The Fundamental Results on Non-Associative Rings with Cyclic Property

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Abstract: This paper describes results on a non-associative ring \( R \) with the cyclic property: 
\((xy)z = (yz)x = (zx)y\) for all \( x, y, z \in R \) along with commutative and /or associative properties mainly.

Key words: Non-Associative ring, Cyclic Property.

I. INTRODUCTION

Schafer, Richard D how they were defined the Cyclic non-associative ring is adopted their assumption [1] and in addition to consider the assumptions of [2]. Mainly, their papers shows when \( R \) is a field or a skew field. Their work show me the way to derive some special results on their defined structures with some properties. Throughout this paper cyclic ring means a cyclic non –associative ring. This paper is organized as follows, section 1: Introduction, section 2: our contribution.

II. OUR CONTRIBUTION

Result 1:
Let \( R \) be a cyclic non-associative ring with identity such that 
\((xy)^2 = xy\), for every \( x, y \in R \). Every non-zero element in \( R \) is invertible if \( R \) satisfies the left(right) cancellation law.

Proof of this result follows by applying cancellation law twice to 
\((xy)^2 = xy \Rightarrow (xy)(xy) = xy \Rightarrow x[y(yx)] = xy\).

Note: Based on the above result, we can easily show that the cyclic non-associative ring \( R \) with identity such that \((xy)^2 = xy\), for every \( x, y \in R \) is a skew field if \( R \) satisfies the left(right) cancellation law and associative property.

Result 2:
Every cyclic commutative ring \( R \) must satisfies associative laws.

This proof follows by commutative and cyclic property respectively on \( x(yz) \) for all \( x, y, z \in R \).

Note: Based on above two results, we can prove that every cyclic commutative ring \( R \) with the properties: 
\((xy)^2 = xy\), for every \( x, y \in R \) and left(right) cancellation law is a field.

Result 3:
In any cyclic non associative (associative) ring \( R \),
\((xy)^2 = x^2y^2\), for all \( x, y \) in \( R \)
This proof follows using cyclic property on \((xy)^2\)

Result 4:
The Cyclic non associative (associative) ring \( R \) with identity \( R \) is a commutative ring.

Put \( y = y + 1 \), where \( 1 \) is the identity element in \( R \) in the \((xy)^2 = (xy)^2 \) of the results 3, and apply the distributive properties, we have \( x(xy) = (xy)x \) ...

Result 5:
Let \( R \) be a cyclic ring with identity such that \((xy)^2 = xy\), for every \( x, y \in R \). Every element in \( R \) is invertible if \( R \) satisfies the cancellation laws.

This proof follows by applying cancellation law twice to \((xy)^2 = xy \Rightarrow (xy)(xy) = xy \Rightarrow y[(xy)x] = yx\)

By applying cancellation laws (left and then right) to the above equation, we get the result

Result 6:
Any cyclic ring with identity \( R \) with the property that satisfies the condition \((xy)^2 = xy\) for any \( x, y \) in \( R \) is commutative cyclic ring.

This proof follows by applying the cyclic property on left hand side of \((xy)^2 = xy\), we get \( y^2x^2 = xy \)

put \( y = e \) in \((xy)^2 = xy\), we get \( x^2 = x \).

Next put \( x = e \) in \((xy)^2 = xy\), we get \( y^2 = y \).

From these equations, we obtain \( xy = yx \).

Note from above two results, we can easily prove that Any cyclic ring with identity \( R \) with the property: \((xy)^2 = xy\) for any \( x, y \) in \( R \) is a field.

Result 7:
A commutative cyclic ring \( R \) is an alternative ring.

This proof follows by applying first cyclic and then commutative properties to \((xy)yz = (yz)xy\)

Result 8:
In any lie ring \( R \) with cyclic property, then \((xy)z = 0\) for every \( x, y, z \in R \)
Since $R$ is a lie ring, \( (xy) = -(yx) \) for any \( x, y \) in \( R \) and \( 2(yz)x = (yz)x + (zx)y = 0 \) for all \( x, y, z \) in \( R \).

Since \( R \) has a cyclic property, so \( (xy)z = (yz)x = (zx)y \) for every \( x, y, z \) in \( R \). By cyclic property and property (2), we get \( (xy)z = 0 \)

**Result 9:**

Any commutative cyclic ring \( R \) is a Jordan Ring.

The ring \( R \) is a Jordan ring if
1. Commutative law: \( xy = yx \), for any \( x, y \) in \( R \).
2. Jordan Identity: \( (xy)^2 = x(xy^2) \), for any \( x, y \) in \( R \).

These two conditions are proved with the help of cyclic and commutative property to \( (xy)^2 \).

**Result 10:**

A Commutative Cyclic Ring Satisfies the Flexible law:
\( (x, y, x) = 0 \).

This proof follows by applying cyclic and then commutative law to right hand side of associator \( (x, y, x) = (xy) x - x (yx) \), we get \( (x, y, x) = 0 \) for any \( x, y \) in \( R \).

**Result 11:**

Every left nucleus becomes a central nucleus and right nucleus in cyclic ring \( R \).

This proofs follows like this:

since \( R \) is a left nucleus, so \( (x, R, R) = 0 \) for every \( x \) in \( R \).

By applying cyclic property to \( (R, x, R) \) and \( (R, R, x) \), we get the result.

**REFERENCES**
