

# Dynamic Simulation of Robust Sensorless Speed Measurement in IM Using MRAC against Variations in Stator Resistance and Rotor-Time Constant

Srikanth Mandarapu, Sreedhar Lolla, Madhu Chandra Popuri

**Abstract**—This paper proposes a Model Reference Adaptive Control (MRAC) which makes use of both back EMF and Reactive Power methods for sensorless rotor speed measurement against variations in rotor time constant in addition to the variations in stator resistance. The adjustable model makes use of these two methodologies eliminates the need to calculate rotor flux, thus avoiding the requirement of synchronously rotating reference angle and the stable speed can also be estimated even at low speeds. The effectiveness of the proposed model is proved with extensive computer simulation.

**Index Terms**—back emf method, reactive power method, Rotor Speed, Sensorless Control, Stator Resistance.

### List of symbols

$B$	Viscous friction coefficient
$J$	Moment of inertia (N-m/rad/sec <sup>2</sup> )
$L_m$	Magnetizing inductance (H)
$L_r$	Rotor self-leakage inductance (H)
$L_{rl}$	Rotor leakage inductance (H)
$L_s$	Stator self-leakage inductance (H)
$L_{sl}$	Stator leakage inductance (H)
$p_p$	Number of pole pairs
$R_s$	Stator resistance (Ω)
$R_r$	Rotor resistance (Ω)
$T_e$	Electromagnetic torque (Nm)
$T_l$	Load torque (Nm)
$T_r$	Rotor Time Constant
$\sigma = 1 - L_m^2 / (L_s L_r)$	Leakage coefficient
$\omega_e$	Synchronous speed (rad/sec)
$\omega_m = \theta \cdot m$	Rotor Mechanical speed (rad/sec)
$\omega_r = p_p \omega_m$	Rotor electrical speed (rad/sec)
$\omega_{sl}$	Slip speed (rad/sec)

## I. INTRODUCTION

Induction motors (IM) with a squirrel cage rotor are the most widely used machines at fixed speed because of their simplicity, ruggedness, efficiency, compactness and reliability. Due to their highly coupled non-linear structure, induction motors were dedicated for years mainly in unregulated drives.

The idea of induction motor control with the principle of Field Oriented Control (FOC) was a big breakthrough which enabled a decoupled control of rotor flux and electromagnetic torque [5].

The MRAC speed estimators are the most attractive approaches due to their design simplicity. Due to the variations of the temperature the detuning of  $R_s$ ,  $T_r = L_r/R_r$  causes the rotor speed ( $\omega_r$ ) and torque response to deteriorate at low speed, where  $T_r$  is a heat dependent [7]. Therefore, the simultaneous estimation of  $\omega_r$  and  $R_s$  is essential.

The drive is without the information of speed during the  $R_s$  identification time interval. For  $R_s$  estimation the steady state condition is required. The reactive power method as a single entity has the disadvantage of  $\theta_s$  calculation for rotor flux ( $\Psi_r$ ) estimation [2].

This paper presents a new MRAC estimator. In the proposed structure  $U_s - I_s$  rotor flux estimator serves as the reference model for  $\omega_r$  estimation with  $I_s \omega_r$  estimator as an adjustable model. The adaptive model does not include the parameter  $R_s$  and  $T_r$  but could be seen with reference model only. The proposed algorithm can readily be implemented in a vector control Sensorless induction motor [1]. Simulation results show the effectiveness of the proposed MRAC observers.

## II. MATHEMATICAL MODEL OF IM

An IM model in the d-q synchronously rotating frame, under commonly used assumptions, can be expressed as,

$$\dot{i}_{ds} = -\left(\frac{R_s}{\sigma L_s} + \frac{L_m^2}{\sigma L_s L_r T_r}\right) i_{ds} + \omega_e i_{qs} + \frac{L_m}{\sigma L_s L_r T_r} \psi_{rd} + \frac{\omega_r L_m}{\sigma L_s L_r} \psi_{rq} + \frac{1}{\sigma L_s} U_{Ds} \quad (1)$$

$$\dot{i}_{qs} = -\left(\frac{R_s}{\sigma L_s} + \frac{L_m^2}{\sigma L_s L_r T_r}\right) i_{qs} + \omega_e i_{ds} + \frac{L_m}{\sigma L_s L_r T_r} \psi_{rd} + \frac{\omega_r L_m}{\sigma L_s L_r} \psi_{rd} + \frac{1}{\sigma L_s} U_{Qs} \quad (2)$$

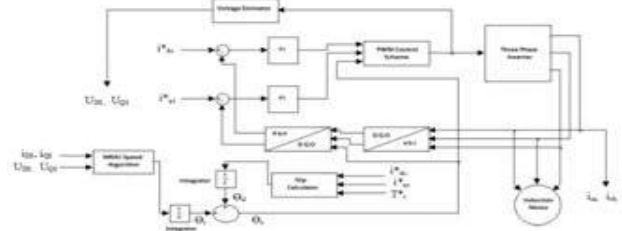


Fig.1. Block diagram of proposed robust control system using the simplified rotor flux oriented vector control of an Induction Motor

Manuscript published on 30 June 2013.

\* Correspondence Author (s)

**Srikanth Mandarapu\***, Electrical & Electronics Engineering, JNTU-K, Pydah college of Engineering & Technology, Visakhapatnam, India.

**Sreedhar Lolla**, Electrical & Electronics Engineering, JNTU-K Pydah college of Engineering & Technology, Visakhapatnam, India.

**Madhu Chandra Popuri**, Electrical & Electronics Engineering, JNTU-K, Pydah college of Engineering & Technology, Visakhapatnam, India.

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>.



## Dynamic Simulation of Robust Sensorless Speed Measurement in IM Using MRAC against Variations in Stator Resistance and Rotor-Time Constant

A sixth-order non-linear model describes the IM in the  $d-q$  system.

$$\dot{\Psi}_{rd} = \frac{L_m}{T_r} I_{DS} + \frac{\Psi_{rd}}{T_r} + (\omega_e - \omega_r) \Psi_{rq} \quad (3)$$

$$\dot{\Psi}_{rq} = \frac{L_m}{T_r} I_{QS} - \frac{\Psi_{rq}}{T_r} - (\omega_e - \omega_r) \Psi_{rd} \quad (4)$$

$$J\ddot{\theta} + B\dot{\theta}_m + T_l = T_e \quad (5)$$

$$T_e = \frac{3PpL_m}{2L_r} (I_{QS}\Psi_{rd} - I_{DS}\Psi_{rq}) \quad (6)$$

where  $u_{ds}$  and  $u_{qs}$  are the  $d-q$  components of stator voltage, respectively;  $i_{ds}$  and  $i_{qs}$  are the stator current components;  $\Phi_{dr}$  and  $\Phi_{qr}$  are the rotor flux components.

The vector control principle, usually implemented by rotorflux-oriented control, ensures decoupling of torque control and rotor flux control [3]. Rotor flux is oriented toward the  $d$ -axis

$$\Psi_{rd} = \Psi_r, \Psi_{rq} = \dot{\Psi}_{rq} = 0 \quad (7)$$

Using (7), (3) and (4) are reduced to

$$T_r \dot{\Psi}_r + \Psi_r = L_m i_{ds} \quad (8)$$

$$\omega_s = \omega_e - \omega_r = \omega_e - p_p m = L_m i_{qs} / (T_r \Psi_r) \quad (9)$$

defining rotor flux dynamics and slip frequency.  $T_r = L_r/R_r$  is a rotor time constant. Rotor flux is generated only by the fluxcurrent component  $i_{ds}$ . Since the rotor flux should be constant, the  $d$ -axis current controller should ensure that  $i_{ds}$  keeps a desired constant value  $i_{ds}^*$ . In steady state, the rotor flux is given by

$$\Psi_r = L_m i_{ds}^* \quad (10)$$

Substituting (7) and (10) into (6), electromagnetic torque becomes

$$T_e = k_t i_{qs}, k_t = \left(\frac{3p_p}{2}\right) \left(\frac{L_m^2}{L_r}\right) i_{ds}^* \quad (11)$$

As a result, the electromagnetic torque is linearly dependent on the torque current component  $I_{QS}$ , indicating that both rotor flux and electromagnetic torque can be controlled separately.

In the proposed IM control scheme, shown in Fig.1, there is only a flux current PI controller. Torque current controller and decoupling circuits are excluded.

### III. SPEED TUNING ALGORITHM

In the new scheme, the speed tuning signal is deliberately chosen to be  $Im(\Delta \bar{e} \bar{i}_s)$ , where  $\Delta \bar{e} = \bar{e} - \hat{\bar{e}}$  and  $\bar{e}, \hat{\bar{e}}$  are the space vectors of the back e.m.f.s in the reference model and adaptive model respectively. It follows that

$$Im(\Delta \bar{e} \bar{i}_s) = \bar{i}_s \times \Delta \bar{e} = \bar{i}_s \times \bar{e} - \bar{i}_s \times \hat{\bar{e}} \text{ and } \bar{e} = e_d + je_q, \bar{U}_s = U_{DS} + jU_{QS}, \bar{i}_s = i_{DS} + ji_{QS},$$

$$y = \bar{i}_s \times \bar{e} = \bar{i}_s \times \left( \bar{U}_s - L'_s \frac{d\bar{i}_s}{dt} \right) \quad (12 a)$$

is obtained, which is the output of the reference model. It can be seen that this does not contain the stator resistance and this

is why  $y$  has been chosen to be a component of the speed tuning signal. In other words, since the stator-voltage space vector  $(\bar{U}_s)$  is equal to the sum of the stator ohmic voltage drop ( $R_s \bar{i}_s$ ) plus  $L'_s d\bar{i}_s/dt$ , plus the back e.m.f.  $\bar{e} = (L_m/L_r) d\bar{\Psi}_r/dt$ , therefore the vectorial product  $\bar{i}_s \times \bar{U}_s$ , does not contain the stator resistance and takes the form  $\bar{i}_s \times \bar{U}_s = \bar{i}_s \times L'_s d\bar{i}_s/dt + \bar{i}_s \times \bar{e}$ , and this gives equation (12 a) as expected. The first term on the right-hand side of equation (12 a) is  $\bar{i}_s \times \bar{U}_s$ , the reactive input power.

Similarly, the stator voltage components  $U_{DS}, U_{QS}$  can be obtained from the monitored line voltages, or in an inverter-fed induction motor drive, they can be reconstructed from the inverter switching states and the monitored value of the d.c. link voltage.

$$\hat{e}_d = \frac{L_m}{L_r} \frac{d\hat{\Psi}_{rd}}{dt} = \frac{L_m}{L_r} \frac{(L_m i_{DS} - \Psi_{rd} - \omega_r T_r \Psi_{rq})}{T_r} \quad (12 b)$$

$$\hat{e}_q = \frac{L_m}{L_r} \frac{d\hat{\Psi}_{rq}}{dt} = \frac{L_m}{L_r} \frac{(L_m i_{QS} - \Psi_{rq} - \omega_r T_r \Psi_{rd})}{T_r} \quad (12 c)$$

The output of the adaptive model is obtained by considering equations (12 b), (12 c) and  $\hat{e} = e_d + je_q$ , as follows:

$$\hat{y} = \bar{i}_s \times \hat{e} = \bar{i}_s \times \left[ \frac{L_m}{L_r} \bar{i}_s + \frac{\bar{\Psi}_r' (j\omega_r - 1)}{T_r} \right] = \frac{L_m}{L_r} \left[ \frac{1}{T_r} \bar{\Psi}_r' \times \bar{i}_s + \omega_r (\bar{i}_s \times j\bar{\Psi}_r') \right] \quad (13)$$

It could be observed that the adaptive model is a function of  $\omega_r$  and  $T_r$  which will not be observed with the reference model. The reference model is represented by equations (14) and the adaptive model by equation (15) [5].

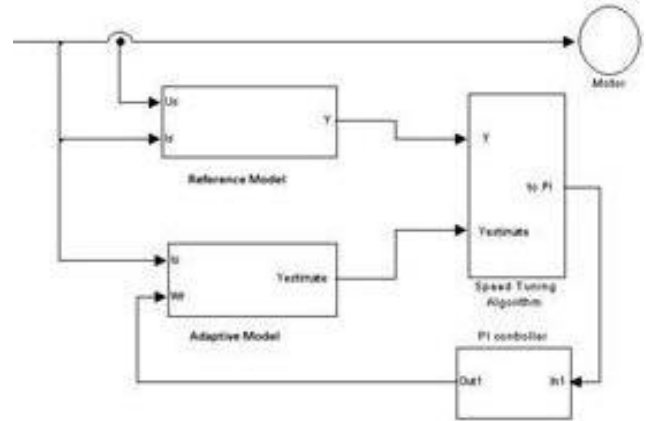


Fig.2 Block diagram of Proposed MRAC Scheme for Speed Estimation

#### A. Reference Model

The reference model is based on the following equation.

$$y = \bar{i}_s \times \bar{e} = U_{QS} i_{DS} - U_{DS} i_{QS} - L'_s \left[ i_{DS} \frac{di_{QS}}{dt} - i_{QS} \frac{di_{DS}}{dt} \right] \quad (14)$$

#### B. Adaptive Model

$$\hat{y} = \bar{i}_s \times \hat{e} = \frac{L_m}{L_r} \left[ i_{QS} - \Psi_{rq} i_{DS} + \omega_r (\Psi_{rd} i_{DS} - \Psi_{rq} i_{QS}) \right] \quad (15)$$

Where  $\Psi_{rd}$  and  $\Psi_{rq}$  are the estimated values from the model itself instead of deploying separate flux calculator for the calculation of the same.

#### IV. FLUX ESTIMATION ALGORITHM

The block diagram in Fig. 3 explains the estimation of flux [3] with the help of the equations from eq. (14) to eq. (17)

$$L'_S = \sigma L_S = 1 - \frac{L_m^2}{L_s L_r} T_s \quad (16)$$

$$\Psi'_r = \frac{L_r}{L_m} (\Psi_s - L'_S i_s) \quad (17)$$

$$T_r = \frac{L_r}{R_r} \quad (18)$$

$$\Psi'_r = \frac{L_r}{L_m} \left[ \int (\overline{U}_s - R_s \overline{i}_s) dt - L'_S i_s \right] \quad (19)$$

Thus the flux is estimated with which the speed is estimated avoiding stator resistance,  $R_s$  and rotor time constant,  $T_r$ . The rotor speed estimation algorithm (adaptation mechanism) is chosen according to *Popov's hyperstability* theory, eq. (18) [2], whereby the transfer function matrix of the linear time invariant subsystem must be strictly positive real and the non-linear time varying feedback subsystem satisfies proportional and integral (PI) inequality according to which in the time interval  $[0, t_1] \in t_1 \geq 0$ .

$$\int V^T W dt \geq 0 \quad (20)$$

A PI controller is obtained from the proposed scheme from which the appropriate speed tuning signal in the MRAC system is designed.

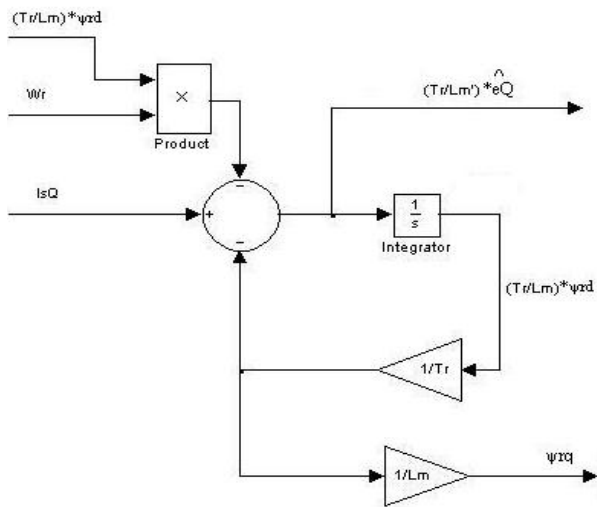


Fig.3 Block diagram of the Flux Estimator

#### V. SIMULATION

The performance of the proposed model of speed estimation is verified with simulation in Matlab 7 software.

#### VI. SIMULATION RESULTS

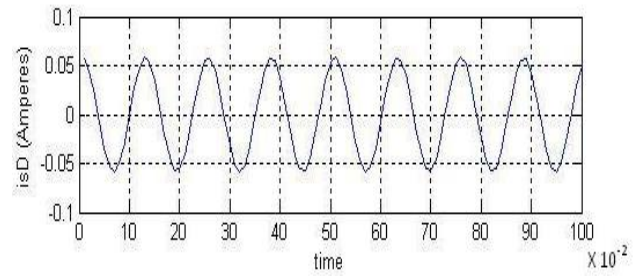


Fig. 4 (a)  $i_{sD} = 0.1$  A

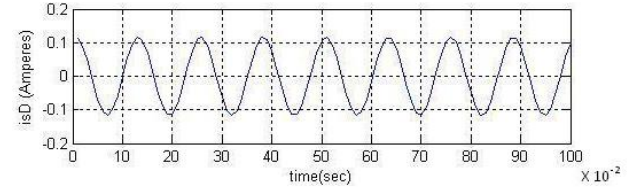


Fig. 4 (b)  $i_{sD} = 0.2$  A

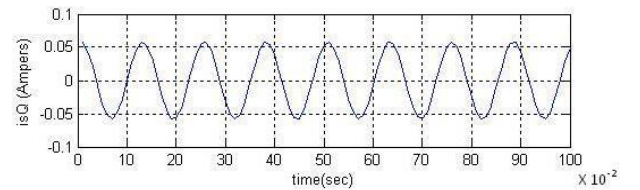


Fig. 4 (c)  $i_{sQ} = 0.1$  A

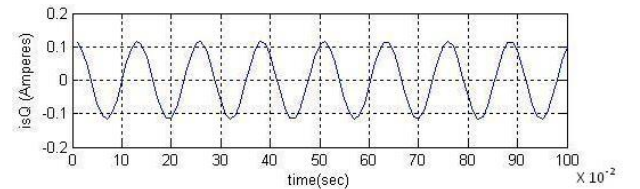


Fig. 4 (d)  $i_{sQ} = 0.2$  A

The simulation results in the evaluation of rotor speed response with the variations in the stator resistance. Fig. 4(a) and Fig. 4(b) shows  $I_{sD}$  and  $I_{sQ}$  respectively, when  $I = 0.1$  A and Fig. 4(c) and Fig. 4(d) shows  $I_{sD}$  and  $I_{sQ}$  respectively, when  $I = 0.2$  A.

Fig 4(e), Fig 4(f) and Fig 4(g) shows the speed response when the current  $i_s$  is 0.1 A for the stator resistance of 10 ohms, 8 ohms and 12 ohms respectively for various rotor time constants

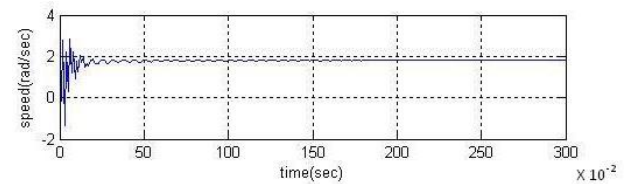


Fig. 4 (e)  $i = 0.1$  A,  $R_s = 10 \Omega$  and  $T_r = 0.0732$  s

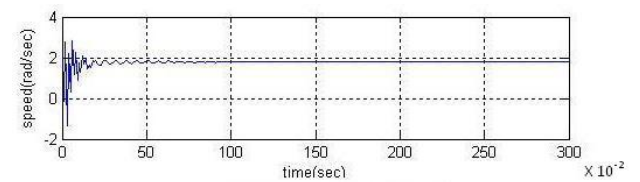


Fig. 4 (f)  $i = 0.1$  A,  $R_s = 8 \Omega$  and  $T_r = 0.0586$  s



# Dynamic Simulation of Robust Sensorless Speed Measurement in IM Using MRAC against Variations in Stator Resistance and Rotor-Time Constant

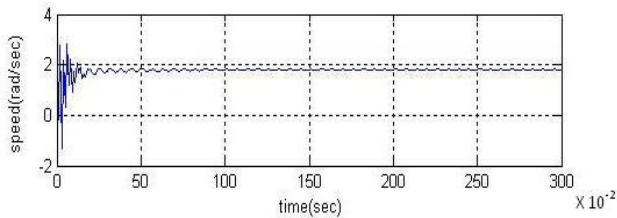


Fig. 4 (g)  $i_s = 0.1$  A,  $R_s = 12 \Omega$  and  $T_r = 0.0878$  s

Fig 4(h), Fig 4(i) and Fig 4(j) shows the speed response when the current  $i_s$  is 0.2 A for the stator resistance is 10 ohms, 8 ohms and 12 ohms respectively also for different rotor time constants. In all the combinations, the speed is expected to be constant irrespective to the variations in stator resistance.

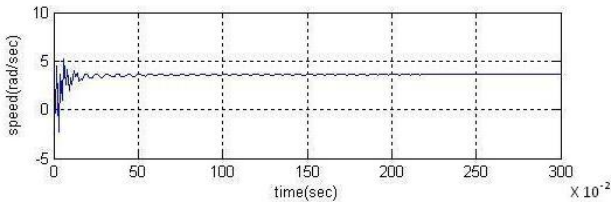


Fig. 4 (h)  $i_s = 0.2$  A,  $R_s = 10 \Omega$  and  $T_r = 0.0732$  s

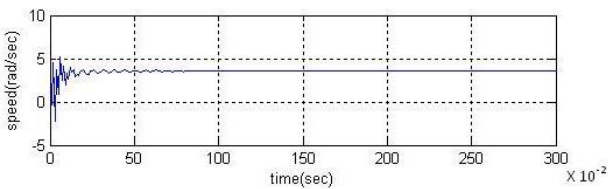


Fig. 4 (i)  $i_s = 0.2$  A,  $R_s = 8 \Omega$  and  $T_r = 0.0586$  s

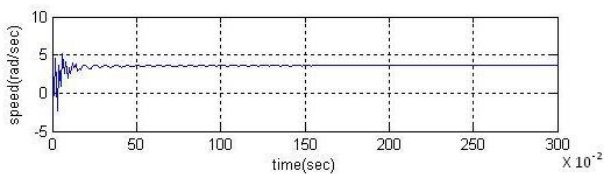


Fig. 4 (j)  $i_s = 0.2$  A,  $R_s = 12 \Omega$  and  $T_r = 0.0878$  s

## VII. CONCLUSION

The adaptive model is a function of  $\omega_r$  and  $T_r$ . These parameters are not observed with the reference model. The error signal which is the difference of the outputs of the adaptive and reference models actuates the rotor speed identification algorithm, which makes the error converge asymptotically to zero. Thus, the estimated rotor speed is made equal to the actual rotor speed.

There is a requirement of synchronous reference frame with reactive power method and there is a problem of speed calculation especially at low speeds with back emf method. The present model takes the advantages of both the methods i.e., back emf method that does not require synchronous reference frame, and the reactive power method for stable speed estimation especially at low speeds, helps greatly in converging giving a better response.

It is worth mentioning that, whilst in [5] where the variation in  $R_s$  only was considered, a similar response of less than 0.25sec has been obtained, in spite of addition of variations in  $T_r$ .

## REFERENCES

- [1] Hassan K. Khalil, Elias G. Strangas, and Sinisa Jurkovic, "Speed Observer and Reduced Nonlinear Model for Sensorless Control of Induction Motors" IEEE Transactions on Control Systems Technology, vol. 17, No. 2, March 2009, pp. 327–339.
- [2] Peter Vas, "Sensorless Vector and Direct Torque Control", New York: Oxford University Press, 1998.
- [3] Bimal K. Bose, "Modern Power Electronics and AC Drives", Prentice-Hall India, 2008.
- [4] Joachim Holtz, "Sensorless control of induction motor drives", Proceedings of Vol. 90, No. 8, Aug. 2002 pp 1359 – 1394.
- [5] M. Srikanth, Madhu Chandra P., G. Manofer Ali, Arun Kumar Rath, "Robust MRAC Based Sensorless Rotor Speed Measurement against Variations in Stator Resistance Using Combination of Back EMF and Reactive Power Method" International Journal of Multidisciplinary Educational Research, Volume 1, Issue 3, August 2012, pp 38–46.
- [6] A. M. El-Sawy, Yehia S. Mohamed and A. A. Zaki, "Stator Resistance and Speed Estimation for Induction Motor Drives as Influenced by Saturation", The Online Journal on Electronics and Electrical Engineering (OJEEE), Vol. 3 – No. 2, pp 416 - 424.
- [7] A. Venkadesan, S.Himavathi and A.Muthuramalingam, "Novel SNC-NN-MRAS Based Speed Estimator for Sensor-Less Vector Controlled IM Drives," International Journal on Electrical and Electronics Engineering 5:2 2011, pp 73 – 78.
- [8] Shady M. Gadoue, Damian Giaouris, and John W. Finch, "MRAS Sensorless Vector Control of an Induction Motor Using New Sliding-Mode and Fuzzy-Logic Adaptation Mechanisms," IEEE J. Quantum Electron., submitted for publication.



**Srikanth Mandarapu** was born in Vizianagaram, India, in 1983. He received his B.Tech degree from JNTU Hyderabad in 2005 and M. Tech in Power Electronics and Drives from BPUT, Rourkela, India in 2012. His field of interest is design of controllers for Industrial Drives. He has four years of teaching and research experience and four years experience as Engineer in electrical equipment installation, Commissioning in Power Generation and Mineral Processing.



**Sreedhar Lolla** was born in Vuyyuru, India in 1963. He received his B.E. (EEE) and M.Tech (AI & Robotics) in 1988 and 2010 respectively from Andhra University, India. His field of Interest is applications of AI techniques in Control Engineering. He has 1 year of Industrial experience in Hindustan Shipyard, 2 years of research in Andhra University, India and 16 years of teaching experience. He is a life member of ISTE and IE (Institute of Engineers, India)



**Madhu Chandra Popuri** was born in Narasaraopet, India, in 1979. He received his B. Tech degree from JNTU Hyderabad in 2001 and M. Tech in Power and Industrial Drives from JNTUK, Kakinada, India in 2013. His field of interest is control of Industrial Drives. He has eight years of teaching and research experience and one year experience as Quality Control Engineer in electrical equipment installation in Ship Building. He is a Member of IEEE.