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Some Classes of Fuzzy I – Convergent Difference Double Sequence Spaces Associated With Multiplier Sequences

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Abstract— In this article we introduce some new classes of fuzzy real-valued difference double sequence spaces associated with a multiplier sequence. We introduce double sequence spaces $_2c^{I(F)}(\Lambda,\Delta,p)$ and $_2c_0^{I(F)}(\Lambda,\Delta,p)$ where $\Lambda=(\lambda_{nk})$ a multiplier sequence of non-zero real numbers is and $p=(p_{nk})$ is a double sequence of bounded strictly positive numbers. We also make an effort to study some algebraic and topological properties of these sequence spaces Also we characterize the multiplier problem and obtain some inclusion relation involving these classes of sequences.

Index Terms— Multiplier sequence, I–convergent, Ddifference sequence spaces, Solid space, Sequence algebra, Convergence free etc.

I. INTRODUCTION

To overcome limitations induced by vagueness and uncertainty of real life data, neoclassical analysis [5] has been developed. It extends the scope and results of classical mathematical analysis by applying fuzzy logic to conventional mathematical objects, such as functions, sequences and series etc. Since the introduction of the concept of fuzzy sets by Zadeh [32] in 1965, fuzzy set theory has become an active area of research in science and engineering. The ideas of fuzzy set theory have been used widely not only in many Engineering applications, such as, Computer programming [10], Quantum physics [18], Population dynamics [2], Control of chaos [9], Bifurcation of non-linear dynamical system [13], but also in various branches of Mathematics, such as, Theory of metric and topological spaces [7], Theory of linear systems [20], Studies of convergence of sequences of functions [6,14] and Approximation theory [1]. Using the notion of fuzzy real numbers, different types of fuzzy real-valued sequence spaces have been introduced and studied by several mathematicians. The initial works on double sequences of real or complex terms are found in Bromwich [4]. Hardy [12] introduced the notion of regular convergence for double sequences of real or complex terms. Moričz [19], Basarir and Solancan [3], Tripathy and Dutta [26], Tripathy and Sarma [28] are a few to be named those who have introduced different types of double sequence spaces.

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In order to generalize the notion of convergence of real sequences, Kostyrko, Šalát and Wilczyński [17] introduced the idea of Ideal convergence (*I*-convergence) for single sequences in 2000-2001 which is an interesting and natural generalization of statistical convergence, based on the structure of the ideal *I* of the subset of the set of natural numbers. Subsequently the notion of *I*-convergence was further developed by many authors such as in [21, 22, 23, 27, 30, 31].
The scope for the studies on sequence spaces was further extended by using the notion of associated multiplier sequences. The notion of multiplier sequences was first studied by Goes and Goes [11] and later on it was followed by many workers, Some works in this direction can be found in [23, 25, 29] . Goes and Goes defined the differentiated sequence space dE and integrated sequence space $\int E$ for a given sequence space E, by using multiplier sequences (k^{-1}) and (k) respectively. For a fuzzy real valued double sequence space E^F, the multiplier sequence space $E^F(\Lambda)$, associated with the multiplier double sequence Λ is defined as $E^F(\Lambda)=\{(X_{nk}):(\lambda_{nk}X_{nk})\in E^F\}$. The notion of difference sequence spaces $c(\Delta)$, $c_0(\Delta)$ and $\ell_\infty(\Delta)$ for complex terms was introduced by Kizmaz [16] as follows: $Z(\Delta)=\{x=(x_k):(\Delta x_k)\in Z\}$, for $Z=\ell_\infty, c, c_0$, where

$$\Delta X_{nk}=X_{n,k}-X_{n,k+1}-X_{n+1,k}+X_{n+1,k+1},\ \text{for all } n,k\in N.$$

The notion was further investigated by Et and Colak [8], Tripathy [24] and many others. The notion of fuzzy real-valued double difference sequence spaces of crisp set was introduced by Tripathy and Sarma [28]. Now we give the fuzzy analogues of the same as follows: $Z(\Delta)=\{(X_{nk}):(\Delta X_{nk})\in Z\}$, for $Z={}_2\ell_\infty{}^F$, ${}_2c^F$ and ${}_2c_0{}^F$, where

$$\Delta X_{nk}=X_{n,k}-X_{n,k+1}-X_{n+1,k}+X_{n+1,k+1},\ \text{for all } n,k\in N.$$

The aim of the present paper is to introduce and investigate some classes of fuzzy real valued difference double sequences associated with a multiplier sequence of non-zero real numbers and to obtain some important results on them.

II. DEFINITIONS AND BACKGROUND

In this section we recall some notation and basic definitions which will be used in this paper.
Throughout N, R and C denote the sets of natural, real and complex numbers respectively. Let X be a non empty set.

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A non-void class $I \subseteq 2^X$ (power set of X) is called an ideal if I is additive $(i.e.\ A, B \in I \Rightarrow A \cup B \in I)$ and hereditary $(i.e.\ A \in I$ and $B \subseteq A \Rightarrow B \in I)$. A non-empty family of sets $F \subseteq 2^X$ is said to be a filter in X if $\phi \notin F$; $A, B \in F \Rightarrow A \cap B \in F$ and $A \in F,\ A \subseteq B \Rightarrow B \in F$.
For each ideal I there is a filter $F(I)$ corresponding to I, given by $F(I) = \{K \subseteq N : N \setminus K \in I\}$.
The usual convergence is a particular case of I-convergence. In this case $I = I_f$ (the ideal of all finite subsets of N).
Throughout the ideals of 2^N and $2^{N \times N}$ will be denoted by I and I_2 respectively.
A fuzzy real number X is a fuzzy set on R, i.e. a mapping $X : R \to L$ (= [0, 1]) associating each real number t with its grade of membership $X(t)$. Every real number r can be expressed as a fuzzy real number $\bar{r}$ as follows:

$$\bar{r}(t) = \begin{cases} 1 & if \quad t = r \\ 0 & otherwise \end{cases}$$

The α-level set of a fuzzy real number X, $0 < \alpha \le 1$ denoted by $[X]^\alpha$ is defined as $[X]^\alpha = \{t \in R : X(t) \ge \alpha\}$.
A fuzzy real number X is called convex if $X(t) \ge X(s) \wedge X(r)$ = min ($X(s)$, $X(r)$), where $s < t < r$. If there exists $t_0 \in R$ such that $X(t_0) = 1$, then the fuzzy real number X is called normal. A fuzzy real number X is said to be upper semi-continuous if for each $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$, for all $a \in L$ is open in the usual topology of R. The set of all upper semi continuous, normal, convex fuzzy number is denoted by $R(L)$.
Let D be the set of all closed bounded intervals $X = [X^L, X^R]$ on the real line R. Then $X \le Y$ if and only if $X^L \le Y^L$ and $X^R \le Y^R$. Also let $d(X, Y) = \max(|X^L - X^R|, |Y^L - Y^R|)$.
Then (D, d) is a complete metric space.
Let $\bar{d} : R(L) \times R(L) \to R$ be defined by $\bar{d}(X, Y) = \sup\limits_{0 \le \alpha \le 1} d([X]^\alpha, [Y]^\alpha)$, for $X, Y \in R(L)$.
Then $\bar{d}$ defines a metric on $R(L)$ and $(R(L), \bar{d})$ is a complete metric space.
A fuzzy real valued double sequence (X_{nk}) is said to be convergent in Pringsheim's sense to the fuzzy real number X, if for every $\varepsilon > 0$, there exists $n_0 = n_0(\varepsilon)$, $k_0 = k_0(\varepsilon) \in N$ such that $\bar{d}(X_{nk}, X) < \varepsilon$ for all $n \ge n_0$, $k \ge k_0$.
A fuzzy real valued double sequence (X_{nk}) is said to be I_2-convergent to the fuzzy number X_0, if for all $\varepsilon > 0$, the set $\{(n,k) \in N \times N : \bar{d}(X_{nk}, X_0) \ge \varepsilon\} \in I_2$.
We write $I_2\text{-}\lim X_{nk} = X_0$.
Throughout ${}_2w^F$, ${}_2\ell_\infty^F$, ${}_2c^F$, ${}_2c_0^F$, ${}_2c^{I(F)}$, ${}_2c_0^{I(F)}$, ${}_2\ell_\infty^I$ denote the spaces of all, bounded, convergent in Pringsheim's sense, null in Pringsheim's sense, I-convergent I-null fuzzy and I-bounded real valued double sequences respectively.

A fuzzy real valued double sequence space E^F is said to be solid if $(Y_{nk}) \in E^F$, whenever $|Y_{nk}| \le |X_{nk}|$ for all $n, k \in N$ and $(X_{nk}) \in E^F$.
A fuzzy real valued double sequence space E^F is said to be monotone if E^F contains the canonical pre-image of all its step spaces.
A fuzzy real valued double sequence E^F is said to be symmetric if $S(X) \subset E^F$, for all $X \in E^F$, where $S(X)$ denotes the set of all permutations of the elements of $X = (X_{nk})$.
A fuzzy real valued double sequence space E^F is said to be sequence algebra if $(X_{nk} \otimes Y_{nk}) \in E^F$, whenever (X_{nk}), $(Y_{nk}) \in E^F$.
A fuzzy real valued double sequence space E^F is said to be convergence free if $(Y_{nk}) \in E^F$, whenever $(X_{nk}) \in E^F$ and $X_{nk} = \bar{0}$ implies $Y_{nk} = \bar{0}$.
A multiplier from a fuzzy real valued double sequence space D^F into another fuzzy real valued double sequence space E^F is a real sequence $u = (u_{nk})$ such that $uX = (u_{nk}X_{nk}) \in E^F$ whenever $X = (X_{nk}) \in D^F$ The linear space of all such multipliers will be denoted by $m(D^F, E^F)$. Bounded multipliers will be denoted by $M(D^F, E^F)$.
Hence $M(D^F, E^F) = {}_2\ell_\infty^F \cap m(D^F, E^F)$.
Let $\Lambda = (\lambda_{nk})$ be a multiplier sequence and $p = (p_{nk})$ be a double sequence of bounded strictly positive numbers. Then the following classes of sequences are introduced:

$${}_2c^F(\Lambda, \Delta, p) = \{X = (X_{nk}) : \lim [\bar{d}(\lambda_{nk}\Delta X_{nk}, X_0)]^{p_{nk}} = 0, \text{ for some } X_0 \in R(L)\},$$

$${}_2c^{I(F)}(\Lambda, \Delta, p) = \{X = (X_{nk}) : I_2\text{-}\lim [\bar{d}(\lambda_{nk}\Delta X_{nk}, X_0)]^{p_{nk}} = 0, \text{ for some } X_0 \in R(L)\},$$

$${}_2c_0^{I(F)}(\Lambda, \Delta, p) = \{X = (X_{nk}) : I_2\text{-}\lim [\bar{d}(\lambda_{nk}\Delta X_{nk}, \bar{0})]^{p_{nk}} = 0\},$$

$${}_2\ell_\infty^F(\Lambda, \Delta, p) = \{X = (X_{nk}) : \sup_{n,k} [\bar{d}(\lambda_{nk}\Delta X_{nk}, \bar{0})]^{p_{nk}} < \infty\}.$$

Also we define

$${}_2m^{I(F)}(\Lambda, \Delta, p) = {}_2c^{I(F)}(\Lambda, \Delta, p) \cap {}_2\ell_\infty^F(\Lambda, \Delta, p)$$

and

$${}_2m_0^{I(F)}(\Lambda, \Delta, p) = {}_2c_0^{I(F)}(\Lambda, \Delta, p) \cap {}_2\ell_\infty^F(\Lambda, \Delta, p).$$

Let (X_{nk}) and (Y_{nk}) be two fuzzy real valued double sequences. Then we say that $X_{nk} = Y_{nk}$ for almost all n and k relative to I_2 (in short *a.a.n* & *k r* I_2) if the set $\{(n,k) \in N \times N : X_{nk} \ne Y_{nk}\} \in I_2$.

Lemma 1. *If a sequence space E^F is solid, then it is monotone.*
For the crisp set case, one may refer to Kamthan and Gupta [15], p.53.

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Lemma 2. *If a fuzzy real valued double sequence space E^F is bounded and solid, then* $(\lambda_{nk}) \in M(E^F, E^F)$ *if and only if* $(\lambda_{nk}) \in {}_2\ell_\infty$.

III. MAIN RESULTS

Theorem 1. *Let $p = (p_{nk})$ be a double sequence of bounded strictly positive real numbers. If $\Lambda = (\lambda_{nk})$ is a given multiplier sequence, then the classes of sequences ${}_2c^{I(F)}(\Lambda, \Delta, p)$ and ${}_2c_0^{I(F)}(\Lambda, \Delta, p)$ are closed under the operations of addition and scalar multiplication.*

Proof. Following standard techniques, one can easily prove the result.

Theorem 2. *Let the double sequence $p = (p_{nk})$ be bounded. Then ${}_2m_0^{I(F)}(\Lambda, \Delta, p) \subset {}_2m^{I(F)}(\Lambda, \Delta, p) \subset {}_2\ell_\infty^{I(F)}(\Lambda, \Delta, p)$ and the inclusions are strict.*

Proof. The inclusion ${}_2m_0^{I(F)}(\Lambda, \Delta, p) \subset {}_2m^{I(F)}(\Lambda, \Delta, p) \subset {}_2\ell_\infty^{I(F)}(\Lambda, \Delta, p)$ is obvious.

In order to show that the inclusion ${}_2m^{I(F)}(\Lambda, \Delta, p) \subset {}_2\ell_\infty^{I(F)}(\Lambda, \Delta, p)$ is strict, we consider the following example.

Example 1. Let $A \in I_2$, and $p_{nk} = \begin{cases} \frac{1}{2}, & \text{if } (n,k) \in A \\ 2, & \text{otherwise} \end{cases}$

We consider the sequence (X_{nk}) defined by:

For all $(n,k) \notin A$, $X_{nk} = \bar{1}$.

For all $(n,k) \notin A$ and $(n+k)$ even

$$X_{nk}(t) = \begin{cases} \frac{nt - n + 1}{n+1}, & \text{for } 1 - n^{-1} \le t \le 2 \\ 3 - t, & \text{for } 2 < t \le 3 \\ 0, & \text{otherwise} \end{cases}$$

otherwise $$X_{nk}(t) = \begin{cases} \frac{nt - 1}{n-1}, & \text{for } n^{-1} \le t \le 1 \\ 2 - t, & \text{for } 1 < t \le 2 \\ 0, & \text{otherwise} \end{cases}$$

Then taking $\lambda_{nk} = \frac{1}{n+k}$, for all $n, k \in N$, we have $(X_{nk}) \in {}_2\ell_\infty^{I(F)}(\Lambda, \Delta, p)$, but $(X_{nk}) \notin {}_2m^{I(F)}(\Lambda, \Delta, p)$.

Hence the inclusions are strict.

Theorem 3. *Let $\sup_{n,k} p_{nk} < \infty$. Then the following statements are equivalent:*

(i) $(X_{nk}) \in {}_2c^{I(F)}(\Lambda, \Delta, p)$.

(ii) *There exists a sequence $(Y_{nk}) \in {}_2c^F(\Lambda, \Delta, p)$ such that $X_{nk} = Y_{nk}$ for a.a.n & k r. I_2.*

(iii) *There exists a subset $M = \{(n_i, k_j) \in N \times N : i, j \in N\}$ of $N \times N$ such that $M \in F(I_2)$ and $(X_{n_i k_j}) \in {}_2c^F(\Lambda, \Delta, p)$.*

Proof. (i) $\Rightarrow$ (ii) Let $(X_{nk}) \in {}_2c^{I(F)}(\Lambda, \Delta, p)$. Then there exists $X_0 \in R(L)$ such that

$$I_2\text{-}\lim\,[\bar{d}(\lambda_{nk}\Delta X_{nk}, X_0)]^{p_{nk}} = 0.$$

So for any $\varepsilon > 0$, we have the set $\{(n,k) \in N \times N : [\bar{d}(\lambda_{nk}\Delta X_{nk}, X_0)]^{p_{nk}} \ge \varepsilon\} \in I_2$.

Let us consider the increasing sequences (T_j) and (M_j) of natural numbers such that if $p > T_j$ and $q > M_j$, then the set

$$\left\{(n,k) \in N \times N : n \le p,\ k \le q \text{ and } [\bar{d}(\lambda_{nk}\Delta X_{nk}, X_0)]^{p_{nk}} \ge \frac{1}{j}\right\} \in I_2.$$

We define the sequence (Y_{nk}) as follows:

$Y_{nk} = X_{nk}$ if $n \le T_1$ or $k \le M_1$.

Also for all (n,k) with $T_j < n \le T_{j+1}$ or $M_j < k \le M_{j+1}$,

let $Y_{nk} = X_{nk}$ if $[\bar{d}(\lambda_{nk}\Delta X_{nk}, X_0)]^{p_{nk}} < \frac{1}{j}$, otherwise $Y_{nk} = \lambda_{nk}{}^{-1} X_0$.

We show that $(Y_{nk}) \in {}_2c^F(\Lambda, \Delta, p)$. Let $\varepsilon > 0$. We choose j such that $\varepsilon > \frac{1}{j}$. We see that for $n > T_j$ and $k > M_j$,

$[\bar{d}(\lambda_{nk}\Delta X_{nk}, X_0)]^{p_{nk}} < \varepsilon$.

Hence $(Y_{nk}) \in {}_2c^F(\Lambda, \Delta, p)$.

We see that for $n > T_j$ and $k > M_j$, $(\bar{d}(\lambda_{nk}\Delta Y_{nk}, X_0))^{p_{nk}} < \varepsilon$. Hence $(Y_{nk}) \in {}_2c^F(\Lambda, \Delta, p)$.

Next we assume that $T_j < n \le T_{j+1}$ and $M_j < k \le M_{j+1}$, then the set

$$A = \{(n,k) \in N \times N : X_{nk} \ne Y_{nk}\} \subseteq \left\{(n,k) \in N \times N : [\bar{d}(\lambda_{nk}\Delta X_{nk}, X_0)]^{p_{nk}} \ge \frac{1}{j}\right\} \in I_2$$

Hence $A \in I_2$ and so $X_{nk} = Y_{nk}$ for a.a.n & k r. I_2.

(ii) $\Rightarrow$ (iii) Suppose there exists a sequence $(Y_{nk}) \in {}_2c^F(\Lambda, \Delta, p)$ such that $X_{nk} = Y_{nk}$ for a.a.n & k r. I_2. Let $M = \{(n,k) \in N \times N : X_{nk} = Y_{nk}\}$. Then $M \in F(I_2)$.

We can enumerate M as

$M = \{(n_i, k_j) \in N \times N : i, j \in N\}$, on neglecting the rows and columns those contain finite number of elements.

Then $(X_{n_i k_j}) \in {}_2c^{I(F)}(\Lambda, \Delta, p)$.

(iii) $\Rightarrow$ (i) is obvious. This completes the proof of the theorem.

The following result can be proved easily using standard technique.

Theorem 4. *If $H = \sup_{n,k} p_{nk} < \infty$, then the classes of sequences ${}_2m^{I(F)}(\Lambda, \Delta, p)$ and ${}_2m_0^{I(F)}(\Lambda, \Delta, p)$ are complete metric spaces with respect to the metric ρ defined by*

$$\rho(X, Y) = \sup_k \bar{d}(\lambda_{1k}X_{1k}, \lambda_{1k}Y_{1k}) + \sup_n \bar{d}(\lambda_{n1}X_{n1}, \lambda_{n1}Y_{n1}) +$$

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$$\sup_{n,k}\left(\bar{d}(\lambda_{nk}\Delta X_{nk},\lambda_{nk}\Delta Y_{nk})\right)^{\frac{p_{nk}}{M}}$$

where $M = \max(1, H)$ *and* $X = (X_{nk})$, $Y = (Y_{nk})$ *are in* ${}_2m^{I(F)}(\Lambda,\Delta,p)$ *and* ${}_2m_0^{I(F)}(\Lambda,\Delta,p)$.

Theorem 4. *The class of sequences* ${}_2m_0^{I(F)}(\Lambda,\Delta,p)$ *is solid as well as monotone*.

Proof. Let $(X_{nk}) \in {}_2m_0^{I(F)}(\Lambda,\Delta,p)$ and (Y_{nk}) be such that $\bar{d}(Y_{nk},\bar{0}) \le \bar{d}(X_{nk},\bar{0})$, for all $n,k \in N$.

Let $\varepsilon > 0$ be given. Then the solidness of ${}_2m_0^{I(F)}(\Lambda,\Delta,p)$ follows from the following relation:

$$\{(n,k)\in N\times N : [\bar{d}(\lambda_{nk}\Delta X_{nk},\bar{0})]^{p_{nk}} \ge \varepsilon\} \supseteq$$
$$\{(n,k)\in N\times N : [\bar{d}(\lambda_{nk}\Delta Y_{nk},\bar{0})]^{p_{nk}} \ge \varepsilon\}.$$

Also by Lemma 1, it follows that the space ${}_2m_0^{I(F)}(\Lambda,\Delta,p)$ is monotone

Theorem 5. *The class of sequences* ${}_2m^{I(F)}(\Lambda,\Delta,p)$ *is neither solid nor monotone in general.*

Proof. The result follows from the following example.

Example 2. Let $A \in I_2$, $p_{nk} = \begin{cases} 2, & \text{if } (n,k)\in A \\ 1, & \text{otherwise}\end{cases}$

We consider the sequence (X_{nk}) defined by:

For all $(n,k) \notin A$,

$$X_{nk}(t) = \begin{cases} 1-(n+k)+\dfrac{t}{n+k}, & \textit{for} \quad (n+k-1)(n+k) \le t \le (n+k)^2 \\ 1+(n+k)-\dfrac{t}{n+k}, & \textit{for} \quad (n+k)^2 \le t \le (n+k+1)(n+k) \\ 0, & \textit{otherwise}\end{cases}$$

otherwise $X_{nk} = \overline{(n+k)^2}$.

Then taking $\lambda_{nk} = \dfrac{1}{(n+k)^2}$, for all $n,k \in N$, we have $(X_{nk}) \in {}_2m^{I(F)}(\Lambda,\Delta,p)$.

Let $K = \{2i : i \in N\}$.

We consider the sequence (Y_{nk}) defined by:

$$Y_{nk} = \begin{cases} X_{nk}, & \text{if } (n,k)\in K \\ \bar{0}, & \text{otherwise}\end{cases}$$

Then (Y_{nk}) belongs to the canonical pre-image of K step space of ${}_2m^{I(F)}(\Lambda,\Delta,p)$.

Bu $(Y_{nk}) \notin {}_2m^{I(F)}(\Lambda,\Delta,p)$. Hence the class of sequences ${}_2m^{I(F)}(\Lambda,\Delta,p)$ is not monotone. Therefore by the class of sequences ${}_2m^{I(F)}(\Lambda,\Delta,p)$ is not solid.

Theorem 6. *The classes of sequences* ${}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0^{I(F)}(\Lambda,\Delta,p)$ *are not symmetric in general.*

Proof. The result follows from the following example.

Example 3. Let $I_2 = I_2(\rho)$,

$$p_{nk} = \begin{cases} 1, & \text{for } n \text{ even and all } k \in N \\ 2, & \text{otherwise}\end{cases}$$

We consider the sequence (X_{nk}) defined by:

For $n = i^2$, $i \in N$ and for all $k \in N$,

$$X_{nk}(t) = \begin{cases} 1+\dfrac{t}{\sqrt{n}-1}, & \text{for} \quad 1-\sqrt{n} \le t \le 0 \\ 1-\dfrac{t}{\sqrt{n}-1}, & \text{for} \quad 0 < t \le \sqrt{n}-1 \\ 0, & \text{otherwise}\end{cases}$$

otherwise $X_{nk} = \bar{0}$.

Then taking $\lambda_{nk} = \dfrac{1}{n}$, for all $n,k \in N$, we have $(X_{nk}) \in {}_2m_0^{I(F)}(\Lambda,\Delta,p)$, ${}_2m^{I(F)}(\Lambda,\Delta,p)$.

We consider the rearrangement (Y_{nk}) of (X_{nk}) defined by:

For k odd and for all $n \in N$,

$$Y_{nk}(t) = \begin{cases} 1+\dfrac{t}{n-1}, & \text{for} \quad 1-n \le t \le 0 \\ 1-\dfrac{t}{n-1}, & \text{for} \quad 0 < t \le n-1 \\ 0, & \text{otherwise}\end{cases}$$

otherwise $Y_{nk} = \bar{0}$.

Then $(Y_{nk}) \notin {}_2m_0^{I(F)}(\Lambda,\Delta,p)$, ${}_2m^{I(F)}(\Lambda,\Delta,p)$. Hence the classes of sequences ${}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0^{I(F)}(\Lambda,\Delta,p)$ are not symmetric in general.

Theorem 7. *The classes of sequences* ${}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0^{I(F)}(\Lambda,\Delta,p)$ *are not sequence algebras in general.*

Proof. The result follows from the following example.

Example 4. Let $A \in I_2$, $p_{nk} = \begin{cases} \dfrac{1}{2}, & \text{if } (n,k)\in A \\ 1, & \text{otherwise}\end{cases}$

We consider the sequences (X_{nk}) and (Y_{nk}) defined as follows:

For all $(n,k) \notin A$,

$$X_{nk}(t) = \begin{cases} 1+\dfrac{t}{(3n+k)^2}, & \text{for} \quad -(3n+k)^2 \le t \le 0 \\ 1-\dfrac{t}{(3n+k)^2}, & \text{for} \quad 0 < t \le (3n+k)^2 \\ 0, & \text{otherwise}\end{cases}$$

otherwise $X_{nk} = \bar{0}$.

For all $(n,k) \notin A$,

$$Y_{nk}(t) = \begin{cases} 1+\dfrac{t-1}{(3n+k)^2}, & \text{for} \quad 1-(3n+k)^2 \le t \le 1 \\ 1-\dfrac{t-1}{(3n+k)^2}, & \text{for} \quad 1 < t \le 1+(3n+k)^2 \\ 0, & \text{otherwise}\end{cases}$$

otherwise $Y_{nk} = \bar{0}$.

Then taking $\lambda_{nk} = \dfrac{1}{(3n+k)^2}$, for all $n,k \in N$, we have $(X_{nk}), (Y_{nk}) \in Z$, for

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$Z = {}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$.

But $(X_{nk} \otimes Y_{nk}) \notin Z$, for $Z = {}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$.

Hence the classes of sequences ${}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$ are not sequence algebras.

Theorem 8. *The classes of sequences* ${}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$ *are not convergence free.*

Proof . The result follows from the following example.

Example 5. Let $A \in I_2$, $p_{nk} = \begin{cases} \frac{1}{2}, & \text{if } (n,k) \in A \\ 2, & \text{otherwise} \end{cases}$

We consider the sequence (X_{nk}) defined by:

For all $(n,k) \notin A$,

$$X_{nk}(t) = \begin{cases} 1 + (n+k)(t-1), & \text{for } 1 - \frac{1}{(n+k)} \le t \le 1 \\ 1 - (n+k)(t-1), & \text{for } 1 < t \le 1 + \frac{1}{(n+k)} \\ 0, & \text{otherwise} \end{cases}$$

otherwise $X_{nk} = \bar{0}$.

Then taking $\lambda_{nk} = \dfrac{1}{n+k}$, for all $n,k \in N$, we have $(X_{nk}) \in Z$, , for $Z = {}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$.

We consider the sequence (Y_{nk}) defined by:

For all $(n,k) \notin A$,

$$Y_{nk}(t) = \begin{cases} 1 + \frac{t-1}{(n+k)^3}, & \text{for } 1-(n+k)^3 \le t \le 1 \\ 1 - \frac{t-1}{(n+k)^3}, & \text{for } 1 < t \le 1+(n+k)^3 \\ 0, & \text{otherwise} \end{cases}$$

otherwise $Y_{nk} = \bar{0}$.

But $(Y_{nk}) \notin Z$, for $Z = {}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$.

Hence the classes of sequences ${}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$ not convergence free.

Theorem 9. *The classes of sequences* ${}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$ *are nowhere dense subsets of* ${}_2\ell_\infty^F(\Lambda,\Delta,p)$.

Proof. By Theorem 2, the sequence spaces ${}_2m^{I(F)}(\Lambda,\Delta,p)$ and ${}_2m_0{}^{I(F)}(\Lambda,\Delta,p)$ are proper subspaces of ${}_2\ell_\infty^F(\Lambda,\Delta,p)$.

Hence the result follows from Theorem 4.

Theorem 10. $(\lambda_{nk}) \in M\left({}_2c_0^{I(F)}(\Lambda,\Delta,p),\ {}_2c_0^{I(F)}(\Lambda,\Delta,p)\right)$ *if and only if* $(\lambda_{nk}) \in {}_2\ell_\infty^I(\Lambda,\Delta,p)$.

Proof. Let $(\lambda_{nk}) \in {}_2\ell_\infty^I(\Lambda,\Delta,p)$ and $(X_{nk}) \in {}_2c_0^{I(F)}(\Lambda,\Delta,p)$.

Then there exists a $J > 0$ such that

$$P = \{(n,k) \in N \times N : |\lambda_{nk}|^{p_{nk}} \le J\} \in F(I_2)$$

and $$Q = \left\{(n,k) \in N \times N : [\bar{d}(\Delta X_{nk}, \bar{0})]^{p_{nk}} < \frac{\varepsilon}{J}\right\} \in F(I_2).$$

Then $P \cap Q = \{(n,k) \in N \times N : [\bar{d}(\lambda_{nk}\Delta X_{nk}, \bar{0})]^{p_{nk}} < \varepsilon\} \in F(I_2)$.

Hence $(\lambda_{nk}\Delta X_{nk}) \in {}_2c_0^{I(F)}(\Lambda,\Delta,p)$ and so $(\lambda_{nk}) \in M\left({}_2c_0^{I(F)}(\Lambda,\Delta,p),\ {}_2c_0^{I(F)}(\Lambda,\Delta,p)\right)$

The converse part is easy, so omitted.

Theorem 11. *If the class of sequences* ${}_2c^{I(F)}(\Lambda,\Delta,p)$ *is not solid, then* $(\lambda_{nk}) \notin M\left({}_2c^{I(F)}(\Lambda,\Delta,p),\ {}_2c^{I(F)}(\Lambda,\Delta,p)\right)$.

Proof. The result follows from Lemma 2.

IV. CONCLUSION

In this paper, we have introduced some classes of fuzzy real valued difference double sequences with a multiplier sequence of non-zero real numbers . We have studied several properties like solidness, symmetry etc. of these sequence spaces. We have proved some inclusion results involving these classes of sequences.

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