

Analysis of Crack Propagation in Thin Metal Sheet, Three Point Bend Specimen, and Double Cantilever Beam

Negarullah Naseebullah Khan, Nitesh P. Yelve

Abstract— *Fracture Mechanics provides a theory background for failure of material and structures containing cracks. Stress intensity factor (SIF) is a key parameter in crack analysis. Because of the importance of SIF, its solutions for crack under different types of loading have been paid considerable attention. In the present study the SIF is calculated for thin metal sheet and three point bend specimen using finite element (FE) method. For the side crack in thin metal sheet, 2-D model is created in FE to calculate the SIF and this SIF is compared with that obtained by analytical method. For three point bend specimen, 3-D model is created in FE to calculate the SIF and this SIF is then compared with that obtained through experiments in the literature. The effect of thickness on the SIF is also estimated for three point bend specimen.*

It is also attempted here to understand crack propagation in layered materials such as composite materials, coated materials, etc. where the individual layers of materials are bonded together. For this purpose, an experiment is conducted on aluminium double cantilever beam (DCB) and results are plotted for load versus displacement. Also the simulation is carried out in FE using cohesive zone modeling (CZM) for the similar aluminium DCB, and the results are compared with these obtained through experiment.

Index Terms— *Stress intensity factor, three point bend specimen, double cantilever beam, traction separation law, cohesive zone modeling.*

I. INTRODUCTION

Engineering structures are designed to withstand the load which they are subjected to while in service. Large stress concentrations are avoided and a reasonable margin of security is taken to ensure that the values close to the maximum admissible stress are never attained. However, material imperfections that arise at the time of production or usage of the material are unavoidable and must be taken into account. Indeed, there are many unfortunate examples of situations where microscopic flaws have caused seemingly safe structures to fail. In the past, when a component of some structure exhibited a crack, it either repaired or simply retired from service. Such precautions are nowadays often deemed unnecessary, not possible to enforce, or may prove

too costly. On one hand, the safety margins assigned to structures have to be smaller, due to increasing demands for energy and material conservation.

On the other hand, the detection of a flaw in a structure does not automatically mean that it is not safe to use anymore.

This is particularly relevant for expensive materials or components of structures whose usage would be inconvenient to interrupt. Fracture mechanics plays a central role, as it provides useful tools allowing an analysis of materials that exhibit cracks. The goal is to predict whether and in which manner failure might occur.

The first person to make a setup to measure the strength of a wire was Leonardo da Vinci [1]. He found out that strength of a wire depends on its length. The quality of a wire in his time was not high and longer wire was likely to have more number of cracks. However, fracture mechanics was not studied as a separate discipline for a long time. The Industrial Revolution opened a new vista and many different kinds of machines and structures have been designed and built, mostly made of metals. Many bridges, boilers, buildings, ships failed due to fracture in nineteenth century [1]. Locomotive, a very important industry in those days, used to have numerous accidents due to failure of wheels, axels of wheels, and rails. Wohler is one of the earliest investigators who conducted stress controlled cyclic loading on fatigue life of axles of locomotives. This led to development of Goodman Diagram and finding endurance limit of steel [2].

World War II accelerated the industrial production at a very rapid rate, due to unusually high demands of the war. Within six years of the war, the know-how of aircraft making improved dramatically and the ships, which were being made earlier by riveting the plates together were changed to all welded frames. Many cargo ships, known as liberty ships were rolled out from American docks within a short span. However, soon it was discovered that welded structure had serious problem. Many of them failed in cold temperature of North Atlantic Ocean [1]. Some of them, in fact, broke into two parts each one floating separately. Ships made by riveting plates together did not have such problems because if a crack is nucleated and grown in a plate it would only split that plate into two parts; the crack is not likely to grow into another plate. A welded structure is a large single continuous part and once a crack becomes critical, it runs through the entire hull of the ship.

As very large welded ships were developed and high capacity jet airplanes were made, new problems were expected. Then a new discipline of engineering fracture mechanics was developed.

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In fact, Griffith [3,4] developed the right ideas for growth of a crack in 1920s. He estimated the strength of a material using atomistic models and found that strength should be of the order of its modulus, whereas in engineering materials the strength is two to three orders lower. Further he developed the ideas of energy requirements in propagating a crack. However, Griffith was not able to invent a convenient parameter that could be used by a practicing engineer or designer in predicting the failure load of a component through the growth of a crack under a given loading condition.

For all practical purposes, the modern fracture mechanics was born in 1948 when George Irwin [5] formulated the fracture mechanics and devised workable parameters like stress intensity factor (SIF) and energy release rate (ERR). Once the breakthrough took place, many investigators jumped into the wagon and fracture mechanics became a separate and important discipline with several journals and text books. Irwin's development was mainly for brittle or less ductile materials. The analysis was conservative for most engineering materials which are generally ductile. Other parameters like crack tip opening displacement by Wells [6] in 1961 and J-integral by Rice [7] in 1968, were developed to account for large plastic zone at the crack tip.

Fracture mechanics is also applied in fields like nuclear engineering, piping, spaceships, rockets, offshore structure, etc., wherein critical parts are made from very tough materials and found to fail catastrophically cracks developed. For such complicated configuration, the intersection of an area crack with a free surface is referred to as a surface flaw, where the exact solutions are difficult to obtain and generally unavailable. With the difficulty of the problem, there is unfortunate fact that such surface flaw is the most commonly encountered defect in many engineering structures. To compute Stress Intensity Factor (SIF) many methods are adopted till now, such as finite difference method (FDM), boundary element method (BEM), and finite element method (FEM), etc. FEM is the most popular tool for computing SIF.

Adhesive/epoxy joints are also a subject of great interest in different fields like aerospace, biomedical, automotive, etc. because of the advantages they provide with respect to the traditional joining techniques, such as more uniform load distribution, less sources of stress concentration, weight reduction and flexibility in design. The strength properties of adhesive bonded joints are commonly evaluated by standard test methods assuming a defect free bondline. However inaccurate bonding or inappropriate curing may cause the occurrence of un-bonded areas [8] which may affect the strength of such joints. This necessitates the development of robust methodologies and testing procedures for adhesive joints in critical structural applications to tackle fracture. Linear Elastic Fracture Mechanics (LEFM) provides an approach for analyzing the fracture mechanics properties of adhesive bonded joints [9]. It requires pre-existing crack like flaws for nucleation. Furthermore, it neglects a detailed description of what happens in the fracture process zone because it lumps all effects into the crack tip. However, a detailed description of the fracture process zone is essential, especially to understand fracture mechanisms and to make suitable design modifications of the material, e.g. toughening by reinforcement in polymeric structural adhesive [10]. To overcome these limitations, a powerful and efficient

computational tool for fracture studies is required, which led to the development of cohesive zone modeling (CZM) of fracture at inter-laminar surfaces. However, to accommodate CZM in Finite element (FE) analysis cohesive elements are placed along the anticipated crack path. According to this model, the zone in front of the physical crack tip opens and then tears progressively following a given traction-separation behavior which is used to predict failure under any loading conditions. The method is based on a softening relationship between stresses and relative displacements between delaminated surfaces, thus simulating a gradual degradation of the material properties.

The state of stress at the crack tip i.e. SIF is required to be estimated to find its value after which crack start propagating. To calculate SIF in the present work two specimens are considered one is thin metal sheet with side crack, and second is three point bend specimen. The SIF in thin metal sheet is calculated using 2Dimensional (2D) modeling in FE software ANSYS® and the obtained SIF is compared with that obtained from empirical formula given by Tada [11]. For three point bend specimen, the SIF is calculated by modeling it in 3 Dimensional (3D) in ANSYS® and the obtained SIF is compared with that obtained experimentally in the literature.

In the present study, CZM is carried out in FE using ANSYS® to find traction separation in a double cantilever beam (DCB). Experiment is also carried out on an aluminium DCB specimen to analyze traction separation. The resulting graph between load and displacement in both FE and experiment are compared.

II. SIF CALCULATION FOR SIDE CRACK IN THIN METAL SHEET

A. Linear Elastic Fracture Mechanics

Linear elastic fracture mechanics is a practical analytical tool for studying structural fracture, where the plastic deformation surrounding a crack tip is small. Fracture mechanics deals with the conditions under which cracks form and grow. As a consequence, fracture mechanics can be used in structural design to determine acceptable stress levels, defect sizes, and material properties for certain working conditions [12].

Linear elastic fracture mechanics is based on an analytical procedure that relates the stress field in the vicinity of the crack tip to the nominal stress in the structure, to the size, shape, and orientation of the crack, and to the material properties of the structure [12]. Consider an elliptical hole of major axis $2a$ and minor axis $2b$, as shown in Fig. 2.1a. If $a \gg b$, the elliptical hole becomes a crack of length $2a$ (Fig. 2.1b). The stress field is given as,

$$\sigma_x = \sigma \frac{\sqrt{a}}{\sqrt{2r}} \cos \frac{\theta}{2} (1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2}), \quad (2.1)$$

$$\sigma_y = \sigma \frac{\sqrt{a}}{\sqrt{2r}} \cos \frac{\theta}{2} (1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2}), \quad (2.2)$$

$$\tau_{xy} = \sigma \frac{\sqrt{a}}{\sqrt{2r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (2.3)$$

where r and θ are shown in Fig. 2.1b. The stress σ_y near the crack tip with $\theta = 0$ becomes

$$\sigma_y = \sigma \frac{\sqrt{a}}{\sqrt{2r}} \quad (2.4)$$



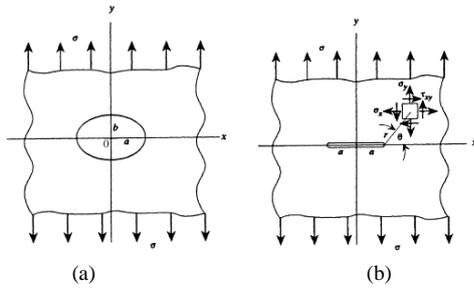


Figure 2.1 Elliptical hole in an infinite plate [12]
(a) $a > b$, (b) $a \gg b$.

It is clear that at the crack tip ($r = 0$) the stress is singular since $\sigma_y \rightarrow \infty$ as $r \rightarrow 0$. Because of this singularity, the usual stress concentration approach is inappropriate for this problem. Alternatively, the stress intensity factor is defined for the specific geometry and loading involved, to assess the safety factor for a solid.

The SIF, in a more general form, is given as [12]

$$K = C \sigma \sqrt{\pi a} \quad (2.5)$$

It has unit $MPa\sqrt{m}$. The stress σ is the nominal stress, a is the flaw size, and C is a constant that depends on the shape and size of the flaw and specimen.

Three types of crack propagations are recognized: opening, sliding, and tearing (Fig. 2.2). These types are called modes I, II, and III, respectively. A flaw may propagate in a particular mode or in a combination of these modes. These modes of fracture are explained in detail in the following section.

B. Modes of Fracture

Consider a cracked plate to distinguish several manners in which a force can be applied on the plate which might enable the crack to propagate. Irwin [5] proposed a classification corresponding to the three situations represented in Figure 2.3. Accordingly, three distinct modes are considered: mode I, mode II and mode III. In the mode I or opening mode, the body or structure is loaded by tensile forces such that the crack surfaces are pulled apart in the y direction. The deformed surfaces are symmetric with respect to the planes perpendicular to the y -axis and the z -axis [1].

In the mode II or sliding mode, the body or structure is loaded by shear forces parallel to the crack surfaces which slide over each other in the x -direction. The deformed surfaces are symmetric with respect to the plane perpendicular to the z -axis and skew symmetric with respect to the plane perpendicular to the y -axis.

Finally, in the mode III or tearing mode, the body or structure is loaded by shear forces parallel to the crack front and the crack surfaces slide over each other in the z -direction. The deformed surfaces are skew-symmetric with respect to the plane perpendicular to the z -axis and the y -axis.

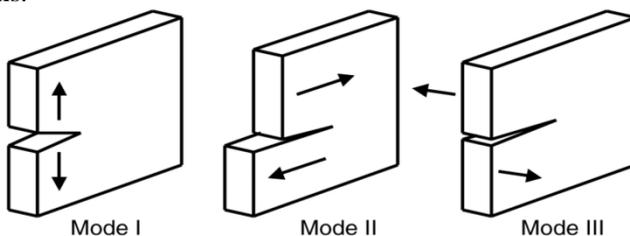


Figure 2.2 Modes of crack [1].

The stress intensity factor is calculated for opening mode (Mode I) as most of the structure tends to fail in opening mode due to tensile loading. This is discussed in following chapter for both 2-D and 3-D.

C. Mathematical Calculation of SIF

The stress intensity factor ' K ' is used in fracture mechanics to predict the stress state (stress intensity) near the tip of a crack caused by a remote load or residual stresses [12]. It is a theoretical construct usually applied to a homogeneous, linear elastic material and is useful for providing a failure criterion for brittle materials, and is a critical technique in the discipline of damage tolerance. The concept can also be applied to materials that exhibit small-scale yielding at the crack tip.

Consider a finite plate in tension with a side crack as shown in Fig. 2.3. The plate is made of steel with Young's modulus $E = 200$ GPa and Poisson's ratio $\nu = 0.3$. Let $b = 0.2$ m, $a = 0.06$ m, $\sigma = 100$ MPa. To determine the stress intensity factor (SIF) in case of infinite stripe with an edge with crack under tension, the generalized equation is given by Tada [11] as

$$K_I = \sigma \sqrt{\pi a} [1.12 - 0.23(a/b) + 10.55(a/b)^2 - 21.72/9a/b)^3 + 30.39(a/b)^4] \quad (2.6)$$

After substituting the values in Eq. 2.6, K_I is found to be 72.08 $MPa\sqrt{m}$.

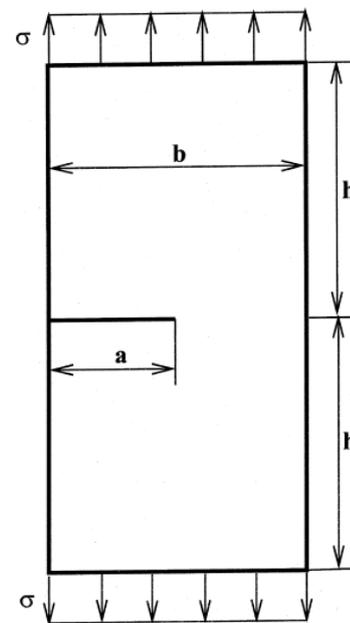


Figure 2.3 Finite plate with side crack [11].

D. Finite Element Simulation

Fracture analysis is a combination of stress analysis and fracture mechanics parameter calculation. The stress analysis is a standard linear elastic or nonlinear elastic plastic analysis. Because high stress gradients exist in the region around the crack tip, the finite element modeling of a component containing a crack requires special attention in that region [13]. Figure 2.4 shows crack tip zone



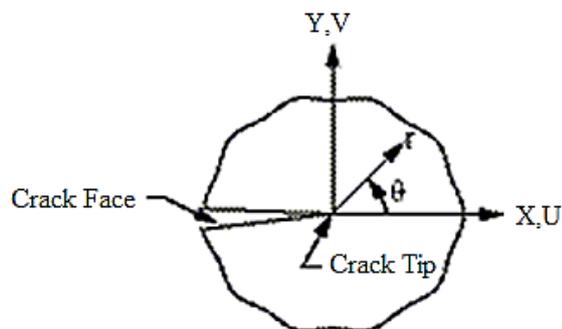


Figure 2.4 Crack tip of 2-D model[13]

To calculate stress intensity factor first local crack-tip or crack-front coordinate system is defined. The X - axis must be parallel to the crack face and the Y-axis perpendicular to the crack face as shown in Fig. 2.4. This coordinate system must be the active model coordinate system (CSYS) and results coordinate system (RSYS) when calculation of SIF is done by executing KCALC command.

Then a path along the crack face is defined. The first node on the path should be the crack-tip node. For a half-crack model, two additional nodes are required, both along the crack face. Fig. 2.5 shows the half crack model in ANSYS®.

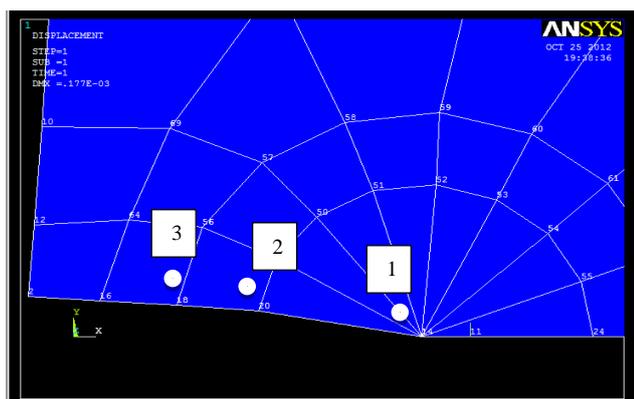


Figure 2.5 Enlarged Half crack model Path definition.

The crack tip field (KPLAN) on the KCALC command specifies whether the model is plane-strain or plane-stress. Except for the analysis of thin plates, the asymptotic or near-crack-tip behavior of stress is usually thought to be that of plane strain. The KCSYM field specifies whether the model is a half-crack model with symmetry boundary conditions, a half-crack model with antisymmetry boundary conditions, or a full-crack model.

The stress intensity factor calculated for half crack model with symmetry boundary conditions in plane stress is found to be $71.596 \text{ MPa}\sqrt{\text{mm}}$ and K_{II} and K_{III} are zero as the geometry is a 2-D model.

III. SIF CALCULATION FOR THREE POINT BEND SPECIMEN

A. Finite Element Modeling

Fig. 3.1 shows crack front zone in 3d

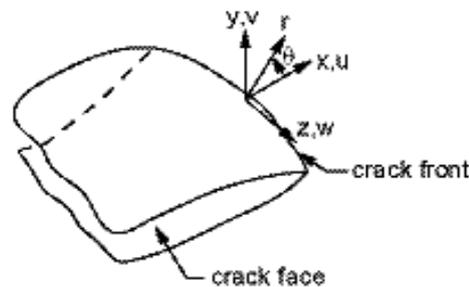


Figure 3.1 Crack front of 3-D model [13]

The three point bend specimen used in the analysis is shown in the Fig. 3.2. Where S is length of the beam between two support points (16 mm), a is crack length (1.6 mm), B is thickness (3 mm), W is width (4 mm), P is force applied (47 N), and f (a/w) is the dimensionless function depending on 'a' and 'W'. The method used for modeling is mode I.

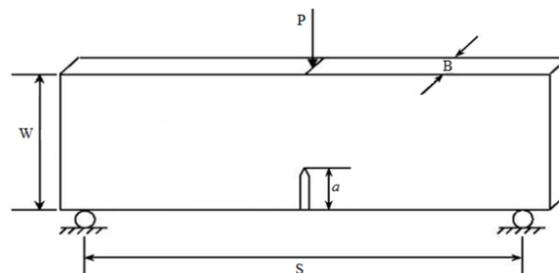


Figure 3.2 Three point bend specimen [15]

The model is generated in 2-D and then extruded with given thickness as shown in Fig. 3.3. The initial area of the geometry should be cleared after extruding so that the applied loads are distributed throughout the volume. If initial area is not cleared after extrusion, the loads would act only on the initial area and not throughout the volume. The loads at the bottom are applied on lines with zero displacement which means that they are fixed. The load at the top is concentrated force -47N on the nodes the negative sign indicates that the force is in the downward direction.

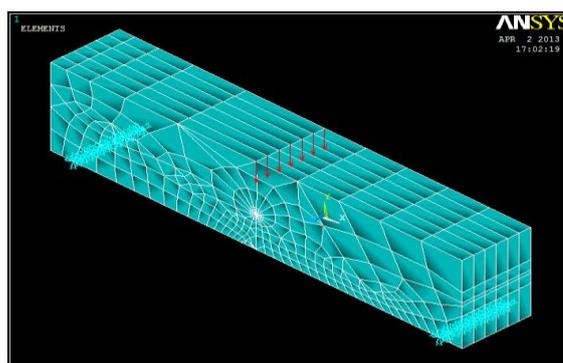


Figure 3.3 FE model with applied load.

To calculate the SIF in three point bend specimen first a local co-ordinate system is defined at the crack tip. The local co-ordinate system is defined by selecting three nodes first node point at the crack tip, second node point on the y-axis parallel to the crack and the last node point on the x-axis perpendicular to the crack as shown in Fig. 3.4.

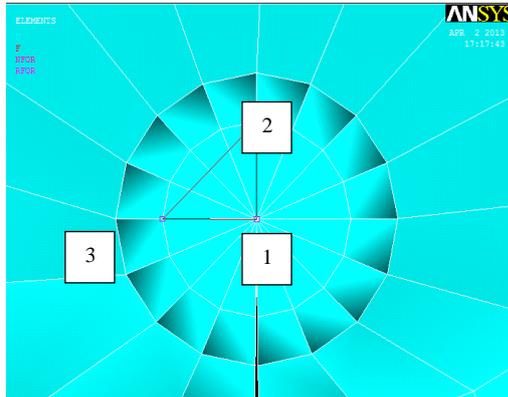


Figure 3.4 Local co-ordinate systems.

The co-ordinate system is then set as active co-ordinate system by selecting the specified co-ordinate system and giving the reference number. The global co-ordinate system is changed to local co-ordinate system and the defined local reference number is specified.

Then the propagation path along the crack face is defined. This path along the crack face can be defined in two ways, one is half crack model which is used when the model is symmetric, and the second is full cracked model which is used when the model is asymmetric. The three point bend specimen is modeled as full-crack model. In full crack model along with crack tip node, four additional nodes are required, two along the left crack face and two along the right crack face as shown in the Fig. 3.5.

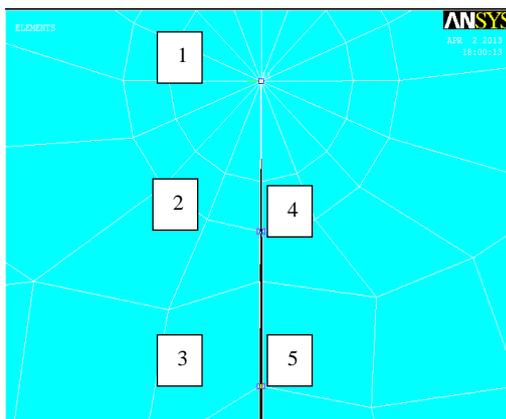


Figure 3.5 Path along the crack face

The SIF is then calculated by using “nodal calcs” command in ANSYS®. A window appears in which displacement extrapolation is changed to plain strain and model type is taken as full crack model as five nodes are selected, one on the crack tip and other four on the crack face. The stress intensity factor found for mode I is $113.66 \text{ MPa}\sqrt{\text{mm}}$.

The experimentally obtained SIF is taken from Literature [16] for comparison. It is found to be 108.14 to $114.47 \text{ MPa}\sqrt{\text{mm}}$.

IV. EFFECT OF THICKNESS ON SIF

The dependence of K_I upon thickness of specimen is given diagrammatically in Fig 4.1 [16]. Beyond a certain thickness B_s a state of plane strain prevails and toughness reaches the plane strain value K_{Ic} , K_I is independent of thickness as long as $B > B_s$. There is an optimum thickness B_0 where the toughness reaches its highest level. This level is usually considered to be the real plane stress fracture toughness. In

the transitional region between B_0 and B_s , the toughness has intermediate values.

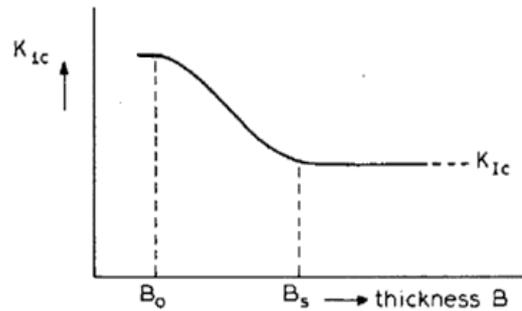


Figure 4.1 Effect of thickness on stress intensity factor [16]

For thicknesses below B_0 there is an uncertainty about the toughness [16]. The effect of thickness on SIF for three point bend specimen is found through FE and is shown in Fig. 4.2. The effect is in accordance with that given in the literature [16].

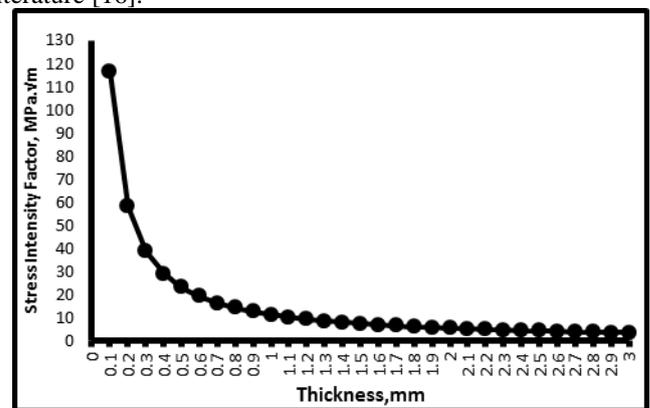


Figure 4.2 Effect of thickness on stress intensity factor for ceramic material.

V. TRACTION SEPARATION IN MULTILAYERED OBJECTS

An interface exists whenever two materials are joined together. The interface between the layers of a composite structure is of special interest, because when this type of structure is subjected to certain types of external loading, delamination takes place. The cohesive zone modeling is used to simulate interface delamination and other fracture phenomenon. This approach introduces failure mechanisms by using the hardening-softening relationships between the separations and incorporating the corresponding tractions across the interface. This technique is also well suited for modeling the fracture process in a homogenous medium, since fracture can be viewed as a separation process between two surfaces. The specimen analyzed here is the aluminium DCB bonded with epoxy adhesive as shown in Fig. 5.1

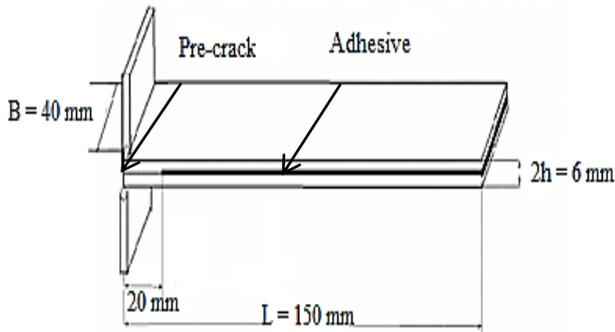


Figure 5.1 DCB Specimen configurations.

The specimen length (L) is 150 mm, width (B) is 40 mm, Thickness ($2h$) is 6 mm, tensile strength (σ_{max}) is 30 MPa, (aluminium AA 2014 T6), Young’s modulus (E) is 70 GPa, Poisson’s ratio (ν) is 0.3, epoxy adhesive - Young’s modulus (E_e) is 0.8GPa, and its poisson’s ratio (ν_e) is 0.15[17].

A. Finite Element Simulation of Fracture in DCB

The DCB specimen consists of two identical adherents (arms or legs), which are joined by an adhesive layer with a finite thickness. The FE model of DCB with CZM is shown in Fig. 5.2. One end of the beam (on right) is fixed, and the other end (on left) is loaded with an external force with the same magnitude but opposite direction. This self-equilibrated pair of forces is modeled by prescribing displacements resulting in an opening mode loading (mode-I).

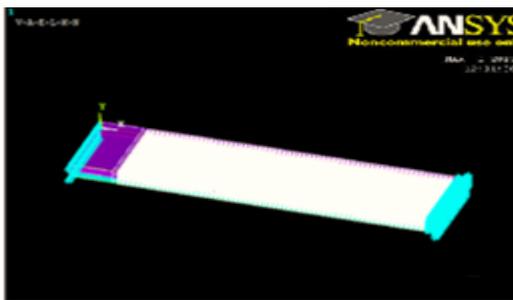


Figure 5.2 DCB modeled in ANSYS®.

The FE analysis of this DCB is carried out in ANSYS®. INTER205 interface elements are used for this purpose. The property of interface element is given by using TBDATA code. The CZM command is used for applying interface element and cohesive meshing of the interface element is done by using CZMESH command.

The nonlinear solution is performed by giving substeps of 40, for a time period of 1sec. The Von-Mises stress distribution is shown in Fig. 5.3 which shows the stresses developed in the DCB specimen at various points. The layer which is seen between two arms is the adhesive which is used for cohesive bonding.

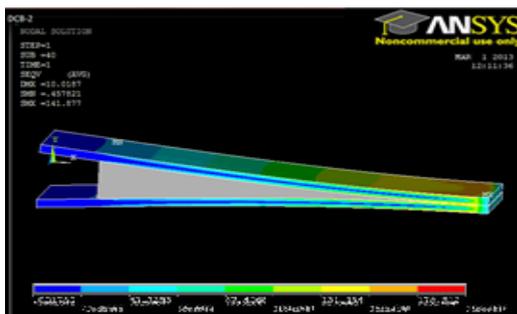


Figure 5.3 Von-Mises stress distribution.

The graph obtained between load and displacement is shown in Fig. 5.7. It is seen that the plot follows the relation of traction separation law. The critical load of the specimen is at the highest peak. Once the critical load is reached, the crack starts propagating and finally specimen breaks. To verify the results obtained through FE analysis of DCB, experiment is also carried out, which is explained in the following section.

B. Experimental Work

In order to carry out mode-I delamination study, a bonded DCB specimen made of aluminium is made as shown in Fig. 5.4.

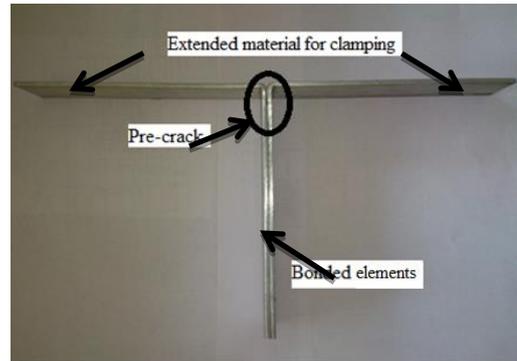


Figure 5.4 Aluminium DCB specimens.

The test specimen is made by bonding two symmetric cantilever beams. The specimen is first cut into desired size and marking is done to have a pre-crack in the specimen. The Teflon tape is used as shown in Fig. 5.5 to prevent cantilever beams from bonding so as to have pre-crack.

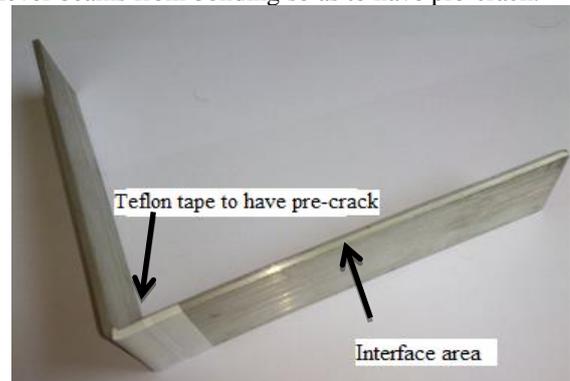


Figure 5.5 Teflon-tape coating for having pre-crack.

A thin layer of epoxy adhesive is applied over both the surfaces of the specimen and bonded together and kept for 24 hour at room temperature under a uniform nominal load.

The bonded DCB specimen is then mounted on the UTM as shown in Fig. 5.6. Load is applied till specimen fractures. Fig. 5.6 also shows the propagation of crack during loading. The load required to open the bond at various displacements is plotted as shown in Fig. 5.7. It can be seen that the plot between load and displacement follows the traction separation law.

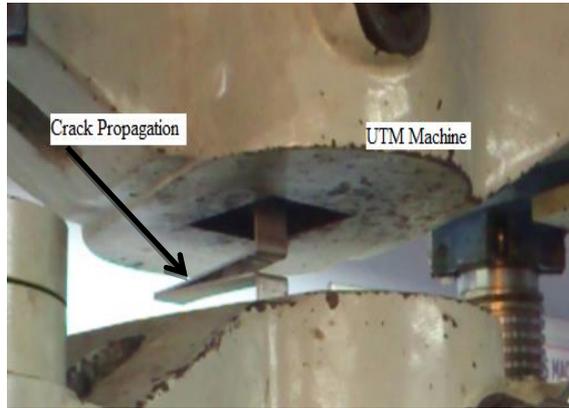


Figure 5.6. Crack propagation in DCB.

The results obtained from FE simulation and experiment, as shown in the Fig. 5.7, are in close agreement. The highest peak is the critical load that the specimen can take during loading. A small peak observed in experimental graph may be because of sudden rise of load as a result of local change of adhesive density.

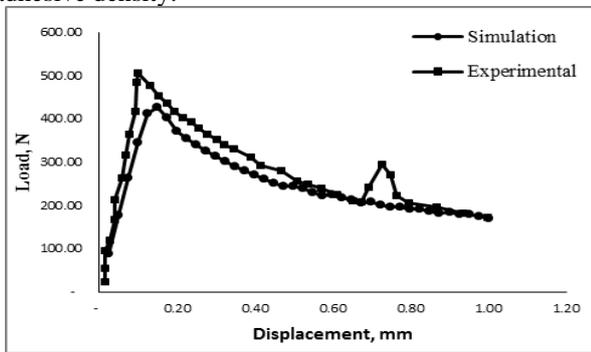


Fig 5.7 Load v/s displacement plot for experiment and simulation

VI. CONCLUSION

Finite element analysis of 2-D crack is carried out for calculating the SIF. The SIF obtained through FE simulation and by using mathematical equation are found to be in good agreement. This validates the method used for FE simulation of side crack in thin metal sheet to find SIF.

The FE simulation is then carried out for finding the SIF in case of a three point bend specimen in 3-D. The SIF obtained through FE simulation is compared with that obtained through experiment in the literature. They are found to be in good agreement, which validates the method used for FE simulation to find SIF in case of three point bend specimen.

The effect of thickness of the three point bend specimen on the SIF is studied. This shows that as thickness increases, SIF decreases and then remain at constant at K_{IC} .

The FE simulation is carried out using CZM in ANSYS® to study the traction separation in DCB. Experiment performed on a DCB shows similar load displacement plot as that obtained in FE simulation. Therefore it can be concluded that CZM can be effectively used to study traction separation in adhesively bonded structures. This may help understand inter-laminar fracture in composites and coated materials.

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