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Performance Analysis of Tikhonov Distributed Phase Error over Wireless Fading Channels

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Abstract— ***There is need to formulate an accurate and thoroughly reproducible error model for wireless mobile channels in order to enhance the quality of communication by using better modulation and coding schemes. However the various physical properties of propagation mode, distortion due to error in physical wireless medium and the synchronization mismatch between transmitter and receiver, leads to difficulty in modelling of the error performance of wireless channels. Phase shift keying (PSK) is one of the best modulation schemes for wireless applications mainly due to its bandwidth efficiency and constant envelope. Inspite of these advantages, PSK systems are prone to phase synchronization error which becomes even more vital issue in wireless systems as calculating correct phase over a random propagation medium is almost impossible. As a result, in addition with AWGN and fading, the synchronization mismatch of the phase between the transmitter and receiver evaluates the error performance of a wireless system. This paper examines the problem of wrong phase evaluation for the BPSK as well as for the case of general MPSK signals over Rayleigh, Nakagami-n (Rician), Nakagami-m and Nakagami-q (Hoyt) fading channels. The phase distortions are assumed to be random, unbiased, i.e. having zero-mean and may be represented by a Tikhonov distribution. The major contributions of related works were surveyed and the method that requires minimum mathematical operations (and thus proves to be less complex, more stable and accurate than others) is also explained. Apart from this, simple alternative approaches for obtaining analytical bit error rate (BER) for BPSK and symbol error rate (SER) through moment generating function (MGF) for Tikhonov distributed phase error have been proposed.***

The MGF methodology has wider applicability, is able to obtain reproducible results, and shows significant improvement in accuracy regarding theoretical BER calculation as seen from the graphical comparisons. Extensive Monte Carlo simulations that builds models of possible results by substituting a range of values were also performed to validate the theoretical results presented in the research paper

Index Terms— ***bit error rate, montecarlo simulation, synchronization, and Tikhonov distribution***

I. INTRODUCTION

In past few years we have seen high growth rates in wireless communication and mobile systems due to the various vitial factors: advancement in microelectronics, high speed networking, positive user response and an encouraging climate worldwide. In order to produce high data rates signal in case of wireless communication the total number of cells, microcells should be increased and frequency reuse should also be maximized. But the allocated area and cell spectrum is restricted. This causes increase in rate of interference of spectrum, crosstalk and degradation of wireless signals.

So the basic need for the communication system is to achieve high data rates with limited spectrum bandwidth and to enhance the performance of signals by using various modulation schemes

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The parameters are amplitude, frequency and phase, any one of these parameters if not copied perfectly at the receiver the signal is distorted. PSK systems suffer most by the phase synchronization mismatch problem between transmitter and receiver . apart from these synchronization errors, the wireless channels are subjected to random multipath fading. So the next challenging issue is that how multipath fading affects the degradation of the *symbol error rate* of the modulated signal in presence of phase error.

II. RESEARCH METHOD

A.Research Works On Bpsk

The analysis of performance of PSK systems in presence of imperfect carrier phase recovery has been studied in the last five decades. At start the interest was ranged to error probability analysis in pure AWGN channels. In 1960, Viterbi was first to give a statistical model for the carrier phase error of a first-order PLL and Lindsey in 1966, gave the performance of BPSK with phase error and AWGN. After that a usable but approximated formula was provided by Kam et al. In above all works carrier recovery through a first order PLL was taken, while Tikhonov distribution was used to describe the statistical modelling of phase error. Because Tikhonov distribution objected as the dominant statistical model to characterize the random phase error due to its effectiveness in phase error modelling of first (and to some extent second) order PLL. Taking into considersation the argument above, the research work till date may be divided into the following three categories, performance analysis of BPSK:

(i) with Tikhonov distributed phase error and AWGN

(ii) with Tikhonov distributed phase error and fading (where $\alpha = K\gamma$)

(iii) with Tikhonov distributed phase error and fading (where $\alpha \neq K\gamma$)

The research work may be found in the recent literature discussing SER performance of PSK constellations. In all these papers perfect carrier phase recovery and symbol timing synchronization have been considered. However, phase and timing synchronization errors can have a significant effect on the receiver's capabil make the correct decisions. The problem gets more. The various research methodologies are:

i.**Direct Integration**

This is the direct numerical integration or may also be called as PDF method of solving the BER values. A direct calculation is generally not opted due to greater computation time and numerical complexity. For general MPSK systems the CEP is stated by an integral equation which requires calculation of a triple integral, which is not supported by most commercial mathematical packages.

ii. Percentage of error calculation

In this paper, we have calculated the BER values through numerical integration wherever possible. These values provided us with a reference points and are used to calculate the percentage of error, given by

$$\text{Percentage of error} = \frac{[P_{e,Th} - P_{e,NI}]}{P_{e,NI}}$$

Where $P_{e,Th}$ is theoretical error probability obtained by any approach and $P_{e,NI}$ is error probability obtained by direct integration approach

iii.Simulation methodology

Besides using the direct numerical integration Monte Carlo simulations were also performed to assess the accuracy of different methods. Monte Carlo is a stochastic simulation process that is used here to estimate the BER by counting the error bits at the receiver and then dividing the count by the total number of bits passed through the system. The number of bits examined at a SNR point is at least 10 times higher than the inverse of the expected error rate, i.e. to test a BER of 10^{-4}, 10^{5} bits were examined.

III. RESULT ANALYSIS

Besides using the direct numerical integration Monte Carlo simulations were also performed to assess the accuracy of different methods. Monte Carlo is a stochastic simulation process that is used here to estimate the BER by counting the error bits at the receiver and then dividing the count by the total number of bits passed through the system. The number of bits examined at a SNR point is at least 10 times higher than the inverse of the expected error rate, i.e. to test a BER of 10^{-4}, 10^{5} bits were examined. The simulation model was generated through Matlab and its various component blocks are described in Figure 1. The first block randomly generates binary digits with equal probability. Next the generated bits are modulated through a BPSK modulator. The simulations were performed in the baseband level and thus the modulator does not involve any frequency translation. The modulated signal is then fed to the channel where three system imperfections, fading, AWGN, and phase error, are introduced. Accordingly, the simulation process involves generation of three different kinds of random variables. Gaussian noise may be realized rather easily with an in-built function.

For arbitrary values of *m*, the RV may be generated by taking square root of samples of a gamma distributed RV. The root mean square value of the generated fading RV is normalized to 1. Finally, to realize phase error we need to generate either Gaussian or Tikhonov distributed random samples. Generation of a Tikhonov distributed RV is, by far, the most difficult task encountered in the simulation process and details of the algorithm used are given later in this section. The output of the channel is passed through a baseband demodulator and its output is compared with the original bits. Subsequently the BER values are computed.

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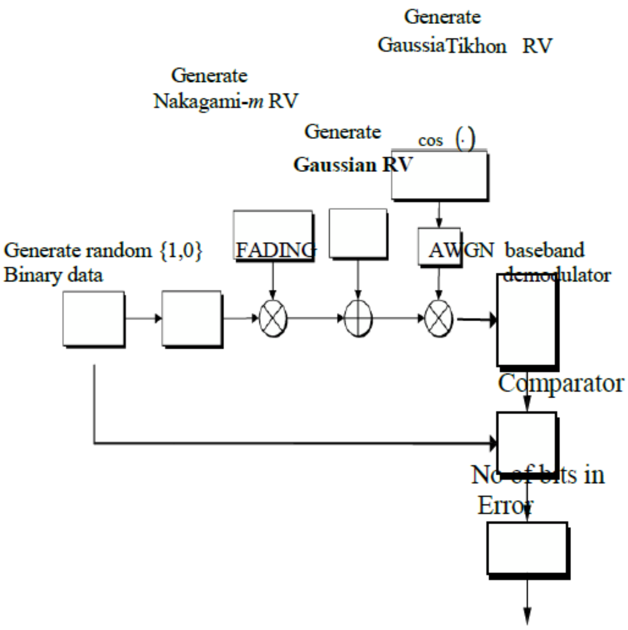
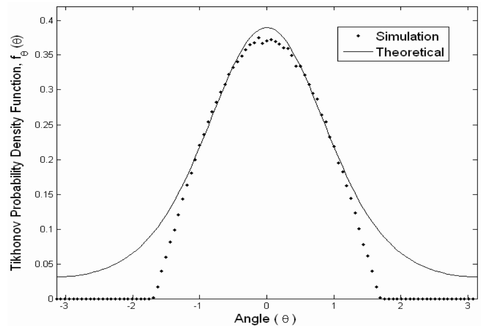
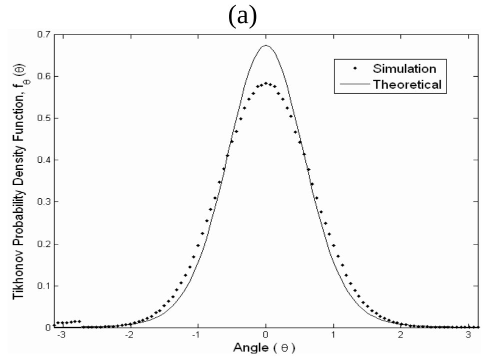


Figure 1 Simulation model for BER evaluation of BPSK with phase error (Tikhonov/ Gaussian), AWGN and Nakagami-*m* distributed fading using Monte Carlo method.

The first method for Tikhonov RV generation was proposed by Mardia while a faster algorithm was developed by Best and Fisher , with different methods given later by Dagpunar , Barabesi , Wood , Yuan and Kalbeisch , Devroye , and De Abreu .



(a)



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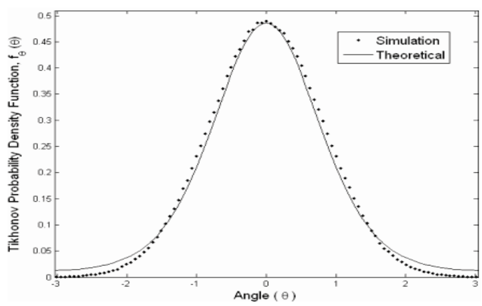

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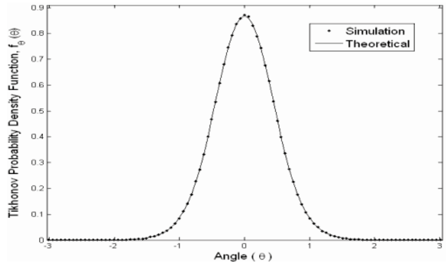

(d)

Figure 2 Comparison of the theoretical Tikhonov PDF with simulated PDFs obtained through four different methods. (a) Best-Fisher's Method , (b) De Abreu's Method , (c) Mardia's Method , and (d) Luc Devroye's Method

To generate Tikhonov distributed RV we have obtained for four methods and the corresponding graphs are shown in Figure 2.2 (a), (b), (c), and (d). It is shown from the graphs, generated for Best and Fisher's method and De Abreu's method , that the simulated PDFs clearly deviate from the theoretical PDF, in the tail and mid ranges respectively. The simulated PDF generated through Mardia's method follows the theoretical plot more or less, but it has been seen that this similarity is continued only for small α values (α< 3Db).The best result was generated when we followed the method given by Devroye as seen from Figure 5.2 (d).

Further, the similarity is maintained for almost all values of α within the range of interest. The simulation algorithm of Devroye's method is as follows

1. generate X ~ Bessel(0,α)
2. generate B ~ Beta(X + $\frac{1}{2}$, $\frac{1}{2}$)
3. generate S , arandon sign.
4. generate U ~ Uniform[0,1]
5. If U < $\frac{1}{1+\exp(-2\alpha\sqrt{B})}$
6. then
   Return θ=arcos($\sqrt{B}$)
7. else
   Return θ=arcos($-\sqrt{B}$)

Figure 3 Generator algorithm for the Tikhonov/ von Mises distribution.

The only complexity with this algorithm is that it uses a Bessel and a beta random variate. Simulating Bessel distribution is generally not easy. Devroye gave a truncated normal approximation for Bessel when the mean is large, and a table sampling procedure when the mean is small. However generation of a beta RV is easy.

IV. CONCLUSION

In this paper we have provided a comprehensive survey of BER for BPSK operating over AWGN and Nakagami-*m* fading channel, when distorted with erroneous phase estimate which is Tikhonov distributed .

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A graphical comparison of BER curves for BPSK suffering from phase synchronization problem is displayed. The plots obtained clearly indicates the degradation in error performance due to phase noise and small scale fading. Besides from surveying the existing methodologies, we also introduced another method based on MGF for evaluating BER of BPSK and SER of MPSK over different fading channels for Tikhonov distributed phase error. Numerical calculation of the derived BER expressions shows great extent of accuracy when compared with the values obtained through direct integration. Extensive simulations were also performed to authenticate the theoretical results. A graphical comparison of SER curves of MPSK for different *M* values is depicted which shows that numerical integration fails to follow the simulation for higher *M* values whereas MGF method still gives accurate results. In short, the MGF method performs more better, not only in terms of accuracy, but also with respect to computation time and applicability.

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ACHIEVEMENTS:

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- Stood Second in B.TECH first year
- Stood third in B.TECH second year
- Got Second pirze in MODEL DISPLAY at college level.
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