The Fundamental Results on Non-Associative Rings with Cyclic Property

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Abstract: *This paper describes results on a non-associative ring R with the cyclic property: $(xy)z = (yz)x = (zx)y$ for all $x, y, z \in R$ along with commutative and /or associative properties mainly.*

Key words: *Non-Associative ring, Cyclic Property.*

I. INTRODUCTION

Schafer, Richard D how they were defined the Cyclic non-associative ring is adopted their assumption [1] and in addition to consider the assumptions of [2]. Mainly, their papers shows when R is a field or a skew field. Their work show me the way to derive some special results on their defined structures with some properties. Throughout this paper cyclic ring means a cyclic non –associative ring. This paper is organized as follows, section 1: Introduction, section 2: our contribution.

II. OUR CONTRIBUTION

Result 1:

Let R be a cyclic non-associative ring with identity such that $(xy)^2 = xy$, for every $x, y \in R$ .Every non-zero element in R is invertible if R satisfies the left(right) cancellation law.

Proof of this result follows by applying cancellation law twice to $(xy)^2 = xy \Rightarrow (xy)(xy) = xy \Rightarrow x[y(xy)] = xy$.

Note: Based on the above result , we can easily show that the cyclic non-associative ring R with identity such that $(xy)^2 = xy$, for every $x, y \in R$ is a skew field if R satisfies the left(right)cancellation law and associative property.

Result 2:

Every cyclic commutative ring R must satisfies associative laws.

This proof follows by commutative and cyclic property respectively on $x(yz)$ for all x, y and z in R .

Note: Based on above two results, we can prove that every cyclic commutative ring R with the properties: $(xy)^2 = xy$, for every $x, y \in R$ and left(right) cancellation law is a field.

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Result 3:

In any cyclic non associative (associative) ring R, $(xy)^2 = x^2y^2$, for all x, y in R

This proof follows using cyclic property on $(xy)^2$

Result 4:

The Cyclic non associative (associative) ring R with identity R is a commutative ring .

Put $y = y + 1$, where 1 is the identity element in R in the $(xy)^2 = (xy^2)x$ of the results 3, and apply the distributive properties ,we have $x(xy) = (xy)x$ …(1)

Put $x = x + 1$ in equation (1) and then apply the distributive properties, we have $x(xy) + xy = (xy)x + yx$ …(2). Substitute equation (1) in (2), we get the result.

Result 5:

Let R be a cyclic ring with identity such that $(xy)^2 = yx$, for every $x, y \in R$ .Every element in R is invertible if R satisfies the cancellation laws.

This proof follows by applying cancellation law twice to $(xy)^2 = yx \Rightarrow (xy)(xy) = yx \Rightarrow y[(xy)x] = yx$

By applying cancellation laws (left and then right) to the above equation, we get the result

Result 6:

Any cyclic ring with identity R with the property that satisfies the condition $(xy)^2 = yx$ for any x, y in R is commutative cyclic ring.

This proof follows by applying the cyclic property on left hand side of $(xy)^2 = yx$, we get $y^2x^2 = yx$

put $y = e$ in $(xy)^2 = yx$, we get $x^2 = x$.

Next put $x = e$ in $(xy)^2 = yx$, we get $y^2 = y$

From these equations , we obtain $xy = yx$.

Note from above two results ,we can easily prove that Any cyclic ring with identity R with the property : $(xy)^2 = yx$ for any x, y in R is a field.

Result 7:

A commutative cyclic ring R is an alternative ring.

This proof follows by applying first cyclic and then commutative properties to $(xx)y$ **&** $y(xx)$

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Result 8:

In any lie ring R with cyclic property, then $(xy)z = 0$ for every x, y, z in R

Since R is a lie ring ,1. $(xy) = -(yx)$ for any x, y in R & 2. $(xy)z + (yz)x + (zx)y = 0$ for all x, y, z in R .Since, R has a cyclic property, so $(xy)z = (yz)x = (zx)y$ for every x, y, z in R. By cyclic property and property (2) , we get $(xy)z = 0$

Result 9:

Any commutative cyclic ring R is a Jordan Ring.

The ring R is a Jordan ring if

1. Commutative law : $xy = yx$, for any x, y in R.
2. Jordan Identity: $(xy)\, x^2 = x\,(yx^2)$, for any x, y in R.

These two conditions are proved with the help of cyclic and commutative property to $(xy)x^2$

Result 10:

A Commutative Cyclic Ring Satisfies the Flexible law: $(x, y, x) = 0$.

This proof follows by applying cyclic and then commutative law to right hand side of associator $(x, y, x) = (xy)x - x(yx)$, we get $(x, y, x) = 0$ for any x, y in R.

Result 11:

Every left nucleus becomes a central nucleus and right nucleus in cyclic ring R

This proofs follows like this :

since R is a left nucleus , so $(x, R, R) = 0$ for every x in R

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By applying cyclic property to (R, x, R) and (R, R, x), we get the result .

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