0$.

The design of the observer carries out to extract the following theorem:

Theorem:
The state estimation error converges towards zero, if all the pairs (A_{ai}, C_a) are observables and if the following conditions hold $\forall\, i \in \{1,...,M\}$:

$$N_i^T X + X N_i < 0 \quad (27a)$$
$$N_i = PA_{ai} - K_i C_a \quad (27b)$$
$$P = I + EC_a \quad (27c)$$
$$PD_r = 0 \quad (27d)$$
$$L_i = K_i - N_i E \quad (27e)$$
$$G_i = PB_{ai} \quad (27f)$$

where $X \in \mathbb{R}^{n*n}$ is a positive definite symmetric matrix.

Using (27b), the expression (27a) can be written as:

$$(PA_{ai} - K_i C_a)^T X + X(PA_{ai} - K_i C_a) < 0 \;,\; \forall\, i \in \{1,...,M\} \quad (28)$$

It is noted that the inequalities (28) are nonlinear in X and K_i. To be reduced to the case of a linear problem, changes of variables are used.

B. Method of resolution

Three steps are needed to resolve the system (22):

1. The matrix E is given, using the expression (27d), as:

$$E = -D_r (C_a D_r)^{(-)} \quad (29)$$

where $(C_a D_r)^{(-)}$ is the pseudo-inverse of $(C_a D_r)$
The matrix P is deduced from (27c):

$$P = I - D_r (C_a D_r)^{(-)} C_a \quad (30)$$

2. By the variable change

$$W_i = X K_i \quad (31)$$

The inequalities (28) are written as:

$$(PA_{ai})^T X + X(PA_{ai}) - C_a^T W_i^T - W_i C_a < 0, \forall i \in \{1,...,M\} \quad (32)$$

The inequalities (32) are of LMI type and LMI Matlab Toolbox can be used for that resolution. Then, one deduces:

$$K_i = X^{-1} W_i \quad (33)$$

3. The other matrices defining the observer are deduced knowing P, K_i and E:

$$N_i = PA_{ai} - K_i C_a \quad (34a)$$
$$L_i = K_i - N_i E \quad (34b)$$
$$G_i = PB_{ai} \quad (34c)$$

C. Pole assignement

The objective in this part is to improve the performances of the multiple observer. For better estimating the state variables of the multiple model, the dynamics of the multiple observer is selected in a manner which is appreciably faster than that of the multiple model.
To ensure a certain dynamics of convergence of the state estimate error, it is necessary to fix the eingenvalues of local models in the left half plane limited by a vertical line of x-coordinate equal to $(-\alpha)$; α is a positive constant. The inequality (32) becomes:

$$(PA_{ai})^T X + X(PA_{ai}) - C_a^T W_i^T - W_i C_a + 2\alpha X < 0 \quad (35)$$

D. Example

The selected system is a three tank system shown in Fig.1. It schematizes a hydraulic process made up of three tanks [23], [24]. It is affected by an actuator fault $v(t)$ and a sensor fault $\bar{u}(t)$. These three tanks T_1, T_2 and T_3 with identical sections A, are connected to each others by cylindrical pipes of identical sections S_n. The output valve is located at the output of tank T_2; it ensures to empty the tank filled by the flow of pumps 1 and 2 with respectively flow rates $Q_1(t)$ and $Q_2(t)$.

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Combinations of the three water levels are measured. The pipes of communication between the tanks are equipped with manually adjustable ball valves, which allow the corresponding pump to be closed or open. The three levels x_1, x_2 and x_3 are governed by the constraint $x_1 > x_3 > x_2$. Taking into account the fundamental laws of conservation of the fluid, one can describe the operating mode of each tank. Then, one obtains a nonlinear model expressed by the following state equations [24]:

$$\begin{cases} A\dfrac{dx_1}{dt}(t) = -\alpha_1 S_n(2G(x_1(t)-x_3(t)))^{1/2} + Q_1(t) + Qf_1\bar{u}(t) \\ A\dfrac{dx_2}{dt}(t) = -\alpha_3 S_n(2G(x_3(t)-x_2(t)))^{1/2} - \alpha_2 S_n(2Gx_2(t))^{1/2} \\ +Q_2(t) + Qf_2\bar{u}(t) \\ A\dfrac{dx_3}{dt}(t) = -\alpha_1 S_n(2G(x_1(t)-x_3(t)))^{1/2} + Qf_3\bar{u}(t) \\ -\alpha_3 S_n(2G(x_3(t)-x_2(t)))^{1/2} \end{cases} \tag{36}$$

where α_1, α_2 and α_3 are constants, $\bar{u}(t)$ is regarded as an unknown input. $Qf/f_i(t)$, $i \in \{1...3\}$ denote the additional mass flows into the tanks caused by leaks and G is the gravity constant.

The structure of the multiple model which approximates the nonlinear system (36), is described by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{4} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + Rv(t) + d_i) \\ y(t) = Cx(t) + D\bar{u}(t) \end{cases}$$

The matrices A_i, B_i and d_i are calculated by linearizing the initial system (36) around four points chosen in the operation range of the system. The following numerical values were obtained:

$$A_1 = \begin{bmatrix} -0.0109 & 0 & 0.0109 \\ 0 & -0.0206 & 0.0106 \\ 0.0109 & 0.0106 & -0.0215 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.0110 & 0 & 0.0110 \\ 0 & -0.0205 & 0.0104 \\ 0.0110 & 0.0104 & -0.0215 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.0084 & 0 & 0.0084 \\ 0 & -0.0206 & 0.0095 \\ 0.0084 & 0.0095 & -0.0180 \end{bmatrix},$$

$$d_1 = 10^{-3} * \begin{bmatrix} -2.86 \\ -0.38 \\ 0.11 \end{bmatrix}, \quad d_2 = 10^{-3} * \begin{bmatrix} -2.86 \\ -0.34 \\ 0.038 \end{bmatrix},$$

$$d_3 = 10^{-3} * \begin{bmatrix} -3.7 \\ -0.14 \\ 0.69 \end{bmatrix}, \quad d_4 = 10^{-3} * \begin{bmatrix} -3.67 \\ -0.18 \\ 0.62 \end{bmatrix},$$

$$B_i = 1/A * \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

where $Qf_i = 10^{-4}$, $\forall\, i \in \{1...4\}$, $\alpha_1 = 0.78$, $\alpha_2 = 0.78$ and $\alpha_3 = 0.75$, $G = 0.98ms^{-2}$, $S_n = 5*10^{-5}m^2$ and $A = 0.0154$.

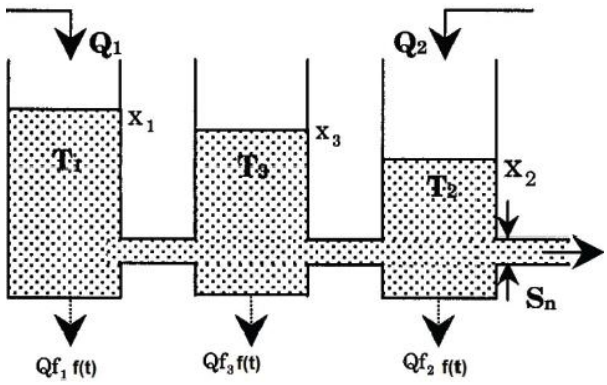

Fig.1 Three tank system

One chooses $\bar{A}_1 = 30 * I$, with $I = eye(3,3)$.

One chooses the following forms of $u(t)$, $v(t)$ and $\bar{u}(t)$.

The known input $u(t)$ is defined by:

$$u(t) = [\, u_1(t) \quad u_2(t)]^T$$

$$\begin{cases} u_1(t) = 0.5 * \sin(0.15\pi t) \\ u_2(t) = 0.25 * \sin(0.25\pi t) \end{cases}$$

The unknown input $v(t)$ is defined by:

$$v(t) = \begin{cases} 0.01 * \sin(\pi t), 0 < t < 40s \\ 0, t > 40s \end{cases}$$

The sensor fault $\bar{u}(t)$ is defined by:

$$\bar{u}(t) = \begin{cases} 0, t \le 10s \\ 0.1 * \sin(0.2\pi t), 10s < t \le 45s \\ 0, t > 45s \end{cases}$$

The simulation results are shown in Fig. (2) and (3). Fig. (2) represents the time evolution of the states and their estimations. The state estimation errors are shown in Fig. (3). The proposed method provides good estimates of the system state.

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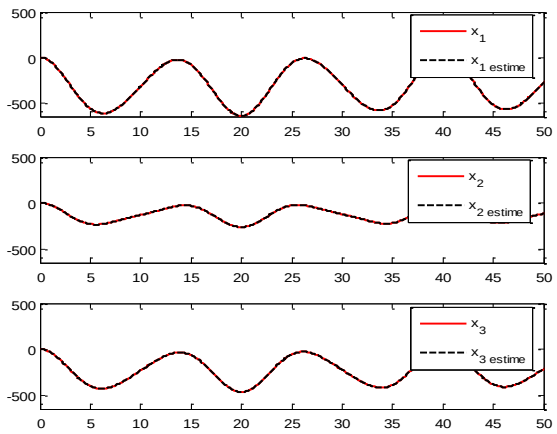

Fig.2 States and their estimates

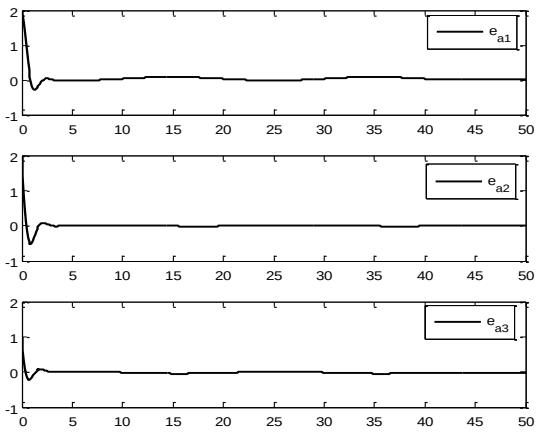

Fig.3 State estimation errors

V. MULTIPLE OBSERVER OF A SYSTEM WITH MODELLING AND MEASURE UNCERTAINTIES

A. *Multiple model representation of an uncertain system*

Generally, process can present uncertainties of inputs, measures and models. In the context of Takagi-Sugeno fuzzy systems, the general representation of uncertain system is given by the following equation:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))((A_i \pm \Delta A_i)x(t) + (B_i \pm \Delta B_i)u(t)) \\ y(t) = (C \pm \Delta C)x(t) \end{cases} \quad (37)$$

where ΔA_i are the matrices of modelling uncertainties, ΔB_i represent the input uncertainties of the system and ΔC represents the measures uncertainties. The weighting functions must satisfy the conditions (2).

By developing the expression given by the equation (37), one obtains:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + (\pm \Delta A_i x(t) \pm \Delta B_i u(t))) \\ y(t) = Cx(t) \pm \Delta C x(t) \end{cases} \quad (38)$$

In this section, the objective is to estimate the state of the nonlinear system modelled by an uncertain multiple model structure (38). A solution suggested with this problem consists in taking account of the input errors as unknown inputs what makes it possible to apply the results obtained in section IV.

Noting $v(t) = (\pm \Delta A_i x(t) \pm \Delta B_i u(t))$ and $\bar{u}(t) = (\pm \Delta C x(t))$, the system (38) becomes:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + v(t)) \\ y(t) = Cx(t) + \bar{u}(t) \end{cases} \quad (39)$$

By comparing the equation (39) with the equation (12), one notices that the two equations are almost identical. The differences are that the matrices R and D are replaced by the matrix identity I. It is possible under these conditions to adapte the results of section IV for the synthesis of an observer in the presence of uncertainties.

B. *Conception of the multiple observer*

The synthesis of the multiple observer is based on the elimination of the uncertainties assumed as unknown inputs affecting the nonlinear system described by the Takagi-Sugeno muliple model structure given by the equation (39).

The structure of the multiple observer is:

$$\begin{cases} \dot{Z}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))(N_i Z(t) + G_i u(t) + L_i Y(t)) \\ \hat{X}(t) = Z(t) - EY(t) \end{cases} \quad (40)$$

where $\hat{X}(t) \in \mathbb{R}^n$ represents the state estimate vector. $Y(t) \in \mathbb{R}^p$ is the measured output. N_i, G_i, L_i is the gain of the local observer and E is a transformation matrix.

To determine the gains of local observers, we utilize the results obtained in part IV-A. The deduction of the matrices N_i, L_i and G_i are given respectively by the equations (34).

C. *Numerical example*

Consider the Takagi-Sugeno model described in (37) with $M = 3$ defined by:

$$A_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 0.5 \\ 2 & 1 & -3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -3 & 2 & 2 \\ 1 & -3 & 0.4 \\ 1 & 0.9 & -2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -5 & 3 & -1 \\ 0.5 & -1 & 4 \\ 3 & -2 & -0.7 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0.5 \\ 2 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 3 \\ 1.5 \\ 2 \end{bmatrix}.$$

The signal $u(t)$ is defined by $u(t) = 0.5 * \sin(0.5\pi t)$.

One takes $\Delta A_i = \pm 0.1 * A_i$, $\Delta B_i = \pm 0.1 * B_i$ and $\Delta C = \pm 0.1 * C$.

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During simulation, one distinguishes the 8 borderline cases: $+\Delta A+\Delta B+\Delta C$, $+\Delta A+\Delta B-\Delta C$, $+\Delta A-\Delta B+\Delta C$, $+\Delta A-\Delta B-\Delta C$, $-\Delta A+\Delta B+\Delta C$, $-\Delta A+\Delta B-\Delta C$, $-\Delta A-\Delta B+\Delta C$ and $-\Delta A-\Delta B-\Delta C$. The study of these cases makes it possible to give an idea on the effectiveness of the state estimation method.

The weighting functions $\mu_i(\xi(t))$ are Gaussian and given in Fig. (4).

The structure of the multiple observer is:

$$\begin{cases} \dot{Z}(t)=\sum_{i=1}^{3}\mu_i(\xi(t))(N_iZ(t)+G_iu(t)+L_iY(t)) \\ \hat{X}(t)=Z(t)-EY(t) \end{cases} \quad (41)$$

The eight studied cases of simulation are considered. These results are shown in the Fig. (5) to (7).

The proposed method provides good estimates of the system state. Indeed, the convergence of the state vector of the multiple observer towards those of the Takagi-Sugeno multiple model is quite good.

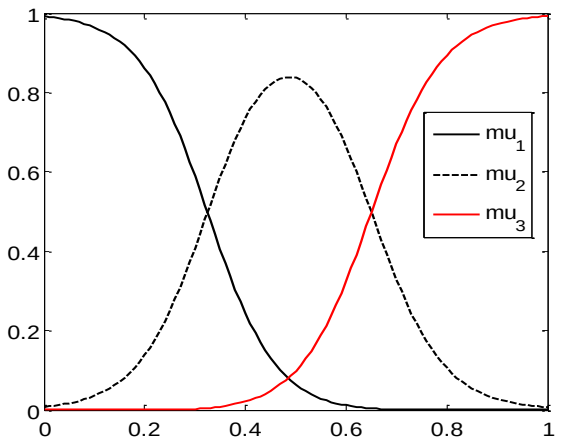

Fig.4 The weighting functions

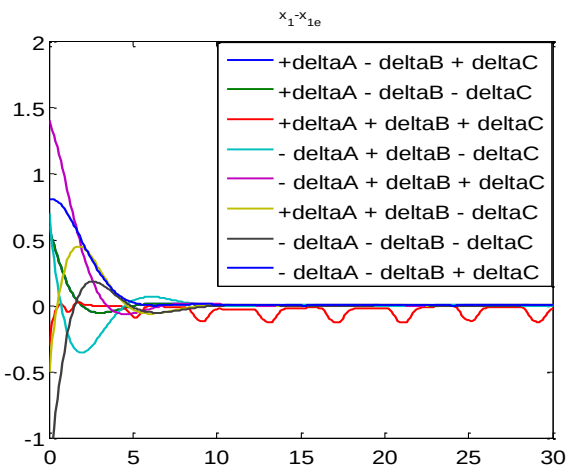

Fig.5 State estimation error for the 8 cases (a)

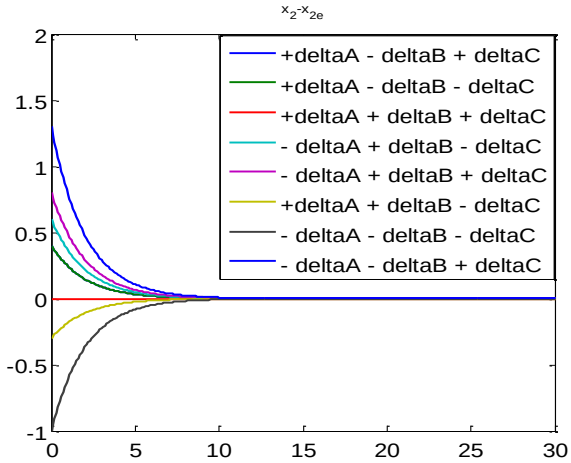

Fig.6 State estimation error for the 8 cases (b)

VI. CONCLUSION

Based on a Takagi-Sugeno uncertain multiple model representation, the conception of a multiple observer using the principle of the interpolation of local observers has been proposed. A mathematical transformation is used in order to formulate the uncertainties as unknown inputs. In this paper, a method of design of multiple observers with unknown inputs and outputs has been presented and extended to the case of modelling and measure uncertainties which can

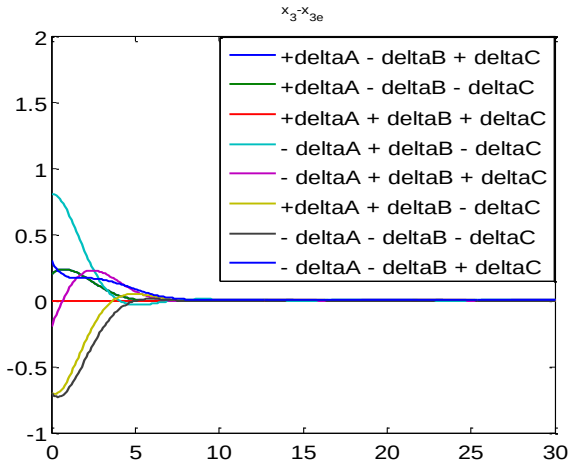

Fig.7 State estimation error for the 8 cases (c)

affect system. The synthesis conditions of that observer are expressed in LMI terms. The simulation results show that one succeeds in making the state estimation in spite of the existence of disturbances.

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