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Nested Sliding Mode Controller for MIMO System Using High Gain Feedback

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Abstract— The paper presents a new algorithm for sliding mode control for MIMO system represented in block companion form. If the system does not posses singularly perturbed structure, it can be made to possess singularly perturbed structure using high gain feedback. Here the block companion form of MIMO is used to design nested sliding mode control using high gain feedback concept. The individual high gain in each stage of decomposition is so applied that η-reachability condition is satisfied, so phenomenon of sliding under sliding occurs till last stage of decomposition and design of controller for higher order system becomes simpler. For maintaining sliding under sliding, we take sliding surface as incremental basis where sliding variable becomes generalized state vector for next sliding surface. Finally, we get a composite controller with robustness through variable structure control design using Lyapunov function. This proposed design method is illustrated with a 6th order two input system.

***Index Terms— Multi timescale decomposition, High Gain Feedback, Incremental sliding mode* control ,** sliding under sliding

I. INTRODUCTION

A fundamental property of high gain feedback system is their relationship with singularly perturbed system (see Young et al. (1977) and Kokotovic et al. (1986)). Even if, the original system is not singularly perturbed, a strong control action can force it to have slow and fast transients (see Young et al. (1977) and Marino (1985)), i.e to behave like an artificial singularly perturbed system (see Wu-Chung (1999)). The fast subsystems tends to equilibrium in a short transient so that average system response is governed by the slow subsystem, of which the dynamics can be predetermined by the designer and are insensitive to system uncertainties such as parameter variations and external disturbances. Another popular method for control design of uncertain systems is variable structure system(VSS) with sliding mode. Both control methods re- duce the entire systems into slow and fast subsystems (see Shang-Teh (1997)).

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The slow motion, approximated by a reduced model, is usually related to a variable structure system as sliding mode and the fast transients, represented by a boundary layer correction portion, correspond to the reaching mode before the state trajectory lies on the sliding surface(Young et al.1977, Marino.1985,Utkin.1992). This relationship is appropriate for relating sliding motion as artificial singular perturbation. A basic problem of control system is the design of controller for large scale MIMO system. There are several approach proposed in control system literature for such type of system. One of them is based on high gain feedback. Hanmandlu et al. (1986) presented a simple multi-time- scale decomposition of the high gain feedback system. Such a decomposition leads to one slow system and several fast systems of different time scales or in other word groups of several partially decomposed systems. This method involves a large amount of mathematical computations, since for each decomposition, transformation of state variables is needed to achieve the particular structure of input matrix. Bandyopadhyay et al. (1993) proposed simplest multi timescale decomposition using the block companion form. The entire system has been transformed into one slow and several fast systems, by a single transformation. But controller design in Bandyopadhyay et al. (1993) has no robustness property. *This paper proposes a new nested sliding mode controller for MIMO system using high gain feedback. In this approach the entire system has been transformed into one slow and several fast systems as done earlier. After the decomposition we can observe that the velocity of state vectors are totally different from one another. Hence we can use incremental based approach for the design of sliding surface which is existing for single input multi output under actuated system (Yinxing et al. (2007)). In this paper incremental approach is extended for MIMO general class of system in block companion form. The incremental sliding variable is needed as a coupling variable for each stage of multi-time-scale decomposition. It is also shown how this incremental sliding variable is related to multi timescale decomposition. Finally composite controller is designed based on Lyapunov function* .

II. MULTI TIMESCALE DECOMPOSITION USING BLOCK COMPANION FORM

For simplicity of illustration of multi timescale decomposition, considering an 6th order two input system, the block companion form will be (see Sheigh and Tsay (1982))

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$$\dot{X} = \begin{pmatrix} 0 & I & 0 \\ 0 & 0 & I \\ -A_0 & -A_1 & -A_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ I \end{pmatrix} u, \quad (1)$$

where

$$X_1 = \begin{pmatrix} X_{11} \\ X_{12} \end{pmatrix}, \quad (2)$$

$$X_2 = \begin{pmatrix} X_{21} \\ X_{22} \end{pmatrix}, \quad (3)$$

$$X_3 = \begin{pmatrix} X_{31} \\ X_{32} \end{pmatrix}. \quad (4)$$

This system can be subjected to two stages of decomposition, resulting into one slow and two fast subsystems, each of them of order two. In the first stage decomposition, the order of slow system is four and that for the fast system is two. The decomposed system can be written as

$$\begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \end{pmatrix} = \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} 0 \\ I \end{pmatrix} X_3, \quad (5)$$

$$\dot{X}_3 = -A_0X_1 - A_1X_2 - A_2X_3 + u_1, \quad (6)$$

where $u = u_1$ is the first stage high gain to the system. The system at the second stage of decomposition can be represented by

$$\dot{X}_1 = 0X_1 + X_2, \quad (7)$$

$$\dot{X}_2 = 0X_1 + 0X_2 + u_2, \quad (8)$$

where $X_3 = u_2$ is the high gain of second stage decomposition. Hence after two stage of decomposition

$$\dot{X}_1 = X_2, \quad (9)$$

$$\dot{X}_2 = u_2, \quad (10)$$

$$\dot{X}_3 = [-A_0X_1 - A_1X_2 - A_2X_3] + u_1. \quad (11)$$

We can see after the first stage of decomposition state vector X_3 is decoupled from whole system. Similarly at the second stage decomposition X_2 is decoupled from remaining subsystem. Hence we can generalize that multi input multi output(MIMO) system after successive decomposition, form a state vectors whose velocities are different from each other. For example if a system has n number of state vector X_1 , X_2 , X_3 .....X_n then after applying HGF to the system all the state vectors behave differently. Hence combination of first two state vectors is different form the next state vector, so combination of first two state vectors becomes generalized state vector for next, similarly generalized state vector and combination of next state vector becomes generalized vector for next. In this fashion whole state space is covered. But this assumption is true only when all state vectors have different velocity which cannot be possible without HGF.

III. SLIDING SURFACES USING INCREMENTAL SLIDING MODE CONCEPT

First sliding surface is defined using the state vectors X_1,

$$s_1 = c_1X_1 + X_2, \quad (12)$$

where

$$s_1 = \begin{pmatrix} s_{11} \\ s_{12} \end{pmatrix}. \quad (13)$$

For defining second sliding surface s_1 can be considered as a general state vector and one of the left state vectors X_3 as a second state vector (see Yinxing et al. (2007))

$$s_2 = c_2X_3 + s_1, \quad (14)$$

where

$$s_2 = \begin{pmatrix} s_{21} \\ s_{22} \end{pmatrix}, \quad (15)$$

c_1 and c_2 are some constant.

IV. METHOD FOR SELECTION OF COEFFICIENT OF SLIDING SURFACES

A group of Lyapunov functions can be defined as:

$$V_1 = \frac{1}{2}s_1^2, \quad (16)$$

$$V_2 = \frac{1}{2}s_2^2. \quad (17)$$

From "(12)" and "(14)" we can find if the coefficients $c_i(i = 1, 2)$ satisfies $c_1X_{11}X_{21} \geq 0$, $c_1X_{12}X_{22} \geq 0$ and $c_1X_1^TX_2 \geq 0$ for first surface and similarly for second surface $c_2s_{11}X_{31} \geq 0$, $c_2s_{12}X_{32} \geq 0$ and $c_2s^TX_3 \geq 0$ then $V_1 \leq V_2$, Therefore , the coefficients can be chosen as

$$c_1 = C_1 sign(X_1^TX_2), \quad (18)$$

$$c_2 = C_2 sign(s_1^TX_3), \quad (19)$$

where C_1 and C_2 are ositive.

High gain feedback is the combination of fast gain and slow gain, where the slow gain is provided by sliding mode control and fast part of high gain in each stage of decomposition is selected according to the requirement. Fast part of high gain in each stage of decomposition is given in the following

$$u_2 = 1/\epsilon_2(-\eta sign(s_1) - \kappa s_1), \quad (20)$$

$$u_1 = 1/\epsilon_1(-\eta sign(s_2) - \kappa s_2), \quad (21)$$

where $g_2 = 1/_2$ and $g_1 = 1/_1$ are constant high gain for second and first decomposition respectively. κ and η are also some other constants. Basically we can choose the high gain in such a fashion that along with slow-fast decomposition it also satisfy $\eta-$ reachability condition, so that after each stage of decomposition trajectory hits the sliding surface in finite time.

V.PROOF FOR RELATION BETWEEN HGF AND ISMC

Differentiating "(14)" w.r.t some time scale t, we get

$$\dot{s}_2 = c_2\dot{X}_3 + \dot{s}_1, \quad (22)$$

$$\dot{s}_2 = c_2\dot{X}_3 + c_1\dot{X}_1 + \dot{X}_2. \quad (23)$$

Putting the value of $\dot{X}_3$,$\dot{X}_2$ and $\dot{X}_1$ in "(23)"

$\dot{s}_2 = c_2(-A_0X_1 - A_1X_2 - A_2X_3 + u_1) + c_1X_2 + u_2$. (24)

Putting the value of fast part of high gain u_1 and u_2 in "(24)"

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$\dot{s_2} = c_2[-A_0X_1 - A_1X_2 - A_2X_3 + 1/\varepsilon_1(-\eta\text{sign}(s_2) - \kappa s_2)] + c_1X_2 + 1/\varepsilon_2(-\eta\text{sign}(s_1) - \kappa s_1)$ (25)

Multiplying both side of "(25)" by ϵ_1 we get

$$\epsilon_1\dot{s_2} = \epsilon_1 c_2[-A_0X_1 - A_1X_2 - A_2X_3] + c_2(-\eta sign(s_2) - \kappa s_2) + \epsilon_1 c_1 X_2 + \epsilon_1 1/\epsilon_2(-\eta sign(s_1) - \kappa s_1). \quad (26)$$

When $\epsilon_1 \to 0$ and ϵ_2 has some finite value, then "(26)" becomes

$$\frac{ds_2}{d\tau_1} = c_2(-\eta sign(s_2) - \kappa s_2), \quad (27)$$

where $\tau_1 = t/$ some other time scale which is faster than t because is less than one for high gain. Hence fast part of system decomposition is equivalent to reaching mode of sliding mode control. Slow part is given by $s_2 = 0$ which further degenerates s_1. Differentiating "(12)" w.r.t t, we get

$\dot{s_1} = c_1\dot{X_1} + \dot{X_2}$. (28)

Putting the value of $\dot{X_1}$ and $\dot{X_2}$ in "(28)"

$\dot{s_1} = c_1X_2 + u_2$. (29)

Putting the value of high gain u_2 in "(29)"

$\dot{s_1} = c_1X_2 + 1/\varepsilon_2(-\eta\text{sign}(s_1) - \kappa s_1)$. (30)

Multiplying both side of "(29)" by ε_2 We get

$\varepsilon_2\dot{s_1} = \varepsilon_2 c_1 X_2 + (-\eta\text{sign}(s_1) - \kappa s_1)$.

(31) When $\varepsilon_2 \to 0$ then "(31)" becomes

$$\frac{ds_1}{d\tau_2} = (-\eta sign(s_1) - \kappa s_1), \quad (32)$$

where τ_2 is again faster than t time scale by similar reason as given above. Hence trajectory again hits the first sliding surface in finite time. Finally slow subsystem is obtained

$c_1X_1 + X_2 = 0$; $X_2 = -c_1X_1$. (33)

Putting X_2 in "(9)" we get

$$\frac{dX_1}{dt} = -c_1X_1. \quad (34)$$

Hence by using HGF, whole system decomposes into one slow and two fast subsystems. The above analysis shows that the states are asymptotically stable using HGF and sliding mode.

VI. COMPOSITE CONTROL DESIGN FOR ORIGINAL SYSTEM

In section 2 using HGF, original system decoupled into several subsystems. Here using the sliding surfaces as coupling variables, the design of final controller for the whole system is done. HGF is a combination of slow and fast subsystem, therefore it can be replaced by sliding mode and reaching mode respectively. The composite controller for closed loop system contains switching control and equivalent control for maintaining the trajectory on reaching mode and sliding mode respectively.

6.1 Equivalent Control of System

After hitting the sliding surface s_2 becomes zero. Hence replacing $u_1 = u_{eq}$, we can get equivalent control. Putting $\dot{s_2} = 0$ in "(24)"

$$\begin{aligned} 0 &= c_2(-A_0X_1 - A_1X_2 - A_2X_3 + u_1) + c_1X_2 + u_2 \\ &= c_2(-A_0X_1 - A_1X_2 - A_2X_3 + u_{eq}) + c_1X_2 \\ &+ 1/\epsilon_2(-\eta sign(s_1) - \kappa s_1) \\ &= c_2(-A_0X_1 - A_1X_2 - A_2X_3) + c_2u_{eq} + c_1X_2 \\ &+ 1/\epsilon_2(-\eta sign(s_1) - \kappa s_1). \end{aligned} \quad (35)$$

$$u_{eq} = -c_2^{-1}[c_2(-A_0X_1 - A_1X_2 - A_2X_3) + c_1X_2 + g_2(-\eta sign(s_1) - \kappa s_1)]. \quad (36)$$

In equivalent control we multiplied some terms by $|sgn(s^T s_1)$ because when second-layer sliding surface converges to zero, they generates equivalent control for first layer. Hence equivalent control after modification becomes

$$u_{eq} = -c_2^{-1}[c_2(-A_0X_1 - A_1X_2 - A_2X_3)|sgn(s_1^T s_1)| + c_1X_2 + g_2(-\eta sign(s_1) - \kappa s_1)]. \quad (37)$$

Remark : The equivalent control shown in "(37)" is not only responsible for maintaining the trajectory along sliding surface but also responsible for switching between consecutive sliding surfaces, therefore discontinuous term appeared. When last surface has been reached then all discontinuous term vanishes and equivalent control becomes continuous.

6.2 SWITCHING CONTROL

For finite time reachability we select switching control u_{sw} as

$$u_{sw} = -g_1c_2^{-1}[\eta sign(s_2) + ks_2]. \quad (38)$$

Proof:- Consider the Lyapunov function $V = \frac{1}{2}s_2^2$ and putting $u_1 = u_{eq} + u_{sw}$

$$\begin{aligned} \dot{V} &= s_2\dot{s_2} \\ &= -s_2(c_2(-A_0X_1 - A_1X_2 - A_2X_3 + u_{eq} \\ &+ u_{sw}) + c_1X_2 + u_2) \\ &= -s_2[c_2(-A_0X_1 - A_1X_2 - A_2X_3) + c_2u_{eq}) \\ &+ c_2u_{sw} + c_1X_2 + g_2(-\eta sign(s_1) - \kappa s_1))] \\ &= -g_1\eta|s_2| - g_1k.s_2^2 \le 0. \end{aligned} \quad (39)$$

Hence composite control is given by $u = u_{eq} + u_{sw}$.

VII. CONCLUSIONS

The method proposed in this paper can be extended to any order of MIMO system. The beauty of this incremental based high gain feedback sliding mode control is that, the whole system can be decomposed into many smaller subsystems so that the sliding mode design becomes simpler and retains the robustness property of sliding mode control. Sliding under sliding is another advantage of this method which is responsible for simpler design for the overall controller. Simulation results show the effectiveness of the proposed controller.

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